CHAPTER 22

LIGHTNING IS AN ELECTRIC PHENOMENA. DURING A LIGHTNING STRIKE, CHARGES ARE TRANSFERRED BETWEEN THE CLOUDS AND THE GROUND. THE VISIBLE LIGHT GIVEN OFF COMES FROM AIR MOLECULES RETURNING TO LOWER ENERGY STATES. *(Photo Disc.)*

The Electric Field II: Continuous Charge Distributions

? How would you calculate the charge on the surface of Earth? (See Example 22-15.)

On a microscopic scale, charge is quantized. However, there are often situations in which many charges are so close together that the charge can be thought of as continuously distributed. We apply the concept of density to charge similarly to the way we use it to describe matter.

In addition to continuous charge distributions, we examine the importance of symmetry within the electric field. The mathematical findings of Carl Friedrich Gauss show that every electric field maintains symmetric properties. It is an understanding of charge distribution and symmetry within the electric field that aids scientists in a vast array of fields.

In this chapter, we show how Coulomb's law is used to calculate the electric field produced by various types of continuous charge distributions. We then introduce Gauss's law and use it to calculate the electric fields produced by charge distributions that have certain symmetries.

22-1 CALCULATING $\vec{E}$ FROM COULOMB'S LAW

Figure 22-1 shows an element of charge $dq = \rho\, dV$ that is small enough to be considered a point charge. The element of charge dq is the amount of charge in volume element dV and ρ is the charge per unit volume. Coulomb's law states that the electric field $d\vec{E}$ at a field point P due to this element of charge is

$$d\vec{E} = dE_r\,\hat{r} = \frac{k\,dq}{r^2}\,\hat{r} \qquad \text{22-1}a$$

where $\hat{r}$ is a unit vector directed away from the charge element dq and toward point P, and dE_r (the component of $d\vec{E}$ in the direction of $\hat{r}$) is given by $k\,dq/r^2$.

The total field $\vec{E}$ at P is calculated by integrating this expression over the entire charge distribution. That is,

$$\vec{E} = \int d\vec{E} = \int \frac{k\hat{r}}{r^2}\,dq \qquad \text{22-1}b$$

ELECTRIC FIELD DUE TO A CONTINUOUS CHARGE DISTRIBUTION

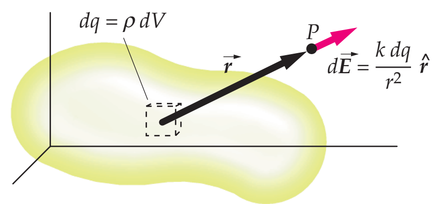


FIGURE 22-1 An element of charge dq produces a field $d\vec{E} = (k\,dq/r^2)\hat{r}$ at point P. The field at P is calculated by integrating Equation 22-1a over the entire charge distribution.

The use of a continuous charge density to describe a large number of discrete charges is similar to the use of a continuous mass density to describe air, which actually consists of a large number of discrete atoms and molecules. In both cases, it is usually easy to find a volume element ΔV that is large enough to contain a multitude of individual charge carriers and yet is small enough that replacing ΔV with a differential dV and using calculus introduces negligible error. If the charge is distributed over a surface or along a line, we use $dq = \sigma\, dA$ or $dq = \lambda\, dL$ and integrate over the surface or line. (In these cases σ and λ are the charge per unit area and charge per unit length, respectively.) The integration usually is done by expressing $\hat{r}$ in terms of its Cartesian components, and then integrating one component at a time.

! The x component of $\hat{r}$ is $\hat{r}\cdot\hat{i} = \cos\theta$, where θ is the angle between $\hat{r}$ and $\hat{i}$.* The y and z components of $\hat{r}$ are calculated in like manner.

PROBLEM-SOLVING STRATEGY

Calculating $\vec{E}$ Using Equations 22-1a and 22-1b

PICTURE Sketch the charge configuration along with a field point P (the point where $\vec{E}$ is to be calculated). In addition, the sketch should include an increment of charge dq at an arbitrary source point S.

SOLVE

1. Add coordinate axes to the sketch. The choice of axes should exploit any symmetry of the charge configuration. For example, if the charge is along a straight line, then select that line as one of the coordinate axes. Draw a second axis that passes through the field point P. In addition, include the coordinates of both P and S, the distance r between P and S, and the unit vector $\hat{r}$ directed away from S toward P.
2. To compute the electric field $\vec{E}$ using Equation 22-1b, we express $d\vec{E} = dE_r\,\hat{r}$ in component form. The x component of $d\vec{E}$ is $dE_x = dE_r\,\hat{r}\cdot\hat{i} = dE_r\cos\theta$, where θ is the angle between $\hat{r}$ and $\hat{i}$ (see Figure 22-2), and the y component of $d\vec{E}$ is $dE_y = dE_r\,\hat{r}\cdot\hat{j} = dE_r\sin\theta$.

* The componet of a vector in a given direction is equal to the scalar product of the vector with the unit vector in the given direction. Scalar products are discussed in Section 6-3.

3. Express $\vec{E}$ in Equation 22-1b in terms of its x and y components:

$$E_x = \int dE_x = \int dE_r \cos\theta = \int \frac{k\,dq}{r^2}\cos\theta$$

$$E_y = \int dE_y = \int dE_r \sin\theta = \int \frac{k\,dq}{r^2}\sin\theta$$

4. To calculate E_x, express dq as $\rho\,dV$ or $\sigma\,dA$ or $\lambda\,dL$ (whichever is appropriate) and integrate. To calculate E_y, follow a procedure similar to that used for calculating E_x.
5. Symmetry arguments are sometimes used to show that one or more components of $\vec{E}$ are equal to zero. (For example, a symmetry argument is used to show $E_y = 0$ in Example 22-5.)

CHECK If the charge distribution is confined to a finite region of space at points far from the charge distribution, the expression for the electric field will approach that of a point charge located at the center of charge. (If the charge configuration is sufficiently symmetric then the location of the center of charge can be obtained by inspection.)

See **Math Tutorial** *for more information on* **Trigonometry**

Example 22-1 Electric Field Due to a Line Charge of Finite Length

A thin rod of length L and charge Q is uniformly charged, so it has a linear charge density $\lambda = Q/L$. Find the electric field at point P, where P is an arbitrarily positioned point.

PICTURE Choose the x axis so the rod is on the x axis between points x_1 and x_2, and choose the y axis to be through the field point P. Let y_P be the radial distance of P from the x axis. To calculate the electric field $\vec{E}$ at P, we separately calculate E_x and E_y. Using Equations 22-1, first find the field increment $d\vec{E}$ at P due to an arbitrary increment dq of the charge distribution. Then integrate each component of $d\vec{E}$ over the entire charge distribution. (Because Q is distributed uniformly, the linear charge density λ equals Q/L.)

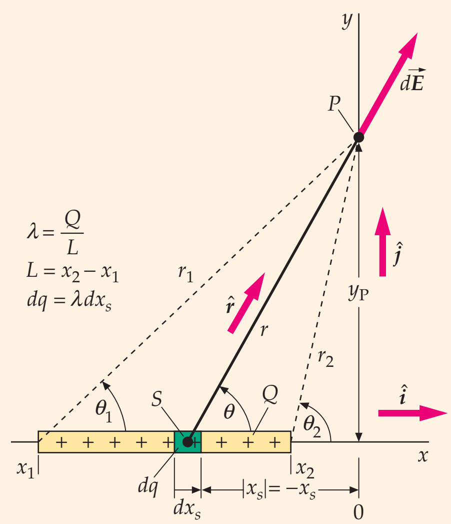


FIGURE 22-2 Geometry for the calculation of the electric field at field point P due to a uniformly charged rod.

SOLVE

1. Sketch the charge configuration and the field point P. Include the x and y axes with the x axis lying along the line of charge and the y axis passing through P. In addition, sketch an *arbitrary* increment of the line charge at point S (at $x = x_S$) that has a length dx_S and a charge dq, and the electric field at P due to dq. Sketch the electric field vector $d\vec{E}$ as if dq is positive (Figure 22-2):

2. $\vec{E} = E_x\hat{i} + E_y\hat{j}$. Find expressions for dE_x and dE_y in terms of dE_r and θ, where dE_r is the component of $d\vec{E}$ in the direction away from S toward P:

$$d\vec{E} = dE_r\,\hat{r}$$

so $dE_x = dE_r\,\hat{r}\cdot\hat{i} = dE_r\cos\theta$

$$dE_y = dE_r\,\hat{r}\cdot\hat{j} = dE_r\sin\theta$$

3. First we solve for E_x. Express dE_r using Equation 21-1a, where r is the distance from the source point S to the field point P. We see (Figure 22-2) that $\cos\theta = |x_S|/r = -x_S/r$. In addition, use $dq = \lambda\,dx_S$:

$$dE_r = \frac{k\,dq}{r^2} \text{ and } \cos\theta = \frac{-x_S}{r}$$

so

$$dE_x = \frac{k\,dq}{r^2}\cos\theta = \frac{k\cos\theta\,\lambda\,dx_S}{r^2}$$

4. Integrate the step-3 result:

$$dE_x = \int_{x_1}^{x_2}\frac{k\cos\theta\,\lambda\,dx_S}{r^2} = k\lambda\int_{x_1}^{x_2}\frac{\cos\theta\,dx_S}{r^2}$$

5. Next change the integration variable from x_S to θ. From Figure 22-2, find the relation between x_S and θ and between r and θ.

$$\tan\theta = \frac{y_P}{|x_S|} = \frac{y_P}{-x_S}, \quad \text{so} \quad x_S = -\frac{y_P}{\tan\theta} = -y_P\cot\theta$$

$$\sin\theta = \frac{y_P}{r}, \quad \text{so} \quad r = \frac{y_P}{\sin\theta}$$

6. Differentiate the step 5 result to obtain an expression for dx_S (the field point P remains fixed, so y_P is constant):

$$dx_S = -y_P\frac{d\cot\theta}{d\theta} = y_P\csc^2\theta\,d\theta$$

7. Substitute $y_P\csc^2\theta\,d\theta$ for dx_S and $y_P/\sin\theta$ for r in the integral in step 4 and simplify:

$$\int_{x_1}^{x_2}\frac{\cos\theta\,dx_S}{r^2} = \int_{\theta_1}^{\theta_2}\frac{\cos\theta\,y_P\csc^2\theta\,d\theta}{y_P^2/\sin^2\theta} = \frac{1}{y_P}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta \qquad (y_P \neq 0)$$

8. Evaluate the integral and solve for E_x:

$$E_x = k\lambda\frac{1}{y_P}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta = \frac{k\lambda}{y_P}(\sin\theta_2 - \sin\theta_1) = \frac{k\lambda}{y_P}\left(\frac{y_P}{r_2} - \frac{y_P}{r_1}\right)$$

$$= k\lambda\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad (r_1 > 0 \text{ and } r_2 > 0)$$

9. E_y can be found using a procedure that parallels the one in steps 3–7 for finding E_x (to find E_y, see Problem 22-21):

$$E_y = -\frac{k\lambda}{y_P}(\cos\theta_2 - \cos\theta_1) = -k\lambda\left(\frac{\cot\theta_2}{r_2} - \frac{\cot\theta_1}{r_1}\right) \qquad (y_P \neq 0)$$

and

$$E_y = 0 \qquad (y_P = 0)$$

10. Combine steps 8 and 9 to obtain and expression for the electric field at P:

$$\boxed{\vec{E} = E_x\hat{i} + E_y\hat{j}}$$

CHECK Consider the plane that is perpendicular to and bisecting the rod. At points on this plane, symmetry dictates that $\vec{E}$ points directly away from the center of the rod. That is, we expect that $E_x = 0$ throughout this plane. At all points on this plane $r_1 = r_2$. The step-8 result gives $E_x = 0$ if $r_1 = r_2$, as expected.

TAKING IT FURTHER The first expression for E_y in the step 9 result is valid everywhere in the xy plane but on the x axis. The two cotangent functions in the expression for E_y are given by

$$\cot\theta_1 = \frac{-x_1}{y_P} \quad \text{and} \quad \cot\theta_2 = \frac{-x_2}{y_P}$$

and neither of these functions is defined on the x axis (where $y_P = 0$). The second expression for E_y in the step-9 result is obtained using Equation 22-1a. By recognizing that on the x axis $\hat{r} = \pm\hat{i}$, we can see that Equation 22-1a tells us that $d\vec{E} = \pm dE\,\hat{i}$, which implies $E_y = 0$.

PRACTICE PROBLEM 22-1 Using the expression for E_x in step 8, show that $E_x > 0$ at all points on the x axis in the region $x > x_2$.

The electric field at point P due to a thin uniformly charged rod (see Figure 22-3) located on the z axis is given by $\vec{E} = E_z\hat{k} + E_R\hat{R}$, where

$$E_z = \frac{k\lambda}{R}(\sin\theta_2 - \sin\theta_1) = k\lambda\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \qquad (r_1 \neq 0) \text{ and } (r_2 \neq 0) \qquad \text{22-2}a$$

$$E_R = -\frac{k\lambda}{R}(\cos\theta_2 - \cos\theta_1) = -k\lambda\left(\frac{\cot\theta_2}{r_2} - \frac{\cot\theta_1}{r_1}\right) \qquad (R \neq 0) \qquad \text{22-2}b$$

These equations are derived in Example 22-1. The expressions for E_z (Equation 22-2a) are undefined at the end points of the thin charged rod and the expressions for E_R (Equation 22-2b) are undefined at all points on the z axis (where $R = 0$). However, $E_R = 0$ at all points where $R = 0$.

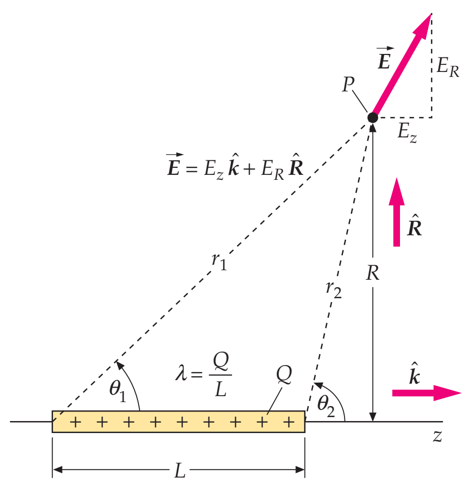


FIGURE 22-3 The electric field due to a uniformly charged thin rod.

Example 22-2 $\vec{E}$ of a Finite Line Charge and Far from the Charge

A charge Q is uniformly distributed along the z axis, from $z = -\frac{1}{2}L$ to $z = +\frac{1}{2}L$. Show that for large values of z the expression for the electric field of the line charge on the z axis approaches the expression for the electric field of a point charge Q at the origin.

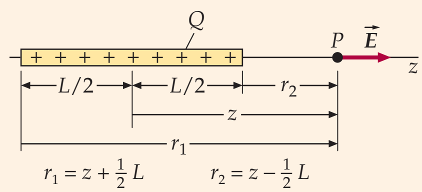


FIGURE 22-4 Geometry for the calculation of the electric field on the axis of a uniform line charge of length L, charge Q, and linear charge density $\lambda = Q/L$.

PICTURE Use Equation 22-2*a* to show that for large values of z the expression for the electric field of the line charge on the z axis approaches that of a point charge Q at the origin.

SOLVE

1. The electric field on the z axis has only a z component, given by Equation 22-2a:
$$E_z = k\lambda\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$
2. Sketch the line charge. Include the z axis, the field point P, and r_1 and r_2 (Figure 22-4):
3. Substitute with $r_1 = z + \frac{1}{2}L$ and $r_2 = z - \frac{1}{2}L$ into the step 1 result and simplify:
$$E_z = k\lambda\left(\frac{1}{z - \frac{1}{2}L} - \frac{1}{z + \frac{1}{2}L}\right) = \frac{kQ}{L}\frac{L}{z^2 - \left(\frac{1}{2}L\right)^2} = \frac{kQ}{z^2 - \left(\frac{1}{2}L\right)^2} \qquad \left(z > \tfrac{1}{2}L\right)$$
4. Find an approximate expression for E_z for $z \gg L$, which is done by neglecting $(\frac{1}{2}L)^2$ in comparison with z^2 in the step 3 result.
$$\boxed{E_z \approx \frac{kQ}{z^2} \qquad (z \gg L)}$$

CHECK The approximate expression (step 4) falls off inversely as the square of z, the distance from the origin. This expression is the same as the expression for the electric field of a point charge Q located at the origin.

