VECTOR CALCULUS

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SAMPLE TEST FOR CHAPTER 2

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1. True or false. If false, explain why.
   (a) If all the partial derivatives of a real-valued function $f(x, y, z)$ exist at the origin, then the function is differentiable at the origin.

   (b) The gradient vector for a function $f\colon \mathbb{R}^2 \to \mathbb{R}$ is parallel to level curves of f.

   (c) A function f is continuous at a point $\mathbf{x}_0$ if
   $$\lim_{\mathbf{x}\to\mathbf{x}_0} f(\mathbf{x}) = f(\mathbf{x}_0).$$

   (d) In general, a continuous function is differentiable.

   (e) Given a function $f(w, x, y, z)$, the directional derivative in the direction of $(0, 0, 1, 0)$ is the same as $\partial f/\partial y$.

2. Let the surface S be given by $x^2 z + xe^y - 6x + y^2 z^2 = 2$. Find a unit vector perpendicular to S at the point $\mathbf{x}_0 = (2, 0, 3)$.

3. Let $f(x, y) = e^{x-2}\sin(3y) + x^2$, and $\mathbf{c}(t) = (t^2 + 5t + 2, t + 2\pi)$. Use the chain rule to find $\frac{d}{dt}(f \circ \mathbf{c})\big|_{t=0}$.

4. Find a parametrization for the tangent line to $\mathbf{c}(t) = (210\cos t, 420\sin t, t)$ at time $t = \frac{\pi}{2}$.

5. Let $z = (x + y)^2 - 5x^3 + 2ye^x$ be the equation of a surface in space. Find the equation of the tangent plane at $x = 0, y = 3$.

6. Let
   $$g(x, y) = \begin{cases} \dfrac{y}{x}, & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}.$$
   Compute $\partial g/\partial x$ and $\partial g/\partial y$ at the origin if they exist there.

7. Let $f(x, y, z) = (x^2 y + z, xyz)$ and $g(u, v) = (2u + v, uv^2, u + 2u^2 v)$. Find $\mathbf{D}(f \circ g)(1, 0)$.

8. (a) Evaluate the following limit for $f(w, x, y, z) = w - x^2 y^3 z$:
   $$\lim_{h\to 0} \frac{f(5, 2, -1, 1) - f(5, 2, -1 + h, 1)}{h}.$$

(b) Is it possible to define $g(0,0)$ so that $g(x,y) = (x^2+y^2)/(x^2+y)$ is continuous on all of $\mathbb{R}^2$? Explain why or why not.

9. Let $u(x,y) = 2x + y^2 + e^x$. At $(1,1)$ show that u increases faster in the direction of the positive x axis than in the direction of the positive y axis.

10. A recent survey showed that patient satisfaction, s, at a pharmacy depended mainly on three factors: the waiting time in minutes (t), the patient's perceived degree of illness (i) on a scale from 0 to 10, and the dollar cost of the prescription (c). The patient satisfaction index is given by $s(t,i,c) = (1000-c)/it^2$.
    (a) In what direction does patient satisfaction increase most rapidly at the point $(10, 0.5, 100)$?

    (b) At the point $(10, 0.5, 100)$, how fast and in what direction (positive or negative) does patient satisfaction change for each extra minute of waiting time?

    (c) The administrators do not want s to decrease by more than 1 per unit change of t, i, or c. Is this goal met with a price decrease of 3/5 dollars and an increase in waiting time of 4/5 minutes? Explain.