PRACTICE PROBLEM 22-2 The validity of the step 3 result is established for the region $L/2 > z > \infty$. Is the step 3 result also valid in the region $-L/2 < z < +L/2$? Explain your answer.

Example 22-3 $\vec{E}$ Due to an Infinite Line Charge

Find the electric field due to a uniformly charged line that extends to infinity in both directions and has linear charge density λ.

PICTURE A line charge is considered infinite if the distances between the ends of the line charge and the field points of interest are much much greater than the distances between any of the radial distances of the field points from the line charge. To calculate the electric field due to such a line charge we take the limit (see Figure 22-2) both as $x_1 \to -\infty$ and as $x_2 \to +\infty$. From the figure, we see that taking the limit as both $\theta_1 \to 0$ and as $\theta_2 \to \pi$ is needed. See Equations 22-2*a* and 22-2*b* for expressions for the electric field.

SOLVE

1. Choose the first expression for the electric field in each of Equations 22-2*a* and 22-2*b*:
$$E_z = \frac{k\lambda}{R}(\sin\theta_2 - \sin\theta_1)$$
$$E_R = -\frac{k\lambda}{R}(\cos\theta_2 - \cos\theta_1)$$
2. Take the limit as both $\theta_1 \to 0$ and as $\theta_2 \to \pi$.
$$E_z = \frac{k\lambda}{R}(\sin\pi - \sin 0) = \frac{k\lambda}{R}(0 - 0) = 0$$
$$E_R = -\frac{k\lambda}{R}(\cos\pi - \cos 0) = -\frac{k\lambda}{R}(-1 - 1) = 2\frac{k\lambda}{R}$$
3. Express the electric field in vector form:
$$\vec{E} = E_z\hat{k} + E_R\hat{R} = 0\hat{k} + \frac{2k\lambda}{R}\hat{R} = \boxed{\frac{2k\lambda}{R}\hat{R}}$$

CHECK The electric field is in the radial direction as expected. We expected this due to the symmetry. (The line charge is uniformly distributed and extends to infinity in both directions.)

TAKING IT FURTHER The magnitude of the electric field decreases inversely with the radial distance from the line charge.

The electric field due to a uniformly charged line that extends to infinity in both directions is given by

$$\vec{E} = \frac{2k\lambda}{R}\hat{R} \qquad 22\text{-}3$$

where λ is the linear charge density, R is the radial distance from the line charge to the field point, and $\hat{R}$ is the unit vector in the radial direction. Equation 22-3 is derived in Example 22-3.

PRACTICE PROBLEM 22-3

Show that if k, λ, and R are in SI units then Equation 22-3 gives the electric field in newtons per coulomb.

It is customary to write the Coulomb constant k in terms of another constant, ϵ_0, called the **electric constant (permittivity of free space):**

$$k = \frac{1}{4\pi\epsilon_0} \qquad 22\text{-}4$$

Using this notation, Coulomb's law for $\vec{E}$ (Equation 21-7) is written

$$\vec{E} = k\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad 22\text{-}5$$

and $\vec{E}$ for a uniformly charged infinite line (Equation 22-3) with linear charge density λ is written

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{R}\hat{R} \qquad 22\text{-}6$$

The value of ϵ_0 in SI units is

$$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\,\mathrm{C^2/(N \cdot m^2)} \qquad 22\text{-}7$$

Example 22-4 Approximating Equations 22-2*a* and 22-2*b* on the Symmetry Plane

A charge Q is uniformly distributed along the z axis, from $z = -\frac{1}{2}L$ to $z = +\frac{1}{2}L$. (*a*) Find an expression for the electric field on the $z = 0$ plane as a function of R, the radial distance of the field point from the z axis. (*b*) Show that for $R \gg L$, the expression found in Part (*a*) approaches that of a point charge at the origin of charge Q. (*c*) Show that for $R \ll L$, the expression found in Part (*a*) approaches that of an infinitely long line charge on the z axis with a uniform linear charge density $\lambda = Q/L$.

PICTURE The charge configuration is the same as that in Example 22-2, and the linear charge density is $\lambda = Q/L$. Sketch the line charge on the z axis and put the field point in the $z = 0$ plane. Then use Equations 22-2*a* and 22-2*b* to find the electric field expression for Part (*a*). The electric field due to a point charge decreases inversely with the square of the distance from the charge. Examine the Part (*a*) result to see how it approaches that of a point charge at the origin for $R \gg L$. The electric field due to a uniform line charge of infinite length decreases inversely with the radial distance from the line (Equation 22-3). Examine the Part (*a*) result to see how it approaches the expression for the electric field of a line charge of infinite length for $R \ll L$.

SOLVE

(*a*) 1. Choose the first expression for the electric field in each of Equations 22-2*a* and 22-2*b*:

$$E_z = \frac{k\lambda}{R}(\sin\theta_2 - \sin\theta_1)$$

$$E_R = -\frac{k\lambda}{R}(\cos\theta_2 - \cos\theta_1)$$

2. Sketch the charge configuration with the line charge on the z axis from $z = -\frac{1}{2}L$ to $z = +\frac{1}{2}L$. Show the field point P in the $z = 0$ plane a distance R from the origin (Figure 22-5):

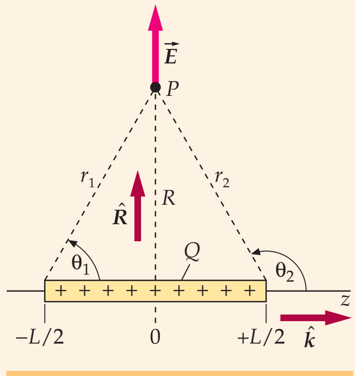


FIGURE 22-5

3. From the figure, we see that $\theta_2 + \theta_1 = \pi$, so $\sin\theta_2 = \sin(\pi - \theta_1) = \sin\theta_1$ and $\cos\theta_2 = \cos(\pi - \theta_1) = -\cos\theta_1$. Substitute into the step 1 results:

$$E_z = \frac{k\lambda}{R}(\sin\theta_1 - \sin\theta_1) = 0$$

$$E_R = -\frac{k\lambda}{R}(-\cos\theta_1 - \cos\theta_1) = \frac{2k\lambda}{R}\cos\theta_1$$

4. Express $\cos\theta_1$ in terms of R and L and substitute into the step-3 result:

$$\cos\theta_1 = \frac{\frac{1}{2}L}{\sqrt{R^2 + \left(\frac{1}{2}L\right)^2}}$$

so

$$E_R = \frac{2k\lambda}{R}\frac{\frac{1}{2}L}{\sqrt{R^2 + \left(\frac{1}{2}L\right)^2}} = \frac{k\lambda L}{R\sqrt{R^2 + \left(\frac{1}{2}L\right)^2}}$$

5. Express the electric field in vector form, and substitute Q for λL:

$$\vec{E} = E_z\hat{k} + E_R\hat{R} = 0\hat{k} + E_R\hat{R}$$

so $$\vec{E} = E_R\hat{R} = \boxed{\frac{kQ}{R\sqrt{R^2 + \left(\frac{1}{2}L\right)^2}}\hat{R}}$$

(*b*) 1. Examine the step-5 result. If $R \gg L$ then $R^2 + (\frac{1}{2}L)^2 \approx R^2$. Substitute R^2 for $R^2 + (\frac{1}{2}L)^2$:

$$\vec{E} \approx \frac{kQ}{R\sqrt{R^2}}\hat{R} = \frac{kQ}{R^2}\hat{R} \qquad (R \gg L)$$

2. This (approximate) expression for the electric field decreases inversely with the square of the distance from the origin, just as it would for a point charge Q at the origin.

$$\vec{E} \approx \boxed{\frac{kQ}{R^2}\hat{R} \qquad (R \gg L)}$$

(*c*) 1. Examine the Part (*a*), step-5 result. If $R \ll L$ then $R^2 + (\frac{1}{2}L)^2 \approx (\frac{1}{2}L)^2$. Substitute $(\frac{1}{2}L)^2$ for $R^2 + (\frac{1}{2}L)^2$. This (approximate) expression for the electric field falls off inversely with the radial distance from the line charge, just as the exact expression for an infinite line charge (Equation 22-3) would.

$$\vec{E} \approx \frac{k\lambda L}{R\sqrt{\left(\frac{1}{2}L\right)^2}}\hat{R} = \boxed{\frac{2k\lambda}{R}\hat{R}} \qquad (R \ll L)$$

CHECK Parts (*b*) and (*c*) are themselves plausibility checks for the Part (*a*) result. They reveal the validity of the Part (*a*) result in two limiting cases, $R \gg L$ and $R \ll L$.

TAKING IT FURTHER Figure 22-6 shows the exact result for a line charge of length $L = 10$ cm and a linear charge density of $\lambda = 4.5$ nC/m. It also shows the limiting cases of an infinite line charge of the same charge density and a point charge $Q = \lambda L$.

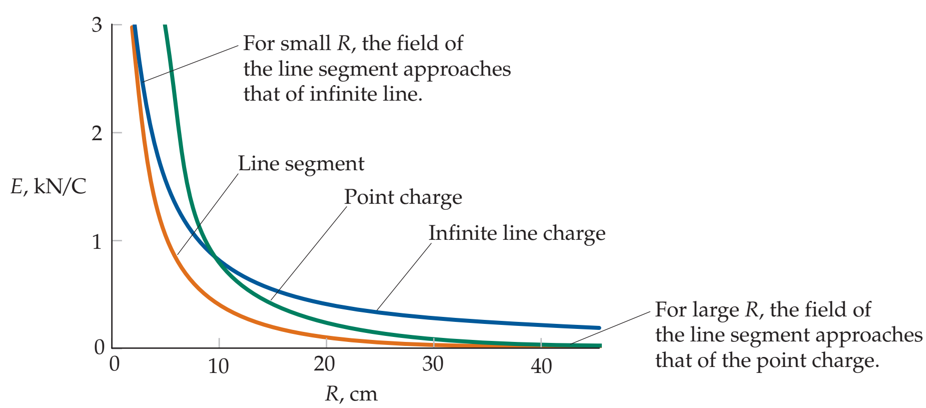


FIGURE 22-6 The magnitude of the electric field is plotted versus distance for a 10-cm-long line charge, a point charge, and an infinite line charge.

Example 22-5 $\vec{E}$ on the Axis of a Charged Ring

A thin ring (a circle) of radius a is uniformly charged with total charge Q. Find the electric field due to this charge at all points on the axis perpendicular to the plane and through the center of the ring.

PICTURE Starting with $d\vec{E} = (k\,dq/r^2)\hat{r}$ (Equation 22-1a), calculate the electric field at an arbitrarily positioned field point on the axis. Sketch the charged ring. Choose the z axis to coincide with the axis of the ring with the ring in the $z = 0$ plane. Label a field point P somewhere on the $+z$ axis, and place a source point S on the ring.

SOLVE

1. Write the equation (Equation 22-1a) giving the electric field due to an element of charge dq:

$$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$$

2. Sketch the ring (Figure 22-7a) and the axis (the z axis), and show the electric field vector at field point P due to an increment of charge dq at source point:

3. Sketch the ring (Figure 22-7b) and show the axial and radial components of $\vec{E}$ for identical charge elements on opposite sides of the ring. The radial components cancel in pairs, as can be seen, so the resultant field is axial:

$$E_R = 0$$

4. Express the z component of the electric field from the step-1 result:

$$dE_z = \frac{k\,dq}{r^2}\cos\theta = \frac{k\,dq}{r^2}\frac{z}{r} = \frac{k\,dq\,z}{r^3}$$

5. Integrate both sides of the step-4 result. Factor constant terms from the integral:

$$E_z = \int \frac{kz\,dq}{r^3} = \frac{kz}{r^3}\int dq = \frac{kz}{r^3}Q$$

6. Using the Pythagorean theorem gives $r = \sqrt{z^2 + a^2}$:

$$\vec{E} = E_z\hat{k} + E_R\hat{R} = E_z\hat{k} + 0 = \boxed{\frac{kQz}{(z^2 + a^2)^{3/2}}\hat{k}}$$

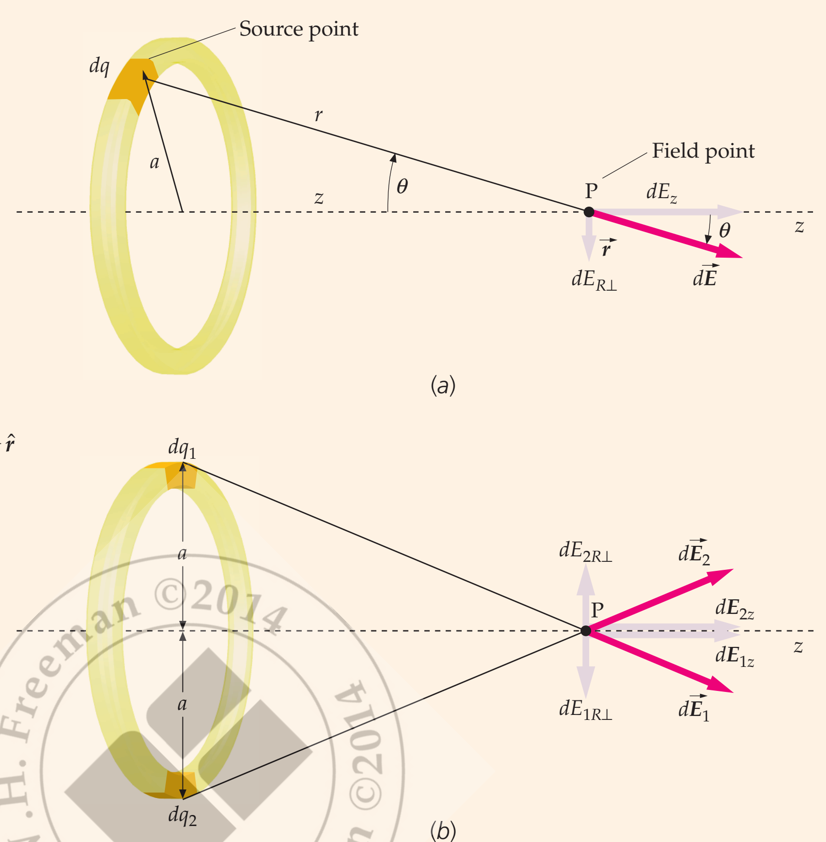


FIGURE 22-7 (a) A ring charge of radius a. The electric field at point P on the z axis due to the charge element dq shown has one component along the z axis and one perpendicular to the z axis. (b) For any charge element dq_1 there is an equal charge element dq_2 opposite it, and the electric field components perpendicular to the z axis sum to zero.

CHECK We expect the direction of the electric field at points on the z axis to be directed away from the origin for $Q > 0$. The step-6 result meets this expectation as z is positive on the $+z$ axis and negative on the $-z$ axis. In addition, for $z \gg a$ we expect E to decrease inversely as the square of the distance from the origin. The step-6 result meets this expectation, giving $E_z \approx kQ/z^2$ if a^2 is negligibly small relative to z^2.

PRACTICE PROBLEM 22-4 A plot of E_z versus z along the axis using the step-6 result is shown in Figure 22-8. Find the point on the axis of the ring where E_z is maximum. *Hint:* $dE_z/dz = 0$ *where* E_z *is maximum.*

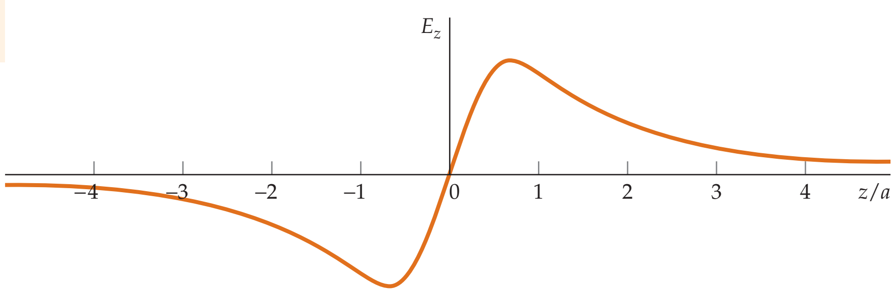


FIGURE 22-8

Example 22-6 $\vec{E}$ on the Axis of a Charged Ring — *Conceptual*

For the charged ring in Example 22-5, why is the magnitude of the electric field small near the origin, even though the origin is closer to the ring than any other points on the z axis (see Figure 22-9)?

PICTURE The key to solving this problem can be found in Figure 22-7*b*. Redraw this figure with the field point P on the z axis, but near the origin.

SOLVE

1. Redraw Figure 22-7*b* with the field point P near the origin:
2. The electric fields near the origin due to the two elements of charge (shown in Figure 22-9) are large but are of equal magnitude and nearly oppositely directed, so they nearly sum to zero.

   Near the origin the resultant electric field is axial and small.

CHECK At the origin, the two electric fields are large, but are oppositely directed and so add to zero. Far from the origin ($|z| \gg a$), the two electric fields (Figure 22-7*b*) are in almost the same direction so they do not add to zero.

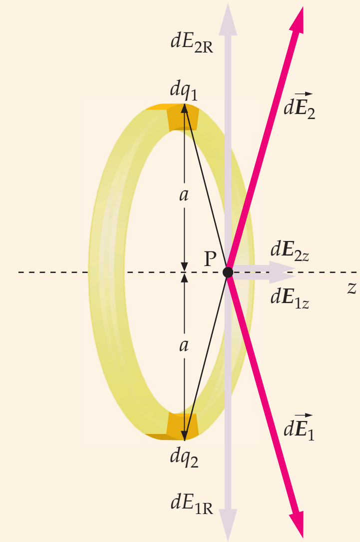


FIGURE 22-9

The electric field on the axis of a uniformly charged circular ring of radius a and charge Q is given by $\vec{E} = E_z\hat{k}$, where

$$E_z = \frac{kQz}{(z^2 + a^2)^{3/2}} \qquad 22\text{-}8$$

Equation 22-8 is derived in Example 22-5.

See **Math Tutorial** *for more information on* **Binomial Expansion**

Example 22-7 $\vec{E}$ on the Axis of a Charged Disk

Consider a uniformly charged thin disk of radius b and surface charge density σ. (*a*) Find the electric field at all points on the axis of the disk. (*b*) Show that for points on the axis and far from the disk, the electric field approaches that of a point charge at the origin with the same charge as the disk. (*c*) Show that for a uniformly charged disk of infinite radius, the electric field is uniform throughout the region on either side of the disk.

PICTURE We can calculate the field on the axis of the disk by treating the disk as a set of concentric, uniformly charged rings.

SOLVE

(*a*) 1. Calculate the field on the axis of the disk by treating the disk as a set of concentric rings of charge. The field of a single uniformly charged ring that has a charge Q and a radius a is shown in Equation 22-8:

$$\vec{E} = E_z\hat{k}, \text{ where } E_z = \frac{kQz}{(z^2 + a^2)^{3/2}}$$

2. Sketch the disk (Figure 22-10) and illustrate the electric field $d\vec{E}$ on its axis due to a single ring of charge dq, radius a, and width da:

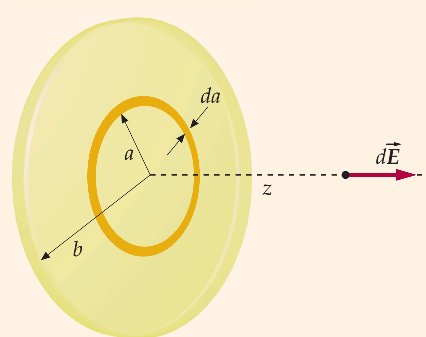


FIGURE 22-10 A uniform disk of charge can be treated as a set of ring charges, each of radius a.

3. Substitute dq for Q and dE_z for E_z in the step-1 result. Then integrate both sides to calculate the resultant field for the entire disk. The field point remains fixed, so z is constant:

$$dE_z = \frac{kz\,dq}{(z^2 + a^2)^{3/2}}$$

$$\text{so} \quad E_z = \int \frac{kz\,dq}{(z^2 + a^2)^{3/2}} = kz\int \frac{dq}{(z^2 + a^2)^{3/2}}$$

4. To evaluate this integral we change integration variables from q to a. The charge $dq = \sigma\, dA$, where $dA = 2\pi a\, da$ is the area of a ring of radius a and width da:

$$dq = \sigma\, dA = \sigma 2\pi a da$$

so $$E_z = \pi k z \sigma \int_0^b \frac{2a da}{(z^2 + a^2)^{3/2}} = \pi k z \sigma \int_{z^2+0^2}^{z^2+b^2} u^{-3/2}\, du$$

where $u = z^2 + a^2$, so $du = 2a da$.

5. Evaluate the integral and simplify the result:

$$E_z = \pi k z \sigma \frac{u^{-1/2}}{-\frac{1}{2}}\Bigg|_{z^2}^{z^2+b^2} = -2\pi k z \sigma\left(\frac{1}{\sqrt{z^2+b^2}} - \frac{1}{\sqrt{z^2}}\right)$$

$$= \boxed{\operatorname{sign}(z)\cdot 2\pi k\sigma\left(1 - \frac{1}{\sqrt{1+\dfrac{b^2}{z^2}}}\right)}$$

where $\operatorname{sign}(z) = z/|z|$. By definition*:

$$\operatorname{sign}(z) = \begin{cases} +1 & z > 0 \\ 0 & z = 0 \\ -1 & z < 0 \end{cases}$$

(b) 1. For $z \gg b$ (on the $+z$ axis far from the disk) we expect the electric field to decrease inversely with z^2, like that of a point charge. To show this we use the binomial expansion:

The binomial expansion (to first order) is $(1 + x)^n \approx 1 + nx$ for $|x| \ll 1$.

2. Apply the binomial expansion to the rightmost term in the step-5 result:

$$\frac{1}{\sqrt{1+\dfrac{b^2}{z^2}}} = \left(1 + \frac{b^2}{z^2}\right)^{-1/2} \approx 1 - \frac{1}{2}\frac{b^2}{z^2} \qquad z^2 \gg b^2$$

3. Substitute into the step-5 result and simplify. [For $z \gg b$, sign $(z) = 1$.] Thus, the approximate expression for the field for $z \gg b$ is the same as that of a point charge $Q = \sigma\pi b^2$ at the origin:

$$E_z \approx 2\pi k\sigma\left(1 - \left[1 - \frac{1}{2}\frac{b^2}{z^2}\right]\right) = 2\pi k\sigma \frac{1}{2}\frac{b^2}{z^2} = \boxed{\frac{kQ}{z^2}} \qquad z \gg b$$

where $Q = \sigma\pi b^2$.

(c) 1. Take the limit of the Part (a) , step-5 result as $b \to \infty$. This result is an expression for E_z that is uniform, both in the region $z > 0$ and in the region $z < 0$:

$$E_z = \operatorname{sign}(z)\cdot 2\pi k\sigma\left(1 - \frac{1}{\sqrt{1+\infty}}\right) = \boxed{\operatorname{sign}(z)\cdot 2\pi k\sigma}$$

CHECK We expect the electric field be in opposite directions on opposite sides of the disk. The Part (a) , step-5 result meets this expectation.

TAKING IT FURTHER According to the Part (c) result the electric field is discontinuous at $z = 0$ (Figure 22-11) where the field jumps from $-2\pi k\sigma\hat{i}$ to $+2\pi k\sigma\hat{i}$ as we cross the $z = 0$ plane. There is thus a discontinuity in E_z in the amount $4\pi k\sigma = \sigma/\epsilon_0$.

PRACTICE PROBLEM 22-5 The electric field due to a uniform surface charge on the entire $z = 0$ plane is given by the Part (c) result. What fraction of the field on the z axis at $z = a$ is due to the surface charge within a circle that has a radius $r = 5a$ centered at the origin? *Hint: Divide the Part (a), step 5 result by the Part (c) result after substituting* 5a *for* b *and* a *for z.*

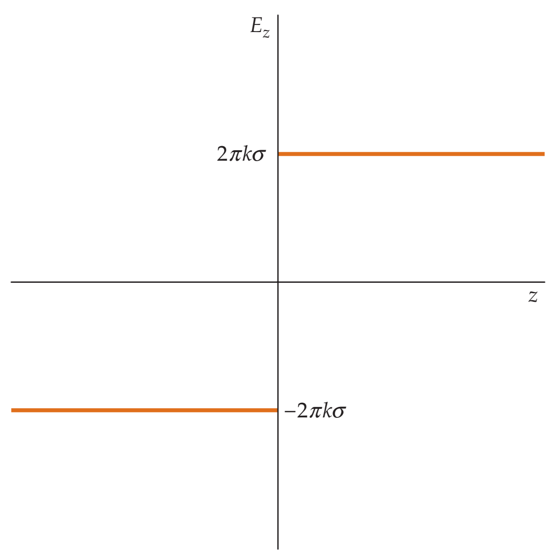


FIGURE 22-11 Graph showing the discontinuity of $\vec{E}$ at a plane charge. Can you see the similarity between this graph and the one in Figure 22-8?

* Both Excel and Mathematica use the definition of the sign function given here. Texas Instruments, however, uses a definition in which sign(0) returns ±1 instead of 0.

The answer to Practice Problem 22-5 depends not on a, but on the ratio $r/a = 5$. Eighty percent of the field at any distance a from a uniformly charged plane surface is due to the charge within a circle whose radius is equal to $5a$ multiplied by that distance.

The formula for the electric field on the axis of a uniformly charged circular disk, established in Example 22-7, is

$$E_z = \text{sign}(z) \cdot 2\pi k\sigma \left(1 - \frac{1}{\sqrt{1 + \dfrac{R^2}{z^2}}}\right) \qquad 22\text{-}9$$

ELECTRIC FIELD ON AXIS OF A UNIFORM DISK OF CHARGE

where sign(z) is defined in Part (a), step 5 of Example 22-7 and R is the radius of the disk. The field of a uniformly charged electric plane of charge can be obtained from Equation 22-9 by letting the ratio R/z go to infinity. Then

$$E_z = \text{sign}(z) \cdot 2\pi k\sigma = \text{sign}(z) \cdot \frac{\sigma}{2\epsilon_0} \qquad 22\text{-}10$$

ELECTRIC FIELD OF A UNIFORM PLANE OF CHARGE

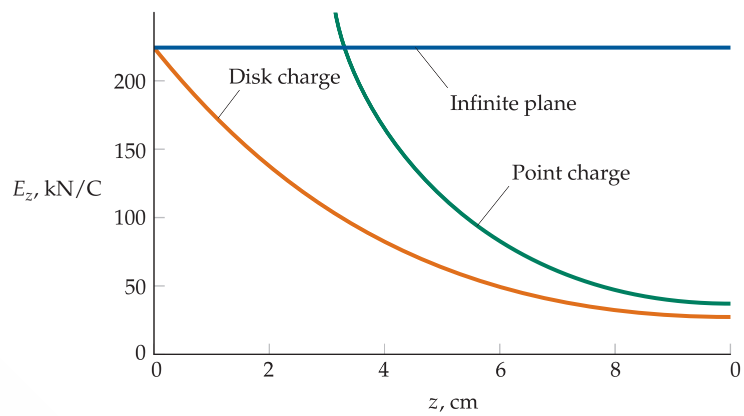


FIGURE 22-12 A disk and a point have equal charges, and an infinite plane and the disk have equal uniform surface-charge densities. Note that the field of the disk charge converges with the field of the point charge as z approaches infinity, and equals the field of the infinite plane charge as z approaches zero.

Figure 22-12 shows the electric fields of a point charge, a uniform disk of charge, and an infinite plane of charge as a function of position.

As we move along the z axis, the electric field jumps from $-2\pi k\sigma\hat{\imath}$ to $+2\pi k\sigma\hat{\imath}$ when we pass through the $z = 0$ plane (Figure 22-11). Thus, at $z = 0$ there is a discontinuity in E_z in the amount $4\pi k\sigma$.

Example 22-8 Electric Field Due to Two Infinite Planes

In Figure 22-13, an infinite plane of surface charge density $\sigma = +4.5\ \text{nC/m}^2$ lies in the $x = 0.00$ m plane, and a second infinite plane of surface charge density $\sigma = -4.50\ \text{nC/m}^2$ lies in the $x = 2.00$ m plane. Find the electric field at (a) $x = 1.80$ m and (b) $x = 5.00$ m.

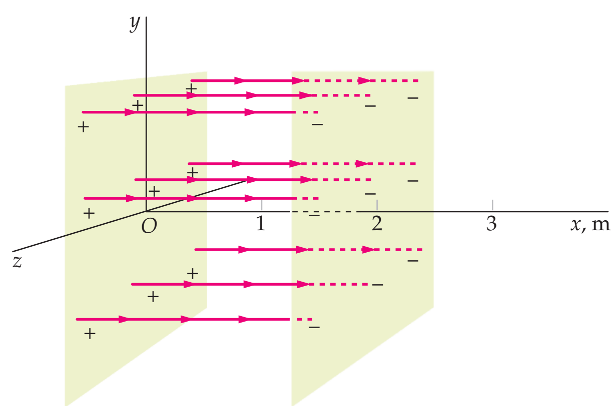


FIGURE 22-13

PICTURE Each charged plane produces a uniform electric field of magnitude $E = \sigma/(2\epsilon_0)$. We use superposition to find the resultant field. Between the planes the fields add, producing a net field of magnitude σ/ϵ_0 in the $+x$ direction. For $x > 2.00$ m and for $x < 0$, the two fields point in opposite directions and thus sum to zero.

SOLVE

(a) 1. Calculate the magnitude of the field E produced by each plane:

$$E = |\sigma|/(2\epsilon_0) = (4.50 \times 10^{-9}\ \text{N/C})/(2 \cdot 8.85 \times 10^{-12}) = 254\ \text{N/C}$$

2. At $x = 1.80$ m, between the planes, the field due to each plane points in the $+x$ direction:

$$E_{x\,\text{net}} = E_1 + E_2 = 254\ \text{N/C} + 254\ \text{N/C} = \boxed{508\ \text{N/C}}$$

(*b*) At $x = 5.00$ m, the fields due to the two planes are oppositely directed:

$$E_{x\,\text{net}} = E_1 - E_2 = \boxed{0.00\ \text{N/C}}$$

CHECK Because the two planes have equal and opposite charge densities, electric field lines originate on the positive plane and terminate on the negative plane. $\vec{E}$ is equal to zero except between the planes.

TAKING IT FURTHER Note that $E_{x\,\text{net}} = 508$ N/C not just at $x = 1.8$ m but at any point in the region between the charged planes. The charge configuration described in this example is that of a parallel-plate capacitor. Capacitors are discussed in Chapter 24.

22-2 GAUSS'S LAW

In Chapter 21, the electric field is described visually by using electric field lines. Here that description is put in rigorous mathematical language called Gauss's law. Gauss's law is one of Maxwell's equations—the fundamental equations of electromagnetism—which are the topic of Chapter 30. In electrostatics, Gauss's law and Coulomb's law are equivalent. Electric fields arising from some symmetrical charge distributions, such as a uniformly charged spherical shell or uniformly charged infinite line, can be easily calculated using Gauss's law. In this section, we give an argument for the validity of Gauss's law based on the properties of electric field lines. A more rigorous derivation of Gauss's law is presented in Section 22-6.

A closed surface—like the surface of a soap bubble—is one that divides the universe into two distinct regions, the region enclosed by surface and the region outside the surface. Figure 22-14 shows a closed surface of arbitrary shape enclosing a dipole. The number of electric field lines beginning on the positive charge and penetrating the surface from the inside depends on where the surface is drawn, but any line penetrating the surface from the inside also penetrates it from the outside. To count the net number of lines out of any closed surface, count any penetration from the inside as $+1$, and any penetration from the outside as -1. Thus, for the surface shown (Figure 22-14), the net number of lines out of the surface is zero. For surfaces enclosing other types of charge distributions, such as that shown in Figure 22-15, *the net number of lines out of any surface enclosing the charges is proportional to the net charge enclosed by the surface.* This rule is a statement of Gauss's law.

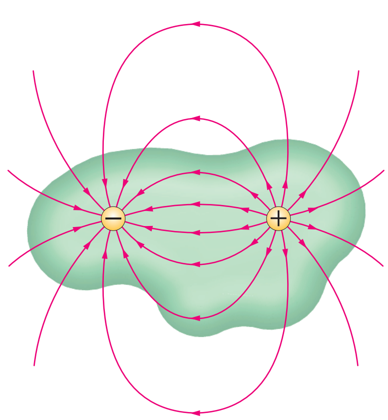


FIGURE 22-14 A surface of arbitrary shape enclosing an electric dipole. As long as the surface encloses both charges, the number of lines penetrating the surface from the inside is exactly equal to the number of lines penetrating the surface from the outside no matter where the surface is drawn.

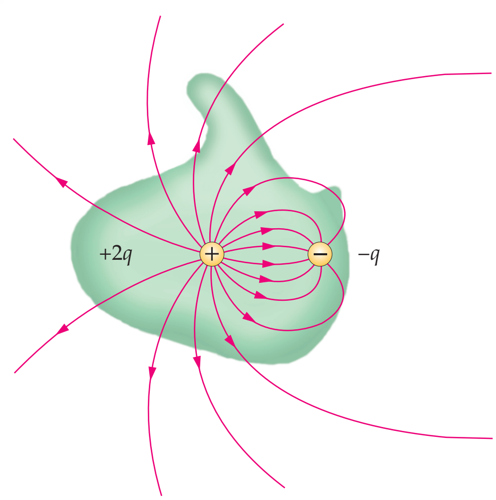


FIGURE 22-15 A surface of arbitrary shape enclosing the charges $+2q$ and $-q$. Either the field lines that end on $-q$ do not pass through the surface or they penetrate it from the inside the same number of times as from the outside. The net number that exit, the same as that for a single charge of $+q$, is equal to the net charge enclosed by the surface.

ELECTRIC FLUX

The mathematical quantity that corresponds to the number of field lines penetrating a surface is called the **electric flux** ϕ. For a surface perpendicular to $\vec{E}$ (Figure 22-16), the electric flux is the product of the magnitude of the field E and the area A:

$$\phi = EA$$

The units of electric flux are $\mathrm{N\cdot m^2/C}$. Because E is proportional to the number of field lines per unit area, the flux is proportional to the number of field lines penetrating the surface.

In Figure 22-17, the surface of area A_2 is not perpendicular to the electric field $\vec{E}$. However, the number of lines that penetrate the surface of area A_2 is the same as the number that penetrate the surface of area A_1, which is normal (perpendicular) to $\vec{E}$. These areas are related by

$$A_2 \cos\theta = A_1 \qquad \text{22-11}$$

where θ is the angle between $\vec{E}$ and the unit vector $\hat{n}$ that is normal to the surface A_2, as shown in the figure. The electric flux through a surface is defined to be

$$\phi = \vec{E}\cdot\hat{n}A = EA\cos\theta = E_n A \qquad \text{22-12}$$

where $E_n = \vec{E}\cdot\hat{n}$ is the component of $\vec{E}$ normal to the surface.

Figure 22-18 shows a curved surface over which $\vec{E}$ may vary. If the area ΔA_i of the surface element that we choose is small enough, it can be modeled as a plane, and the variation of the electric field across the element can be neglected. The flux of the electric field through this element is

$$\Delta\phi_i = E_{ni}\,\Delta A_i = \vec{E}_i\cdot\hat{n}_i\,\Delta A_i$$

where $\hat{n}_i$ is the unit vector perpendicular to the surface element and $\vec{E}_i$ is the electric field on the surface element. If the surface is curved, the unit vectors for the different small surface elements will have different directions. The total flux through the surface is the sum of $\Delta\phi_i$ over all the elements making up the surface. In the limit, as the number of elements approaches infinity and the area of each element approaches zero, this sum becomes an integral. The general definition of electric flux is thus

$$\phi = \lim_{\Delta A_i \to 0}\sum_i \vec{E}_i\cdot\hat{n}_i\,\Delta A_i = \int_S \vec{E}\cdot\hat{n}\,dA \qquad \text{22-13}$$

DEFINITION—ELECTRIC FLUX

where the S stands for the surface we are integrating over.* The sign of the flux depends on the choice for the direction of the unit normal $\hat{n}$. By choosing $\hat{n}$ to be out of one side of a surface we are determining the sign of $\vec{E}\cdot\hat{n}$, and thus the sign of the flux through the surface.

On a *closed* surface we are interested in the electric flux through the surface, and by convention, we always choose the unit vector $\hat{n}$ to be out of the surface at each point. The integral over a closed surface is indicated by the symbol $\oint$. The total or net flux through a closed surface S is therefore written

$$\phi_{\text{net}} = \oint_S \vec{E}\cdot\hat{n}\,dA = \oint_S E_n\,dA \qquad \text{22-14}$$

* The flux of a vector field through a surface is a mathematical operation used to describe the flow rates of fluids and rates of heat transfers. In addition, it is used to relate electric fields with the charges that produce them.

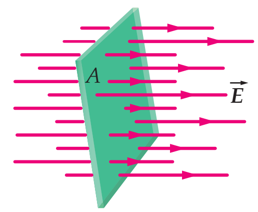


FIGURE 22-16 Electric field lines of a uniform field penetrating a surface of area A that is oriented perpendicular to the field. The product EA is the electric flux through the surface.

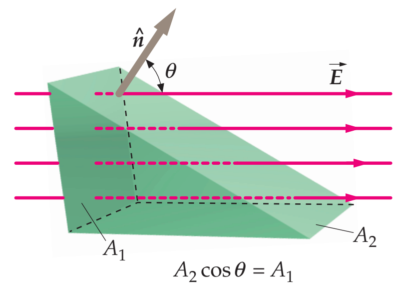


FIGURE 22-17 Electric field lines of a uniform electric field that is perpendicular to the surface of area A_1 but makes an angle θ with the unit vector $\hat{n}$ that is normal to the surface of area A_2. Where $\vec{E}$ is not perpendicular to the surface, the flux is E_nA, where $E_n = E\cos\theta$ is the component of $\vec{E}$ that is perpendicular to the surface. The flux through the surface of area A_2 is the same as that through the surface of area A_1.

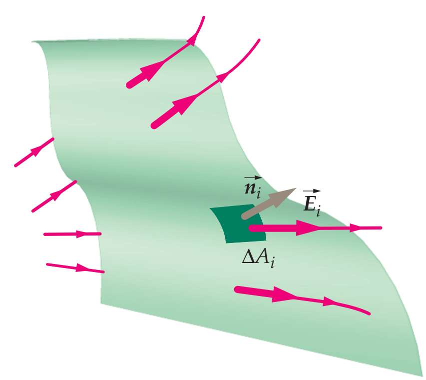


FIGURE 22-18 If E_n varies from place to place on a surface, either because the magnitude E varies or because the angle between $\vec{E}$ and $\hat{n}$ varies, the area of the surface is divided into small elements of area ΔA_i. The flux through the surface is computed by summing $\vec{E}_i\cdot\hat{n}_i\,\Delta A_i$ over all the area elements.

The net flux ϕ_{net} through the closed surface is positive or negative, depending on whether $\vec{E}$ is predominantly outward or inward at the surface. At points on the surface where $\vec{E}$ is inward, E_n is negative.

QUANTITATIVE STATEMENT OF GAUSS'S LAW

Figure 22-19 shows a spherical surface of radius R that has a point charge Q at its center. The electric field everywhere on this surface is normal to the surface and has the magnitude

$$E_n = \frac{kQ}{R^2}$$

The net flux of $\vec{E}$ out of this spherical surface is

$$\phi_{net} = \oint_S E_n \, dA = E_n \oint_S dA$$

where we have taken E_n out of the integral because it is constant everywhere on the surface. The integral of dA over the surface is just the total area of the surface, which for a sphere of radius R is $4\pi R^2$. Using this and substituting kQ/R^2 for E_n, we obtain

$$\phi_{net} = \frac{kQ}{R^2} 4\pi R^2 = 4\pi kQ = Q/\epsilon_0 \qquad 22\text{-}15$$

Thus, the net flux out of a spherical surface that has a point charge Q at its center is independent of the radius R of the sphere and is equal to Q divided by ϵ_0. This is consistent with our previous observation that the net number of lines through a closed surface is proportional to the net charge inside the surface. *This number of lines is the same for all closed surfaces surrounding the charge, independent of the shape of the surface.* Thus, the net flux out of *any surface* surrounding a point charge Q equals Q/ϵ_0.

We can extend this result to systems containing multiple charges. In Figure 22-20, the surface encloses two point charges, q_1 and q_2, and there is a third point charge q_3 outside the surface. Because the electric field at any point on the surface is the vector sum of the electric fields produced by each of the three charges, the net flux $\phi_{net} = \oint_S (\vec{E}_1 + \vec{E}_2 + \vec{E}_3) \cdot \hat{n} \, dA$ out of the surface is just the sum of the fluxes ($\phi_{net} = \Sigma\phi_i$, where $\phi_i = \oint_S \vec{E}_i \cdot \hat{n} \, dA$) due to the individual charges. The flux ϕ_3 (due to charge q_3 which is outside the surface) is zero because every field line from q_3 that enters the region bounded by the surface at one point leaves the region surface at some other point. The flux out of the surface due to charge q_1 is $\phi_1 = q_1/\epsilon_0$ and the flux due to charge q_2 is $\phi_2 = q_2/\epsilon_0$. The net flux out of the surface therefore equals $\phi_{net} = (q_1 + q_2)/\epsilon_0$, which may be positive, negative, or zero depending on the signs and magnitudes of q_1 and q_2.

The net outward flux through any closed surface equals the net charge inside the surface divided by ϵ_0:

$$\phi_{net} = \oint_S \vec{E} \cdot \hat{n} \, dA = \oint_S E_n \, dA = \frac{Q_{inside}}{\epsilon_0} \qquad 22\text{-}16$$

GAUSS'S LAW

This is **Gauss's law.** It reflects the fact that the electric field due to a single point charge varies inversely with the square of the distance from the charge. It was this property of the electric field that made it possible to draw a fixed number of electric field lines from a charge and have the density of lines be proportional to the field strength.

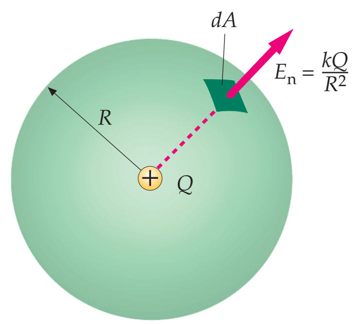


FIGURE 22-19 A spherical surface enclosing a point charge Q. The net flux is easily calculated for a spherical surface. It equals E_n multiplied by the surface area, or $E_n 4\pi R^2$.

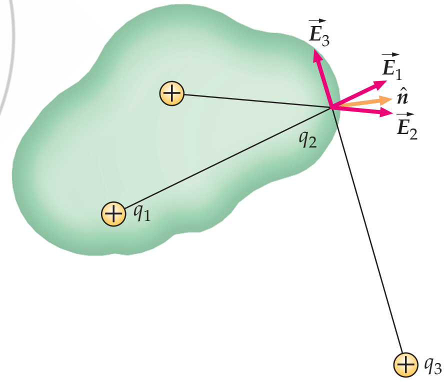


FIGURE 22-20 A surface enclosing point charges q_1 and q_2, but not q_3. The net flux out of this surface is $4\pi k(q_1 + q_2)$.

Gauss's law is valid for all surfaces and all charge distributions. For charge distributions that have high degrees of symmetry, it can be used to calculate the electric field, as we illustrate in the next section. For static charge distributions, Gauss's law and Coulomb's law are equivalent. However, Gauss's law is more general in that it is always valid whereas the validity of Coulomb's law is restricted to static charge distributions.

Example 22-9 Flux through a Piecewise-Continuous Closed Surface

An electric field is given by $\vec{E} = +(200\ \text{N/C})\hat{k}$ throughout the region $z > 0$ and by $\vec{E} = -(200\ \text{N/C})\hat{k}$ throughout the region $z < 0$. An imaginary soup-can-shaped surface that has a length equal to 20 cm and a radius R equal to 5.00 cm has its center at the origin and its axis along the z axis, so that one end is at $z = +10$ cm and the other is at $z = -10$ cm (Figure 22-21). (*a*) What is the net outward flux through the closed surface? (*b*) What is the net charge inside the closed surface?

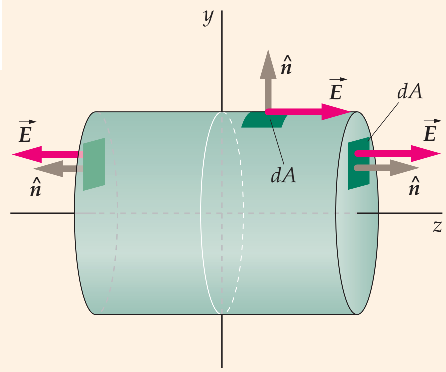


FIGURE 22-21

PICTURE The closed surface described, which is piecewise continuous, consists of three pieces—two flat ends and a curved side. Separately calculate the flux of $\vec{E}$ out of each piece of this surface. To calculate the flux out of a piece draw the outward normal $\hat{n}$ at an arbitrarily chosen point on the piece and draw the vector $\vec{E}$ at the same point. If $E_n = \vec{E}\cdot\hat{n}$ is the same everywhere on the piece, then the outward flux through the piece is E_nA, where A is the area of the piece. The net outward flux through the entire closed surface is obtained by summing the fluxes through the individual pieces. The net outward flux is related to the charge inside by Gauss's law (Equation 22-16).

SOLVE

(*a*) 1. Sketch the soup-can-shaped surface. On each piece of the surface draw the outward normal $\hat{n}$ and the vector $\vec{E}$ (Figure 22-21):

2. Calculate the outward flux through the right end of the "can" (the piece of the surface at $z = +10$ cm). On this piece $\hat{n} = \hat{k}$:

$$\phi_{\text{right}} = \vec{E}_{\text{right}}\cdot\hat{n}_{\text{right}}A = \vec{E}_{\text{right}}\cdot\hat{k}\pi R^2 = +(200\ \text{N/C})\hat{k}\cdot\hat{k}(\pi)(0.0500\ \text{m})^2$$
$$= 1.57\ \text{N}\cdot\text{m}^2/\text{C}$$

3. Calculate the outward flux through the left end of the "can" (the piece of the surface at $z = -10$ cm), where $\hat{n} = -\hat{k}$:

$$\phi_{\text{left}} = \vec{E}_{\text{left}}\cdot\hat{n}_{\text{left}}A = \vec{E}_{\text{left}}\cdot(-\hat{k})\pi R^2$$
$$= -(200\ \text{N/C})\hat{k}\cdot(-\hat{k})(\pi)(0.0500\ \text{m})^2$$
$$= 1.57\ \text{N}\cdot\text{m}^2/\text{C}$$

4. Calculate the outward flux through the curved surface. On the curved surface $\hat{n}$ is in the radial direction, perpendicular to the z axis:

$$\phi_{\text{curved}} = \vec{E}_{\text{curved}}\cdot\hat{n}_{\text{curved}}A = 0$$
($\phi_{\text{curved}} = 0$ because $\vec{E}\cdot\hat{n} = 0$ everywhere on the curved piece.)

5. The net outward flux is the sum through all the individual surfaces:

$$\phi_{\text{net}} = \phi_{\text{right}} + \phi_{\text{left}} + \phi_{\text{curved}} = 1.57\ \text{N}\cdot\text{m}^2/\text{C} + 1.57\ \text{N}\cdot\text{m}^2/\text{C} + 0$$
$$= \boxed{3.14\ \text{N}\cdot\text{m}^2/\text{C}}$$

(*b*) Gauss's law relates the charge inside to the net flux:

$$Q_{\text{inside}} = \epsilon_0\phi_{\text{net}} = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.14\ \text{N}\cdot\text{m}^2/\text{C})$$
$$= \boxed{2.78\times10^{-11}\ \text{C} = 27.8\ \text{pC}}$$

CHECK The flux through either end of the can does not depend on the length of the can. This result is expected for an electric field that does not vary with distance from the $z = 0$ plane.

TAKING IT FURTHER The net flux does not depend on the length of the can. Thus, the charge inside resides entirely on the $z = 0$ plane.

22-3 USING SYMMETRY TO CALCULATE $\vec{E}$ WITH GAUSS'S LAW

Given a highly symmetrical charge distribution, the electric field can often be calculated more easily by using Gauss's law than it can by using Coulomb's law. There are three classes of symmetry to consider. A charge configuration has **cylindrical (or line) symmetry** if the charge density depends only on the distance from a line, **plane symmetry** if the charge density depends only on the distance from a plane, and **spherical (or point) symmetry** if the charge density depends only on the distance from a point.

PROBLEM-SOLVING STRATEGY

Calculating $\vec{E}$ Using Gauss's Law

PICTURE Determine if the charge configuration belongs to one of the three symmetry classes. If it does not, then try another method to calculate the electric field. If is does, then sketch the charge configuration and establish the magnitude and direction of the electric field $\vec{E}$ using symmetry considerations.

SOLVE

1. On the sketch draw an imaginary closed surface, called a **Gaussian surface** (for example, the soup can in Example 22-9). This surface is chosen so that on each piece of the surface $\vec{E}$ is either zero, normal to the surface with E_n the same everywhere on the piece, or parallel to the surface ($E_n = 0$) everywhere on the piece. For a configuration that has cylindrical (line) symmetry, the Gaussian surface is a cylinder coaxial with the symmetry line. For a configuration that has plane symmetry, the Gaussian surface is a cylinder bisected by the symmetry plane and with its symmetry axis normal to the symmetry plane. For a configuration that has spherical (point) symmetry, the Gaussian surface is a sphere centered on the symmetry point. On each piece of the Gaussian surface sketch an area element dA, an outward normal $\hat{n}$, and the electric field $\vec{E}$.
2. Closed cylindrical surfaces are piecewise continuous, with the surface divided into three pieces. Spherical surfaces consist of a single piece. The flux through each piece of a properly chosen Gaussian surface equals E_nA, where E_n is the component of $\vec{E}$ normal to the piece and A is the area of the piece. Add the fluxes to obtain the total outward flux through the closed surface.
3. Calculate the total charge inside the Gaussian surface.
4. Apply Gauss's law to relate E_n to the charges inside the closed surface and solve for E_n.

CONCEPT CHECK 22-1

Is the electric field $\vec{E}$ in Gauss's law only that part of the electric field due to the charges inside a surface, or is it the total electric field due to all the charges both inside and outside the surface?

Example 22-10 $\vec{E}$ Due to a Uniformly Charged Slab

A very large (infinite), uniformly charged slab of plastic of thickness $2a$ occupies the region between the $z = -a$ plane and the $z = +a$ plane. Find the electric field everywhere due this charge configuration. The charge per unit volume of the plastic is ρ.

PICTURE The charge configuration has plane symmetry, with the $z = 0$ plane as the symmetry plane. Use symmetry arguments to determine the direction of the electric field everywhere. Then, apply Gauss's law and solve for the electric field.

SOLVE

1. Use symmetry considerations to determine the direction of $\vec{E}$. Because the sheet is infinite, there is no preferred direction parallel to the sheet:

For $\rho > 0$, $\vec{E}$ points directly away from the $z = 0$ plane, and for $\rho < 0$, $\vec{E}$ points directly toward the $z = 0$ plane. On the $z = 0$ plane $\vec{E} = 0$.

2. Sketch the charge configuration that has a suitable Gaussian surface—a cylinder bisected by the symmetry plane (the $z = 0$ plane with its axis normal to the $z = 0$ plane). The cylinder extends from $-z$ to $+z$ (Figure 22-22):

3. Write down Gauss's law (Equation 22-16):

$$\phi_{\text{net}} = \oint_S \vec{E}\cdot\hat{n}\,dA = \frac{Q_{\text{inside}}}{\epsilon_0}$$

4. The outward flux $\vec{E}$ through the surface is equal to the sum of the fluxes through each piece of the surface. Draw both $\hat{n}$ and $\vec{E}$ at an area element on each piece of the surface (Figure 22-22):

$$\phi_{\text{net}} = \phi_{\text{left end}} + \phi_{\text{right end}} + \phi_{\text{curved side}}$$

where $\phi_{\text{left end}} = \int_{\text{left end}} \vec{E}\cdot\hat{n}dA$

$$\phi_{\text{right end}} = \int_{\text{right end}} \vec{E}\cdot\hat{n}\,dA$$

$$\phi_{\text{curved side}} = \int_{\text{curved side}} \vec{E}\cdot\hat{n}\,dA$$

5. Because $\vec{E}\cdot\hat{n}$ is zero everywhere on the curved piece of the surface, the flux through the curved piece is zero:

$$\phi_{\text{curved side}} = 0$$

6. $\vec{E}$ is uniform on the right end of the surface, so $\vec{E}\cdot\hat{n} = E_n$ can be factored from the integral. Let A be the area of the end of right end of the surface:

$$\phi_{\text{right end}} = \int_{\text{right end}} \vec{E}\cdot\hat{n}\,dA = \int_{\text{right end}} E_n\,dA = E_n\int_{\text{right end}} dA = E_nA$$

7. The two ends of the surface are the same distance from the symmetry plane (the $z = 0$ plane), so $\vec{E}$ on the left end is equal and opposite to $\vec{E}$ on the right end. The normals on the two ends are equal and opposite as well. Thus, $\vec{E}\cdot\hat{n} = E_n$ is the same on both ends. It follows that the flux out of both ends is the same as well:

$\vec{E}\cdot\hat{n} = E_n$ is the same on the two ends,

$$\therefore\ \phi_{\text{left end}} = \phi_{\text{right end}} = E_nA$$

8. Add the individual fluxes to get the net flux out of the surface:

$$\phi_{\text{net}} = \phi_{\text{left end}} + \phi_{\text{right end}} + \phi_{\text{curved side}} = E_nA + E_nA + 0 = 2E_nA$$

9. Solve for the charge inside the Gaussian surface. The volume of a cylinder is the cross-sectional area multiplied by the length. The cylinder has a length of $2z$.

$$Q_{\text{inside}} = \rho A2a \qquad (z \geq a)$$
$$Q_{\text{inside}} = \rho A2z \qquad (z \leq a)$$

10. Substitute the step-8 and step-9 results into $\phi_{\text{net}} = Q_{\text{inside}}/\epsilon_0$ (the step-3 result) and solve for E_n on the right end of the surface:

For $|z| \geq a$, $2E_nA = \rho A2a/\epsilon_0$, so $E_n = \rho a/\epsilon_0$.
For $-a \leq z \leq a$, $2E_nA = \rho A2|z|/\epsilon_0$, so $E_n = \rho|z|/\epsilon_0$.

11. Solve for $\vec{E}$ as a function of z. In the region $z < 0$, $\hat{n} = -\hat{k}$, so $E_z = -E_n$; this means $\vec{E}$ is in the $-z$ direction so E_z is negative:

$$\boxed{\vec{E} = E_z\hat{k} = \begin{cases} -(\rho a/\epsilon_0)\hat{k} & (z \leq -a) \\ (\rho z/\epsilon_0)\hat{k} & (-a \leq z \leq a) \\ +(\rho a/\epsilon_0)\hat{k} & (z \geq +a) \end{cases}}$$

or

$$\boxed{\vec{E} = E_z\hat{k} = \begin{cases} \text{sign}(z)\cdot(\rho a/\epsilon_0)\hat{k} & (|z| \geq a) \\ \text{sign}(z)\cdot(\rho|z|/\epsilon_0)\hat{k} & (|z| \leq a) \end{cases}}$$

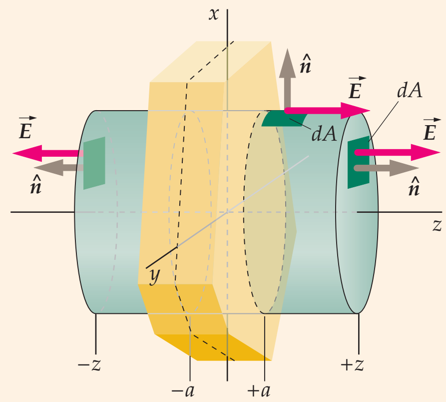


FIGURE 22-22 Gaussian surface for the calculation of $\vec{E}$ due to an infinite plane of charge. (Only the part of the plane that is inside the Gaussian surface is shown.) On the flat faces of this soup-can-shaped surface, $\vec{E}$ is perpendicular to the surface and constant in magnitude. On the curved surface $\vec{E}$ is parallel with the surface.

CHECK The electric field has units of N/C. According to our step 11 results, $\rho a/\epsilon_0$ should have the same units. It does, as $\epsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$, ρ has units of C/m^3, and a has units of m.

TAKING IT FURTHER Outside the slab the electric field is the same as that of the uniformly charged plane of Equation 22-10, with $\sigma = 2\rho a$. Figure 22-23 shows a graph of E_z versus z for the charged slab. Compare this graph with that of Figure 22-11 which shows a graph of E_z versus z for the charged plane. These graphs are readily compared if you recognize that $2\pi k = 1/(2\epsilon_0)$.

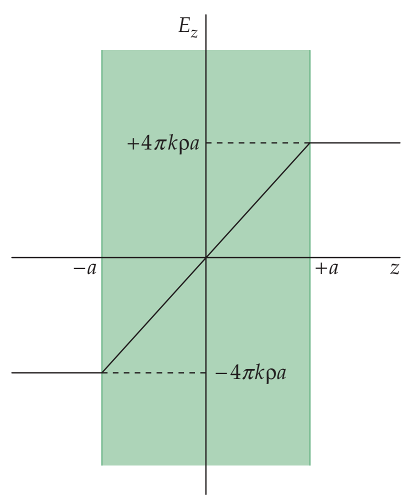


FIGURE 22-23 A graph of E_z versus z for a uniformly charged infinite slab of thickness $2a$ and charge density ρ.

We can use Gauss's law to derive Coulomb's law. This is accomplished by applying Gauss's law to find the electric field a distance r from a point charge q. Place the origin at the location of the point charge and choose a spherical Gaussian surface of radius r centered on the point charge. The outward normal $\hat{n}$ to this surface is equal to the unit vector $\hat{r}$. By symmetry, $\vec{E}$ is directed either radially outward or radially inward, so $\vec{E} = E_r\hat{r}$. It follows that E_n, the component of $\vec{E}$ normal to the surface, equals the radial component $\vec{E}$. That is, $E_n = \vec{E}\cdot\hat{n} = \vec{E}\cdot\hat{r} = E_r$. Also, the magnitude of $\vec{E}$ can depend on the distance from the charge but not on the direction from the charge. It follows that E_n has the same value everywhere on the surface. The net flux of $\vec{E}$ through the spherical surface of radius r is thus

$$\phi_{\text{net}} = \oint_S \vec{E}\cdot\hat{n}\,dA = \oint_S E_n\,dA = E_n\oint_S dA = E_r 4\pi r^2$$

where $\oint_S dA = 4\pi r^2$ (the area of the spherical surface). Because the total charge inside the surface is just the point charge q, Gauss's law gives

$$E_r 4\pi r^2 = \frac{q}{\epsilon_0}$$

Solving for E_r gives

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

which is Coulomb's law. We have thus derived Coulomb's law from Gauss's law. Because for static charges Gauss's law can also be derived from Coulomb's law (see Section 22-6), we have shown that the two laws are equivalent (for static charges).

Example 22-11 $\vec{E}$ Due to a Thin Spherical Shell of Charge

Find the electric field due to a uniformly charged thin spherical shell of radius R and total charge Q.

PICTURE This charge configuration depends only on the distance from a single point—the center of the spherical shell. Thus, the configuration has spherical (point) symmetry. This symmetry dictates that $\vec{E}$ must be radial and have a magnitude that depends only on the distance r from the center of the spherical shell. A spherical Gaussian surface that has an arbitrary radius r and is concentric with the charge configuration is needed.

SOLVE

1. Sketch the charge configuration and a spherical Gaussian surface S of radius $r > R$. Include an area element dA, the normal $\hat{n}$, and the electric field $\vec{E}$ on the area element (Figure 22-24):

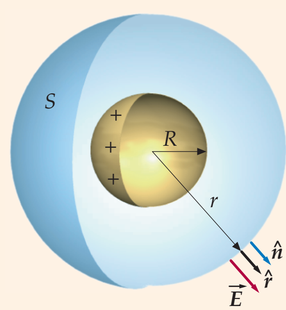


FIGURE 22-24 Spherical Gaussian surface of radius $r > R$ for the calculation of the electric field outside a uniformly charged thin spherical shell of radius R.

2. Express Gauss's law (Equation 22-16):
$$\phi_{\text{net}} = \oint_S E_n\,dA = \frac{Q_{\text{inside}}}{\epsilon_0}$$

3. The value of E_n is the same everywhere on S. Thus we can factor it from the integral:
$$E_n\oint_S dA = \frac{Q_{\text{inside}}}{\epsilon_0}$$

4. The integral of the area element over the surface S is just the area of the sphere. The area of the sphere is $4\pi r^2$:

$$E_n 4\pi r^2 = \frac{Q_{\text{inside}}}{\epsilon_0}$$

5. Due to the symmetry, $E_n = E_r$. Substitute E_r for E_n and solve for E_r:

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q_{\text{inside}}}{r^2}$$

6. For $r > R$, $Q_{\text{inside}} = Q$. For $r < R$, $Q_{\text{inside}} = 0$:

$$\vec{E} = E_r\hat{\boldsymbol{r}}, \quad \text{where} \quad \boxed{\begin{aligned} E_r &= \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad r > R \\ E_r &= 0 \qquad r < R \end{aligned}}$$

CHECK Outside the charged shell, the electric field is the same as that of a point charge Q at the shell's center. This result is expected for $r \gg R$.

TAKING IT FURTHER The step-6 result can also be obtained by direct integration of Coulomb's law, but that calculation is much more challenging.

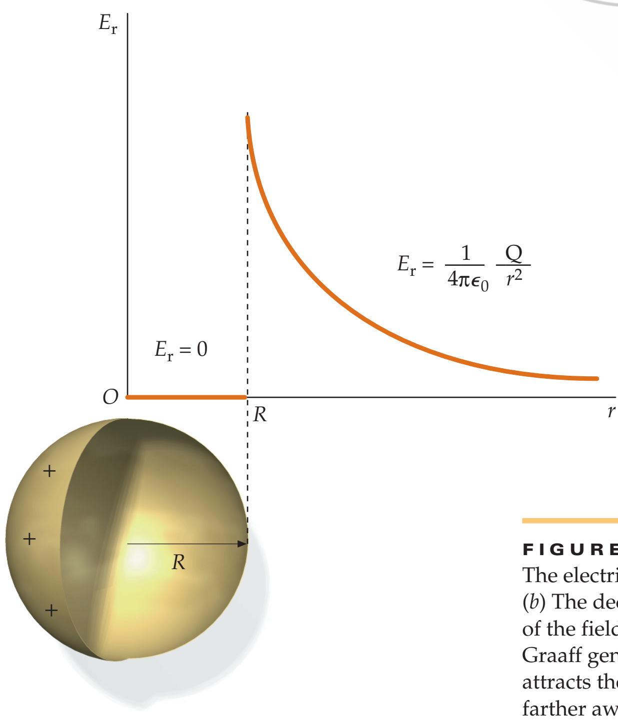
Figure 22-25 shows E_r versus r for a spherical-shell charge distribution. Again, note that the electric field is discontinuous at $r = R$, where the surface charge density is $\sigma = Q/(4\pi R^2)$. Just outside the shell, the electric field is $E_r = Q/(4\pi\epsilon_0 R^2) = \sigma/\epsilon_0$, because $\sigma = Q/4\pi R^2$. Because the field just inside the shell is zero, the electric field is discontinuous at $r = R$ by the amount σ/ϵ_0.

The electric field of a uniformly charged thin spherical shell is given by $\vec{E} = E_r\hat{\boldsymbol{r}}$, where

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad r > R \qquad \text{22-17}a$$

$$E_r = 0 \qquad r < R \qquad \text{22-17}b$$

(a)



(b)

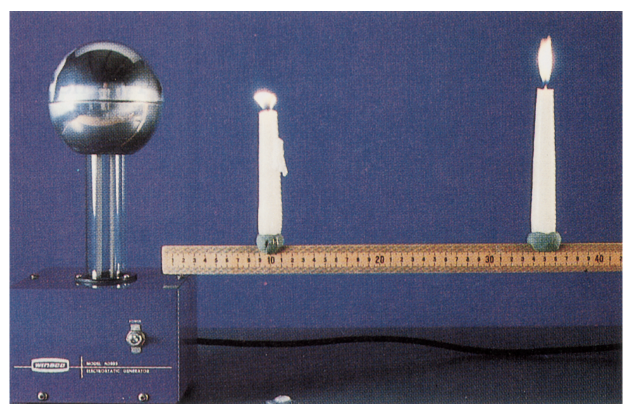


FIGURE 22-25 (a) A plot of E_r versus r for a thin spherical shell charge distribution. The electric field is discontinuous at $r = R$, where there is a surface charge of density σ. (b) The decrease in E_r over distance due to a charged spherical shell is evident by the effect of the field on the flames of two candles. The spherical shell at the left (part of a Van de Graaff generator, a device that is discussed in Chapter 23) has a large negative charge that attracts the positive ions in the nearby candle flame. The flame at right, which is much farther away, is not noticeably affected. *(Runk/Schoenberger from Grant Heilmann.)*

Example 22-12 Electric Field Due to a Point Charge and a Charged Spherical Shell

A spherical shell of radius $R = 3.00$ m has its center at the origin and has a surface charge density of $\sigma = 3.00\ \text{nC/m}^2$. A point charge $q = 250$ nC is on the y axis at $y = 2.00$ m. Find the electric field on the x axis at (*a*) $x = 2.00$ m and (*b*) $x = 4.00$ m.

PICTURE We separately find the field due to the point charge and that due to the spherical shell and sum the field vectors in accord with the principle of superposition. For Part (*a*), the field point is inside the shell, so the field is due only to the point charge (Figure 22-26*a*). For Part (*b*) , the field point is outside the shell, so the field due to the shell can be calculated as if the charge were a point charge at the origin. We then add the fields due to the two point charges (Figure 22-26*b*).

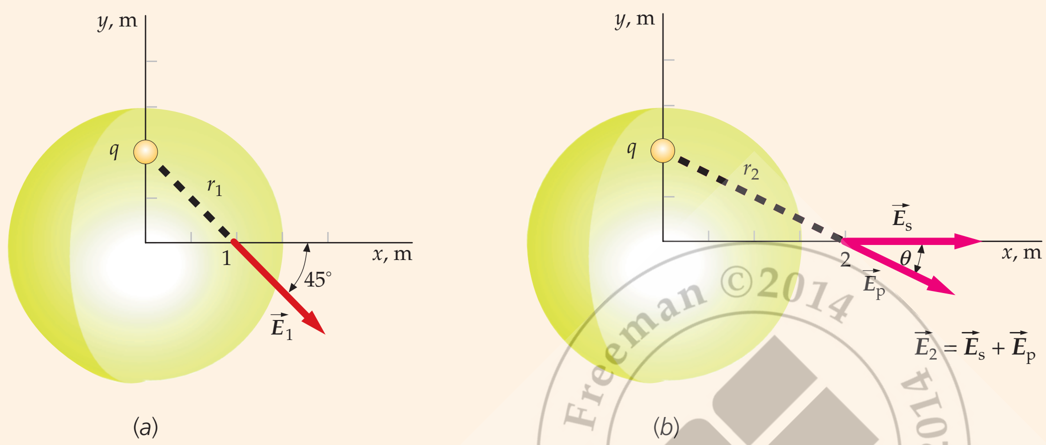


FIGURE 22-26

SOLVE

(*a*) 1. Inside the shell, $\vec{E}_1$ is due only to the point charge: $\quad \vec{E}_1 = \dfrac{kq}{r_1^2}\hat{r}_1$

2. Calculate the square of the distance r_1: $\quad r_1^2 = (2.00\ \text{m})^2 + (2.00\ \text{m})^2 = 8.00\ \text{m}^2$

3. Use r_1 to calculate the magnitude of the field: $\quad E_1 = \dfrac{kq}{r_1^2} = \dfrac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(250 \times 10^{-9}\ \text{C})}{8.00\ \text{m}^2} = 281\ \text{N/C}$

4. From Figure 22-26*a*, we can see that the field makes an angle of 45° with the x axis: $\quad \theta_1 = 45.0°$

5. Express $\vec{E}_1$ in terms of its components:

$$\vec{E}_1 = E_{1x}\hat{i} + E_{1y}\hat{j} = E_1\cos 45.0°\ \hat{i} - E_1 \sin 45.0°\ \hat{j}$$
$$= (281\ \text{N/C})\cos 45.0°\ \hat{i} - (281\ \text{N/C})\sin 45.0°\ \hat{j}$$
$$= \boxed{(199\hat{i} - 199\hat{j})\ \text{N/C}}$$

(*b*) 1. Outside of its perimeter, the field of the shell can be calculated as if the shell were a point charge at the origin, and the field due to the shell $\vec{E}_S$ is therefore along the x axis: $\quad \vec{E}_S = \dfrac{kQ}{x_2^2}\hat{i}$

2. Calculate the total charge Q on the shell: $\quad Q = \sigma 4\pi R^2 = (3.00\ \text{nC/m}^2)4\pi(3.00\ \text{m})^2 = 339\ \text{nC}$

3. Use Q to calculate the field due to the shell: $\quad E_S = \dfrac{kQ}{x_2^2} = \dfrac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(339 \times 10^{-9}\text{C})}{(4.00\ \text{m})^2} = 190\ \text{N/C}$

4. The field due to the point charge is: $\quad \vec{E}_P = \dfrac{kq}{r_2^2}\hat{r}_2$

5. Calculate the square of the distance from the point charge q on the y axis to the field point at $x = 4.00$ m:
$$r_2^2 = (2.00\text{ m})^2 + (4.00\text{ m})^2 = 20.0\text{ m}^2$$

6. Calculate the magnitude of the field due to the point charge:
$$E_P = \frac{kq}{r_2^2} = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(250 \times 10^{-9}\text{ C})}{20.0\text{ m}^2} = 112\text{ N/C}$$

7. This field makes an angle θ with the x axis, where:
$$\tan\theta = \frac{2.00\text{ m}}{4.00\text{ m}} = 0.500 \Rightarrow \theta = \tan^{-1}0.500 = 26.6°$$

8. The x and y components of the net electric field are thus:
$$E_x = E_{Px} + E_{Sx} = E_P\cos\theta + E_S$$
$$= (112\text{ N/C})\cos 26.6° + 190\text{ N/C} = 290\text{ N/C}$$
$$E_y = E_{Py} + E_{Sy} = -E_P\sin\theta + 0$$
$$= -(112\text{ N/C})\sin 26.6° = -50.0\text{ N/C}$$
$$\vec{E}_2 = \boxed{(290\hat{i} - 50.0\hat{j})\text{ N/C}}$$

CHECK The Part (*b*), step 8 result is qualitatively in agreement with Figure 22-26*b*. That is, E_x is positive, E_y is negative, and $|E_y| < E_x$.

TAKING IT FURTHER Specifying the x, y, and z components of a vector completely specifies the vector. In these cases, the z component is zero.

$\vec{E}$ DUE TO A UNIFORMLY CHARGED SPHERE

Example 22-13 $\vec{E}$ Due to a Uniformly Charged Solid Sphere

Find the electric field everywhere for a uniformly charged solid sphere that has a radius R and a total charge Q that is uniformly distributed throughout the volume of the sphere that has a charge density $\rho = Q/V$, where $V = \frac{4}{3}\pi R^3$ is the volume of the sphere.

PICTURE The charge configuration has spherical symmetry. By symmetry, the electric field must be radial. We choose a spherical Gaussian surface of radius r (Figure 22-27*a* and Figure 22-27*b*). On the Gaussian surface, E_n is the same everywhere, and $E_n = E_r$. Gauss's law thus relates E_r to the total charge inside the Gaussian surface.

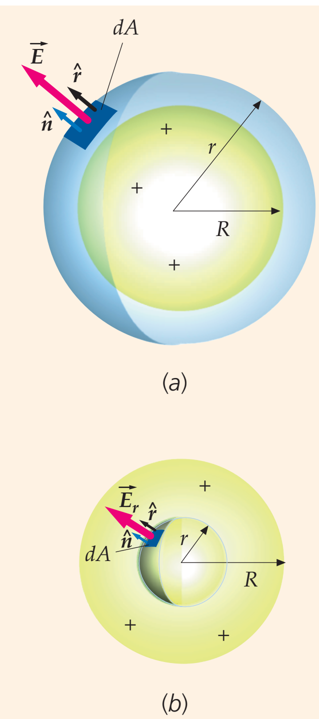


FIGURE 22-27

SOLVE

1. Draw a charged sphere of radius R and draw a spherical Gaussian surface with radius r (Figure 22-27*a* for $r > R$ and Figure 22-27*b* for $r < R$):

2. Relate the flux through the Gaussian surface to the electric field E_r on it. At every point on this surface $\hat{n} = \hat{r}$ and E_r has the same value:
$$\phi_{\text{net}} = \vec{E}\cdot\hat{n}A = \vec{E}\cdot\hat{r}A = E_r 4\pi r^2$$
(The surface area of a sphere of radius r is $4\pi r^2$.)

3. Apply Gauss's law to relate the field to the total charge inside the surface:
$$E_r 4\pi r^2 = \frac{Q_{\text{inside}}}{\epsilon_0}$$

4. Find Q_{inside} for all values of r. The charge density $\rho = Q/V$, where $V = \frac{4}{3}\pi R^3$:

For $r \geq R$, $Q_{\text{inside}} = Q$

For $r \leq R$, $Q_{\text{inside}} = \rho V'$,

where $V' = \frac{4}{3}\pi r^3$

so
$$Q_{\text{inside}} = \frac{Q}{V}V' = \frac{Q}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3}$$

5. Substitute into the step 3 result and solve for $\vec{E}$:

$\vec{E} = E_r\hat{r}$, where

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad r \geq R$$

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\frac{r^3}{R^3} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}r \qquad r \leq R$$

CHECK At the center of the charged sphere the electric field is zero, as symmetry suggests. For $r > R$, the field is identical to the field of a point charge Q at the center of the sphere, also as expected.

TAKING IT FURTHER Figure 22-28 shows E_r versus r for the charge distribution in this example. Inside the sphere of charge, E_r increases with r. Note that E_r is continuous at $r = R$. A uniformly charged sphere is sometimes used as a model to describe the electric field of an atomic nucleus.

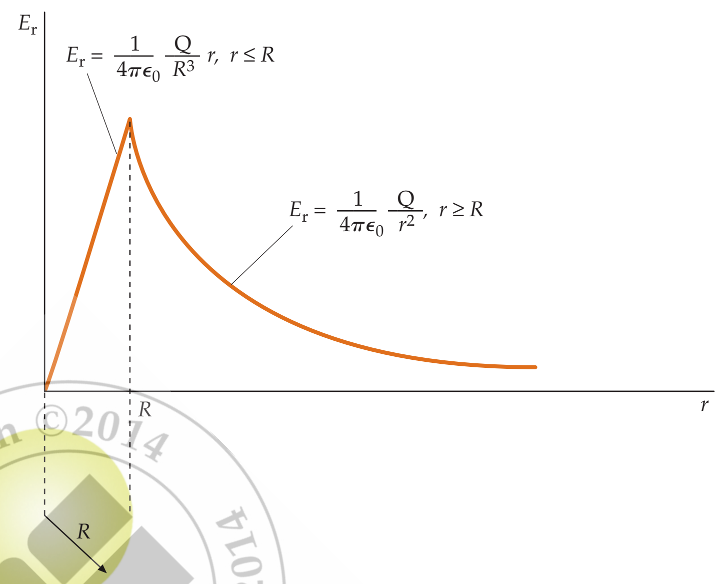


FIGURE 22-28

We see from Example 22-13 that the electric field a distance r from the center of a uniformly charged sphere of radius R is given by $\vec{E} = E_r\hat{r}$, where

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \qquad r \geq R \qquad \text{22-18}a$$

$$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}r \qquad r \leq R \qquad \text{22-18}b$$

and Q is the total charge of the sphere.

Example 22-14 Electric Field Due to Infinite Line Charge

Use Gauss's law to find the electric field everywhere due to an infinitely long line charge of uniform charge density λ. (This problem was already solved in Example 22-3 using Coulomb's law.)

PICTURE Because of the symmetry, we know the electric field is directed away from the line if λ is positive (directly toward it if λ is negative), and we know the magnitude of the field depends only on the radial distance from the line charge. We therefore choose a cylindrical Gaussian surface coaxial with the line charge. We calculate the outward flux of $\vec{E}$ through each piece of the surface, and, using Gauss's law, relate the net outward flux of $\vec{E}$ to the charge inside the cylinder.

SOLVE

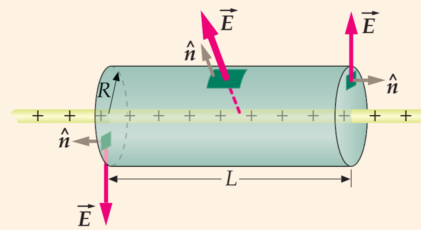


FIGURE 22-29

1. Sketch the wire and a coaxial cylindrical Gaussian surface (Figure 22-29) that has a length L and a radius R. The closed surface consists of three pieces: the two flat ends and the curved side. At a randomly chosen point on each piece, draw an area element and the vectors $\vec{E}$ and $\hat{n}$. Because of the symmetry, we know that the direction of $\vec{E}$ is radial (either toward or away from the line charge), and we know that the magnitude E depends only on the distance from the line charge.

2. Calculate the outward flux through the curved piece of the Gaussian surface. At each point on the curved piece $\hat{R} = \hat{n}$, where $\hat{R}$ is the unit vector in the radial direction.

$$\phi_{\text{curved}} = \vec{E} \cdot \hat{n} A_{\text{curved}} = \vec{E} \cdot \hat{R} A_{\text{curved}} = E_R 2\pi R L$$

3. Calculate the outward flux through each of the flat ends of the Gaussian surface. On these pieces the direction of $\hat{n}$ is parallel with the line charge (and thus perpendicular to $\vec{E}$):

$$\phi_{\text{left}} = \vec{E} \cdot \hat{n} A_{\text{left}} = 0$$

$$\phi_{\text{right}} = \vec{E} \cdot \hat{n} A_{\text{right}} = 0$$

4. Apply Gauss's law to relate the field to the total charge inside the surface Q_{inside}. The net flux out of the Gaussian surface is the sum of the fluxes out of the three pieces of the surface, and Q_{inside} is the charge on a length L of the line charge:

$$\phi_{\text{net}} = \frac{Q_{\text{inside}}}{\epsilon_0}$$

$$E_R 2\pi R L = \frac{\lambda L}{\epsilon_0} \quad \text{so} \quad \vec{E} = E_R \hat{R}, \quad \text{where} \quad \boxed{E_R = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{R}}$$

CHECK Because $1/(2\pi\epsilon_0) = 2k$, the step-4 result can also be written $2k\lambda/R$. This is the same expression for E_R that was obtained by using Coulomb's law (see Example 22-3).

In the calculation of $\vec{E}$ for a line charge (Example 22-14), we needed to assume that the field point was very far from the ends of the line charge so that E_n would be constant everywhere on the cylindrical Gaussian surface. If we are near the end of a finite line charge, we cannot assume that $\vec{E}$ is perpendicular to the curved surface of the cylinder, or that E_n is constant everywhere on it, so we cannot use Gauss's law to calculate the electric field.

It is important to realize that although Gauss's law holds for any closed surface and any charge distribution, it is particularly useful for calculating the electric fields of charge distributions that have cylindrical, spherical, or plane symmetry. It is also particularly useful doing calculations involving conductors in electrostatic equilibrium, as we shall see in Section 22-5.

22-4 DISCONTINUITY OF E_n

We have seen that the electric field for an infinite plane of charge and a thin spherical shell of charge is discontinuous by the amount σ/ϵ_0 at a surface having charge density σ. We now show that this is a general result for the component of the electric field that is perpendicular to a surface having a charge density of σ.

Figure 22-30 shows an arbitrary surface having a surface charge density σ. The surface is arbitrary in that it is arbitrarily curved, although it does not have any sharp folds where the normal direction is ambiguous, and σ may vary continuously on the surface from place to place. We consider electric field $\vec{E}$ in the vicinity of a point P on the surface as the superposition of electric field $\vec{E}_{\text{disk}}$, due just to the charge on a small disk centered at point P, and the electric field $\vec{E}'$ due to all other charges in the universe. Thus,

$$\vec{E} = \vec{E}_{\text{disk}} + \vec{E}' \qquad \text{22-19}$$

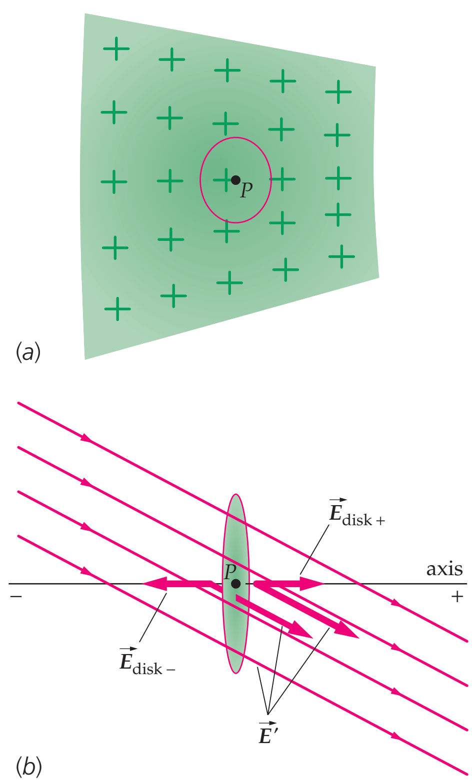


FIGURE 22-30 (*a*) A surface having surface charge. (*b*) The electric field $\vec{E}_{\text{disk}}$ due to the charge on a circular disk, plus the electric field $\vec{E}'$ due to all other charges.

The disk is small enough that it may be considered both flat and uniformly charged. On the axis of the disk, the electric field $\vec{E}_{\text{disk}}$ is given by Equation 22-9. At points on the axis very close to the disk, the magnitude of this field is given by $E_{\text{disk}} = |\sigma|/(2\epsilon_0)$. The direction of $\vec{E}_{\text{disk}}$ is away from the disk if σ is positive, and toward it if σ is negative. The magnitude and direction of the electric field $\vec{E}\,'$ are unknown. In the vicinity of point P, however, this field is continuous. Thus, at points on the axis of the disk and very close to it, $\vec{E}\,'$ is essentially uniform.

The axis of the disk is normal to the surface, so vector components along this axis can be referred to as normal components. The normal components of the vectors in Equation 22-19 are related by $E_n = E_{\text{disk}\,n} + E'_n$. If we refer to one side of the surface as the + side, and the other side as the − side, then $E_{n+} = \frac{\sigma}{2\epsilon_0} + E'_{n+}$ and $E_{n-} = -\frac{\sigma}{2\epsilon_0} + E'_{n+}$. Thus, E_n changes discontinuously from one side of the surface to the other. That is,

$$\Delta E_n = E_{n+} - E_{n-} = \frac{\sigma}{2\epsilon_0} - \left(-\frac{\sigma}{2\epsilon_0}\right) = \frac{\sigma}{\epsilon_0} \qquad \text{22-20}$$

DISCONTINUITY OF E_n AT A SURFACE CHARGE

where we have made use of the fact that near the disk $E'_{n+} = E'_{n-}$ (because $\vec{E}\,'$ is continuous and uniform).

Note that the discontinuity of E_n occurs at a finite disk of charge, an infinite plane of charge (see Figure 22-12), and a thin spherical shell of charge (see Figure 22-25). However, it does not occur at the perimeter of a solid sphere of charge (see Figure 22-28). The electric field is discontinuous at any location with an infinite volume charge density. These include locations that each have a finite point charge, locations that each have a finite line charge density, and locations that each have a finite surface charge density. At all locations with a finite surface charge density, the normal component of the electric field is discontinuous—in accord with Equation 22-20.

22-5 CHARGE AND FIELD AT CONDUCTOR SURFACES

A conductor contains an enormous amount of charge that can move freely within the conductor. If there is an electric field within a conductor, there will be a net force on this free charge causing a momentary electric current (electric currents are discussed in Chapter 25). However, unless there is a source of energy to maintain this current, the free charge in a conductor will merely redistribute itself to create an electric field that cancels the external field within the conductor. The conductor is then said to be in **electrostatic equilibrium.** Thus, in electrostatic equilibrium, the electric field inside a conductor is zero everywhere. The time taken to reach equilibrium depends on the conductor. For copper and other metal conductors, the time is so small that in most cases electrostatic equilibrium is reached in a few nanoseconds.*

We can use Gauss's law to show that for a conductor in electrostatic equilibrium, any net electric charge on the conductor resides entirely on the surface of the conductor. Consider a Gaussian surface completely inside the material of a conductor in electrostatic equilibrium (Figure 22-31). The size and shape of the Gaussian surface do not matter, as long as the entire surface is embedded within the material of the conductor. The electric field is zero everywhere on the Gaussian surface because the surface is completely within the conductor, where the field is everywhere zero. The net flux of the electric field through the surface must therefore be zero, and, by Gauss's law, the net charge inside the surface must be zero. Thus, there can be no

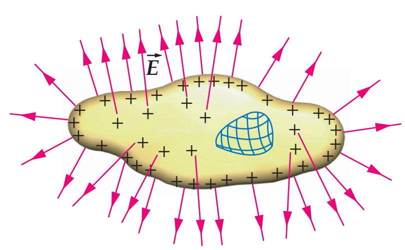


FIGURE 22-31 A Gaussian surface completely within the material of a conductor. Because the electric field is zero inside the material a conductor in electrostatic equilibrium, the net flux through this surface must also be zero. Therefore, the net charge density ρ must be zero everywhere within the material of a conductor.

* At very low temperatures some metals become superconducting. In a superconductor, a current is sustained for a much longer time, even without an energy source. Superconducting metals are discussed in Chapters 27 and 38.

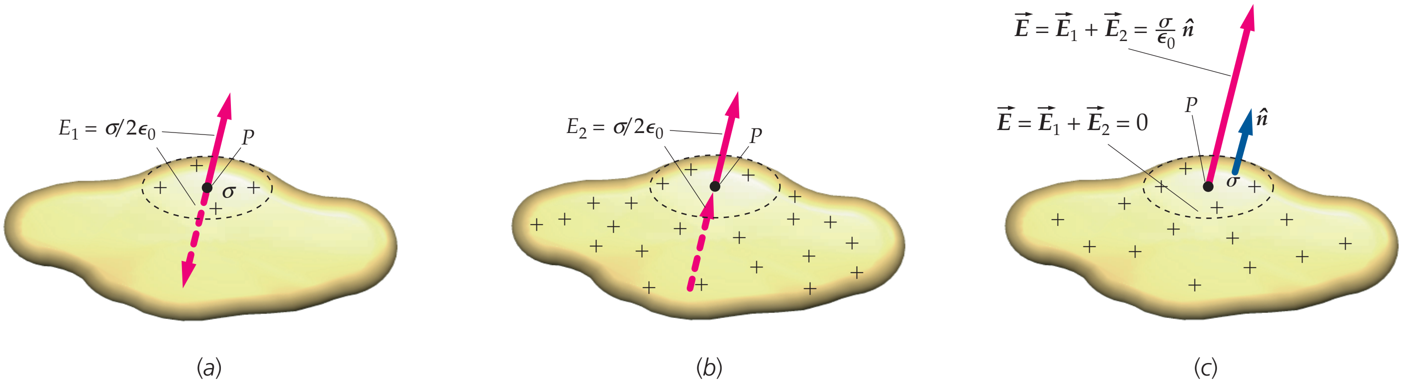


FIGURE 22-32 An arbitrarily shaped conductor having a charge on its surface. (*a*) The charge in the vicinity of point *P* near the surface looks like a small uniformly charged circular disk centered at *P*, giving an electric field of magnitude $\sigma/(2\epsilon_0)$ pointing away from the surface both inside and outside the surface. Inside the conductor, this field points away from point *P* in the opposite direction. (*b*) Because the net field inside the conductor is zero, the rest of the charges in the universe must produce a field of magnitude $\sigma/(2\epsilon_0)$ in the outward direction. The field due to these charges is the same just inside the surface as it is just outside the surface. (*c*) Inside the surface, the fields shown in (*a*) and (*b*) cancel, but outside they add to give $E_n = \sigma/\epsilon_0$.

net charge inside any surface lying completely within the material of the conductor. Therefore, if a conductor has a net charge, it must reside on the conductor's surface. At the surface of a conductor in electrostatic equilibrium, $\vec{E}$ must be perpendicular to the surface. (If the electric field did have a tangential component at the surface, the free charge would be accelerated tangential to the surface until electrostatic equilibrium was reestablished.)

Because E_n is discontinuous at any charged surface by the amount σ/ϵ_0, and because $\vec{E}$ is zero inside the material of a conductor, the field just outside the surface of a conductor is given by

$$E_n = \frac{\sigma}{\epsilon_0} \qquad 22\text{-}21$$

E_n JUST OUTSIDE THE SURFACE OF A CONDUCTOR

This result is exactly twice the field produced by a uniform disk of surface charge density σ. We can understand this result from Figure 22-32. The charge on the conductor consists of two parts: (1) the charge near point *P* and (2) all the rest of the charge. The charge near point *P* looks like a small, uniformly charged circular disk centered at *P* that produces a field near *P* of magnitude $\sigma/(2\epsilon_0)$ just inside and just outside the conductor. The rest of the charges in the universe must produce a field of magnitude $\sigma/(2\epsilon_0)$ that exactly cancels the field inside the conductor. This field due to the rest of the charge in the universe adds to the field due to the small charged disk just outside the conductor to give a total field of σ/ϵ_0.

Example 22-15 The Charge of Earth *Context-Rich*

While watching a science show on the atmosphere, you find out that on average the electric field of Earth is about 100 N/C directed vertically downward. Given that you have been studying electric fields in your physics class, you wonder if you can determine what the total charge on Earth's surface is.

PICTURE Earth is a conductor, so any charge it has resides on the surface of Earth. The surface charge density σ is related to the normal component of the electric field E_n by Equation 22-21. The total charge Q equals the charge density σ multiplied by the surface area A.

SOLVE

1. The surface charge density σ is related to the normal component of the electric field E_n by Equation 22-21: $\quad E_n = \dfrac{\sigma}{\epsilon_0}$

2. On the surface of Earth, $\hat{n}$ is upward and $\vec{E}$ is downward so E_n is negative: $\quad E_n = \vec{E}\cdot\hat{n} = E\cos 180° = -E = -100\text{ N/C}$

3. The charge Q is the charge per unit area multiplied by the area. Combine this with the step 1 and 2 results to obtain an expression for Q: $\quad Q = \sigma A = \epsilon_0 E_n A = -\epsilon_0 EA$

4. The surface area of a sphere of radius r is given by $A = 4\pi r^2$: $\quad Q = -\epsilon_0 EA = -\epsilon_0 E4\pi R_E^2 = -4\pi\epsilon_0 ER_E^2$

5. The radius of Earth is 6.37×10^6 m:

$$Q = -4\pi\epsilon_0 ER_E^2$$
$$= -4\pi(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(100\ \mathrm{N/C})(6.37 \times 10^6\ \mathrm{m})^2$$
$$= \boxed{-4.51 \times 10^5\ \mathrm{C}}$$

CHECK We will check to see if units in the step 5 calculation are correct. In multiplying the three quantities, both the newtons and the meters cancel out, leaving only coulombs as expected.

TAKING IT FURTHER Is -4.53×10^5 C a large amount of charge? In Example 21-1 we calculated that the total charge of all the electrons in a copper penny amounts to -1.37×10^5 C, so the total charge on the surface of Earth is only 3.3 times larger than the total charge of all the electrons in a single copper penny.

Figure 22-33 shows a positive point charge q at the center of a spherical cavity inside a spherical conductor. Because the net charge must be zero within any Gaussian surface drawn within the material of the conductor, there must be a negative charge $-q$ induced on the surface of the cavity. In Figure 22-34, the point charge has been moved so that it is no longer at the center of the cavity. The field lines in the cavity are altered, and the surface charge density of the induced negative charge on the inner surface is no longer uniform. However, the positive surface charge density on the outside surface is not disturbed—it is still uniform—because it is electrically shielded from the cavity by the conducting material. The electric field of the point charge q and that of the surface charge $-q$ on the inner surface of the cavity superpose to produce an electric field that is exactly zero everywhere outside the cavity. This is obviously true if the point charge is at the center of the cavity, but it is true even if the point charge is somewhere else in the cavity. In addition, the surface charge on the outer surface of the conductor produces an electric field that is exactly zero everywhere inside the outer surface of the conductor. Furthermore, these statements are valid even if both the outer surface and the inner surface of the conductor are nonspherical.

Electric field lines for an oppositely charged cylinder and plate, shown by bits of fine thread suspended in oil. Note that the field lines are normal to the surfaces of the conductors and that there are no lines inside the cylinder. The region inside the cylinder is electrically shielded from the region outside the cylinder. *(Harold M. Waage.)*

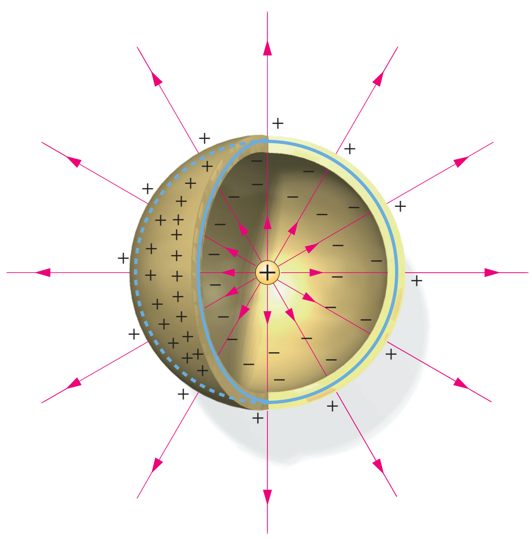


FIGURE 22-33 A point charge q in the cavity at the center of a thick spherical conducting shell. Because the net charge within the Gaussian surface (indicated in blue) must be zero, we know a surface charge $-q$ is induced on the inner surface of the shell, and because the conductor is neutral, an equal but opposite charge $+q$ is induced on the outer surface. Electric field lines begin on the point charge and end on the inner surface. Field lines begin again on the outer surface.

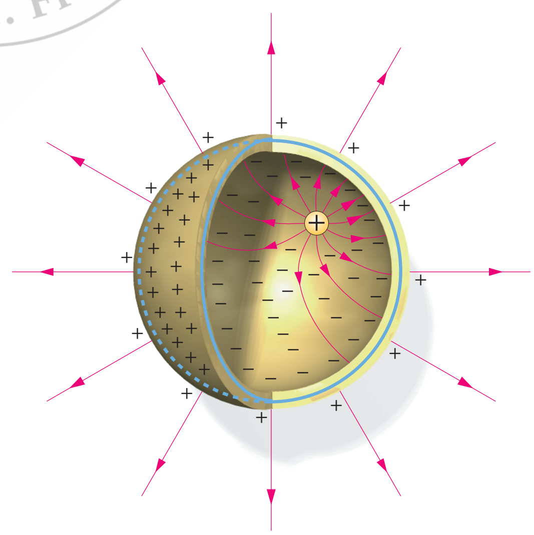


FIGURE 22-34 The same conductor as in Figure 22-33 with the point charge moved away from the center of the sphere. The charge on the outer surface and the electric field lines outside the sphere are not affected.

* 22-6 THE EQUIVALENCE OF GAUSS'S LAW AND COULOMB'S LAW IN ELECTROSTATICS

Gauss's law can be derived mathematically from Coulomb's law for the electrostatic case using the concept of the **solid angle.** Consider an area element ΔA on a spherical surface. The solid angle $\Delta\Omega$ subtended by ΔA at the center of the sphere is defined to be

$$\Delta\Omega = \frac{\Delta A}{r^2}$$

where r is the radius of the sphere. Because ΔA and r^2 both have dimensions of length squared, the solid angle is dimensionless. The SI unit of the solid angle is the **steradian** (sr). Because the total area of a sphere is $4\pi r^2$, the total solid angle subtended by a spherical surface is

$$\frac{4\pi r^2}{r^2} = 4\pi \text{ steradians}$$

There is a close analogy between the solid angle and the ordinary plane angle $\Delta\theta$, which is defined to be the ratio of an element of arc length of a circle Δs to the radius of the circle:

$$\Delta\theta = \frac{\Delta s}{r} \text{ radians}$$

The total plane angle subtended by a circle is 2π radians.

In Figure 22-35, the area element ΔA is not perpendicular to the radial lines from point O. The unit vector $\hat{n}$ normal to the area element makes an angle θ with the radial unit vector $\hat{r}$. In this case, the solid angle subtended by ΔA at point O is

$$\Delta\Omega = \frac{\Delta A\, \hat{\boldsymbol{n}} \cdot \hat{\boldsymbol{r}}}{r^2} = \frac{\Delta A \cos\theta}{r^2} \qquad 22\text{-}22$$

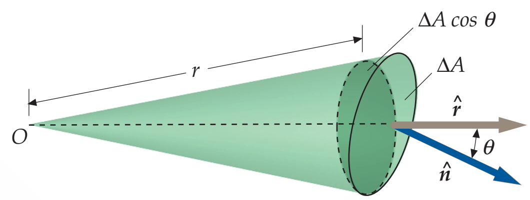


FIGURE 22-35 An area element ΔA whose normal is not parallel to the radial line from O to the center of the element. The solid angle subtended by this element at O is defined to be $(\Delta A \cos\theta)/r^2$.

The solid angle $\Delta\Omega$ is the same as that subtended by the corresponding area element of a spherical surface of any radius.

Figure 22-36 shows a point charge q surrounded by a surface of arbitrary shape. To calculate the flux of $\vec{E}$ through this surface, we want to find $\vec{E} \cdot \hat{n}\Delta A$ for each element of area on the surface and sum over the entire surface. The electric field at the area element shown is given by

$$\vec{E} = \frac{kq}{r^2}\hat{\boldsymbol{r}}$$

so the flux through the element is

$$\Delta\phi = \vec{E} \cdot \hat{\boldsymbol{n}}\Delta A = \frac{kq}{r^2}\hat{\boldsymbol{r}} \cdot \hat{\boldsymbol{n}}\Delta A = kq\, \Delta\Omega$$

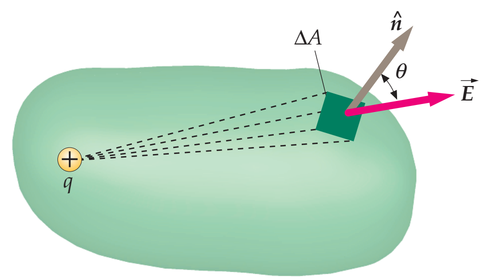


FIGURE 22-36 A point charge enclosed by an arbitrary surface. The flux through an area element ΔA is proportional to the solid angle subtended by the area element at the charge. The net flux through the surface, found by summing over all the area elements, is proportional to the total solid angle 4π at the charge, which is independent of the shape of the surface.

The sum of the fluxes through the entire surface is kq multiplied by the total solid angle subtended by the closed surface, which is 4π steradians:

$$\phi_{\text{net}} = \oint_S \vec{E} \cdot \hat{\boldsymbol{n}}\, dA = kq \oint d\Omega = kq4\pi = 4\pi kq = \frac{q}{\epsilon_0} \qquad 22\text{-}23$$

which is Gauss's law.

Physics Spotlight

Charge Distribution—Hot and Cold

Electrical dipole moment, or *polarity*, affects the solubility of substances. Because water has such a strong electric dipole moment, it works very well as a solvent for other molecules that have both weak and strong dipole moments and ions. On the other hand, molecules without dipole moments, or molecules that are so big that they have large regions without dipole moments, do not dissolve well in water. Some oils, for example, do not have dipole moments and are immiscible with water.

Charge distributions that molecules can have also control whether substances that are not strictly classified as oils dissolve well in water. Anyone who has ever bitten into a very spicy-hot pepper and then taken a large drink of water can testify that water does not wash away the sensation of pain. Capsaicin, the active chemical in spicy-hot peppers such as habañeros, serranos, and piquins, does not dissolve well in cold water because of its charge distribution.[*] However, capsaicin's solubility in water is increased with the addition of ethyl alcohol, as demonstrated by people who cool their mouths with beer after spicy-hot peppers. Alcohol molecules have weak dipole moments, and mix well with both water and capsaicin. Capsaicin also mixes well with oils, some starches, and proteins. In many cultures, rice or meat, rather than alcohol, is used to dissolve capsaicin.

The molecules of the active ingredient in these spicy hot peppers do not dissolve in water because they do not have electric dipole moments. *(Stockbyte Platinum/Getty Images.)*

The sensation of pain that people who eat peppers feel is also due to the charge distributions in molecules. The protein TRPV1 is a neuron receptor in humans that signals how hot—temperature wise—something is. This protein has a charge distribution that is changed by a temperature above 43°C. Proteins change their shapes (fold and unfold) as the charge distribution changes across the molecule.[†] Many protein functions are determined by folding and unfolding caused by changes in charge distributions.[‡] A change in charge distribution on a TRPV1 protein folds the protein and passes information about how hot a human's surroundings are to neurons. Capsaicin creates the same changes as heat does in the charge distributions of TRPV1 proteins,[#] which is why people perceive peppers as hot. Ginger, a "warm" spice, contains gingerols, which trigger similar receptors by means of changing charge distributions.[°] Menthol causes similar charge distribution changes in proteins that are neuron receptors in humans and signal how cold surroundings are.[§] This is why people perceive mint as cool.

Changes in charge distributions of proteins can cause textural changes in proteins. The salting of caviar, for example, changes the charge distribution of proteins inside the fish eggs. As the proteins unfold, they thicken the formerly thin fluid inside the egg to a creamy texture.[¶]

* Turgut, C., Newby, B., and Cutright, T., "Determination of Optimal Water Solubility of Capsaicin for Its Usage as a Non-Toxic Antifoulant." *Environmental Science Pollution Research International*, Jan.-Feb. 2004, Vol. 11, No. 1, pp. 7–10.

† Suydam, I. T., et al., "Electric Fields at the Active Site of an Enzyme: Direct Comparison of Experiment with Theory." *Science*, Jul. 14, 2006, Vol. 313, No. 5784, pp. 200–204.

‡ Honig, B., and Nicholls, A., "Classical Electrostatics in Biology and Chemisty." *Science*, May 26, 1995, Vol. 268, p. 1144.

\# Montell, C., "Thermosensation: Hot Findings Make TRPNs Very Cool." *Current Biology*, Jun. 17, 2003, Vol. 13, No. 12, pp. R476–R478.

° Dedov, V. N., et al., "Gingerols: A Novel Class of Vanilloid Receptor (VR1) Agonists." *British Journal of Pharmacology*, 2002, Vol. 137, pp. 793–798.

§ Montell, C., op. cit.

¶ Sternin, V., and Dorè, I, *Caviar: The Resource Book.* Moscow: Cultura, 1993, in McGee, H., *On Food and Cooking: The Science and Lore of the Kitchen.* New York: Scribner, 2004.

Summary

1. Gauss's law is a fundamental law of physics that is equivalent to Coulomb's law for static charges.
2. For highly symmetric charge distributions, Gauss's law can be used to calculate the electric field.

TOPIC	RELEVANT EQUATIONS AND REMARKS	
1. Electric Field for a Continuous Charge Distribution	$\vec{E} = \int d\vec{E} = \int \frac{k\hat{r}}{r^2}\,dq$ (Coulomb's law)	22-1*b*
	where $dq = \rho\,dV$ for a charge distributed throughout a volume, $dq = \sigma\,dA$ for a charge distributed on a surface, and $dq = \lambda\,dL$ for a charge distributed along a line.	
2. Electric Flux	$\phi = \lim_{\Delta A_i \to 0} \sum_i \vec{E}_i \cdot \hat{n}_i \Delta A_i = \int_S \vec{E} \cdot \hat{n}\,dA$	22-13
3. Gauss's Law	$\phi_{\text{net}} = \oint_S \vec{E} \cdot \hat{n}\,dA = \oint_S E_n\,dA = \frac{Q_{\text{inside}}}{\epsilon_0}$	22-16
	The net outward electric flux through a closed surface equals the net charge within the surface divided by ϵ_0.	
4. Coulomb Constant k and Electric Constant (Permittivity of Free Space) ϵ_0	$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$	
	$\epsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)$	22-7
5. Coulomb's Law and Gauss's Law	$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$	22-5
	$\phi_{\text{net}} = \oint_S E_n\,dA = \frac{Q_{\text{inside}}}{\epsilon_0}$	22-16
6. Discontinuity of E_n	At a surface having a surface charge density σ, the component of the electric field normal to the surface is discontinuous by σ/ϵ_0.	
	$E_{n+} - E_{n-} = \frac{\sigma}{\epsilon_0}$	22-20
7. Charge on a Conductor	In electrostatic equilibrium, the charge density is zero throughout the material of the conductor. All excess or deficit charge resides on the surfaces of the conductor.	
8. $\vec{E}$ Just Outside a Conductor	The resultant electric field just outside the surface of a conductor is normal to the surface and has the magnitude σ/ϵ_0, where σ is the local surface charge density on the conductor:	
	$E_n = \frac{\sigma}{\epsilon_0}$	22-21
9. Electric Fields for Selected Uniform Charge Distributions		
Of a line charge of infinite length	$E_R = 2k\frac{\lambda}{R} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{R}$	22-6
On the axis of a charged ring	$E_z = \frac{kQz}{(z^2 + a^2)^{3/2}}$	22-8
On the axis of a charged disk	$E_z = \text{sign}(z) \cdot \frac{\sigma}{2\epsilon_0}\left[1 - \left(1 + \frac{R^2}{z^2}\right)^{-1/2}\right]$	22-9

TOPIC	RELEVANT EQUATIONS AND REMARKS		
Of a charged infinite plane	$E_z = \text{sign}(z) \cdot \dfrac{\sigma}{2\epsilon_0}$		22-10
Of a charged thin spherical shell	$E_r = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}$	$r > R$	22-17*a*
	$E_r = 0$	$r < R$	22-17*b*

Answers to Concept Checks

22-1 The $\vec{E}$ in Gauss's law is the electric field due to all charges. However, the flux of the electric field due to all the charges outside the surface equals zero, so the flux of the electric field due to all charges equals the flux of the field due to the charges inside the surface alone.

Answers to Practice Problems

22-1 $E_x = k\lambda\left(\dfrac{1}{r_2} - \dfrac{1}{r_1}\right)$. For $x > x_2$, $r_2 < r_1$ so $\dfrac{1}{r_2} > \dfrac{1}{r_1}$ which means $E_x > 0$.

22-2 No. Symmetry dictates that E_z is zero at $z = 0$ whereas the equation in step 3 gives a negative value for E_z at $z = 0$. These contradictory results cannot both be valid.

22-3 The SI units for k, λ, and R are $\text{N} \cdot \text{m}^2/\text{C}^2$, C/m, and m, respectively. It follows that $k\lambda/R$ has units of $(\text{N} \cdot \text{m}^2/\text{C}^2)(\text{C/m})(1/\text{m}) = \text{N/C}$.

22-4 $z = a/\sqrt{2}$

22-5 80%

Problems

In a few problems, you are given more data than you actually need; in a few other problems, you are required to supply data from your general knowledge, outside sources, or informed estimate.

Interpret as significant all digits in numerical values that have trailing zeros and no decimal points.

- • Single-concept, single-step, relatively easy
- •• Intermediate-level, may require synthesis of concepts
- ••• Challenging
- SSM Solution is in the *Student Solutions Manual*
- Consecutive problems that are shaded are paired problems.

CONCEPTUAL PROBLEMS

1 • Figure 22-37 shows an L-shaped object that has sides which are equal in length. Positive charge is distributed uniformly along the length of the object. What is the direction of the electric field along the dashed 45° line? Explain your answer. SSM

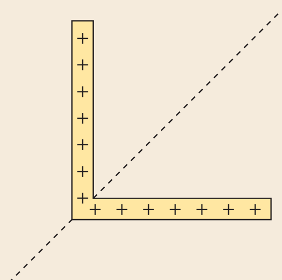


FIGURE 22-37 Problem 1

2 • Positive charge is distributed uniformly along the entire length of the x axis, and negative charge is distributed uniformly along the entire length of the y axis. The charge per unit length on the two axes is identical, except for the sign. Determine the direction of the electric field at points on the lines defined by $y = x$ and $y = -x$. Explain your answer.

3 • True or false:

(*a*) The electric field due to a hollow uniformly charged thin spherical shell is zero at all points inside the shell.
(*b*) In electrostatic equilibrium, the electric field everywhere inside the material of a conductor must be zero.
(*c*) If the net charge on a conductor is zero, the charge density must be zero at every point on the surface of the conductor.

4 • If the electric flux through a closed surface is zero, must the electric field be zero everywhere on that surface? If not, give a specific example. From the given information can the net charge inside the surface be determined? If so, what is it?

5 • True or false:

(*a*) Gauss's law holds only for symmetric charge distributions.
(*b*) The result that $E = 0$ everywhere inside the material of a conductor under electrostatic conditions can be derived from Gauss's law.

6 •• A single point charge q is located at the center of both an imaginary cube and an imaginary sphere. How does the electric flux through the surface of the cube compare to that through the surface of the sphere? Explain your answer.

7 •• An electric dipole is completely inside a closed imaginary surface and there are no other charges. True or false:
(*a*) The electric field is zero everywhere on the surface.
(*b*) The electric field is normal to the surface everywhere on the surface.
(*c*) The electric flux through the surface is zero.
(*d*) The electric flux through the surface could be positive or negative.
(*e*) The electric flux through a portion of the surface might not be zero. SSM

8 •• Explain why the electric field strength increases linearly with r, rather than decreases inversely with r^2, between the center and the surface of a uniformly charged solid sphere.

9 •• Suppose that the total charge on the conducting spherical shell in Figure 22-38 is zero. The negative point charge at the center has a magnitude given by Q. What is the direction of the electric field in the following regions? (*a*) $r < R_1$, (*b*) $R_2 > r > R_1$, (*c*) and $r > R_2$. Explain your answer. SSM

10 •• The conducting shell in Figure 22-38 is grounded, and the negative point charge at the center has a magnitude given by Q. Which of the following statements is correct?
(*a*) The charge on the inner surface of the shell is $+Q$ and the charge on the outer surface is $-Q$.
(*b*) The charge on the inner surface of the shell is $+Q$ and the charge on the outer surface is zero.
(*c*) The charge on both surfaces of the shell is $+Q$.
(*d*) The charge on both surfaces of the shell is zero.

11 •• The conducting shell in Figure 22-38 is grounded, and the negative point charge at the center has a magnitude given by Q. What is the direction of the electric field in the following regions? (*a*) $r < R_1$, (*b*) $R_2 > r > R_1$, (*c*) and $r > R_2$. Explain your answers.

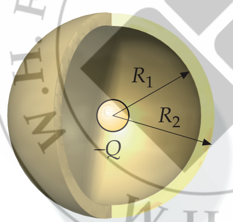


FIGURE 22-38 Problems 9, 10, and 11

ESTIMATION AND APPROXIMATION

12 •• In the chapter, the expression for the electric field due to a uniformly charged disk (on its axis), was derived. At any location on the axis, the field magnitude is given by $|E| = 2\pi k\sigma\left[1-\left(1+\frac{R^2}{z^2}\right)^{-1/2}\right]$. At large distances ($|z| \gg R$), it was shown that this equation approaches $E \approx kQ/z^2$. Very near the disk ($|z| \ll R$), the field strength is approximately that of an infinite plane of charge or $|E| \approx 2\pi k\sigma$. Suppose you have a disk of radius 2.5 cm that has a uniform surface charge density of 3.6 μC/m^2. Use both the exact and approximate expression from those given above to find the electric field strength on the axis at distances of (*a*) 0.010 cm, (*b*) 0.040 cm, and (*c*) 5.0 m. Compare the two values in each case and comment on how well the approximations work in their region of validity.

CALCULATING $\vec{E}$ FROM COULOMB'S LAW

13 • A uniform line charge that has a linear charge density λ equal to 3.5 nC/m is on the x axis between $x = 0$ and $x = 5.0$ m. (*a*) What is its total charge? Find the electric field on the x axis at (*b*) $x = 6.0$ m, (*c*) $x = 9.0$ m, and (*d*) $x = 250$ m. (*e*) Estimate the electric field at $x = 250$ m, using the approximation that the charge is a point charge on the x axis at $x = 2.5$ m, and compare your result with the result calculated in Part (*d*). (To do this, you will need to assume that the values given in this problem statement are valid to more than two significant figures.) Is your approximate result greater or smaller than the exact result? Explain your answer. SSM

14 • Two infinite nonconducting sheets of charge are parallel to each other, with sheet A in the $x = -2.0$ m plane and sheet B in the $x = +2.0$ m plane. Find the electric field in the region $x < -2.0$ m, in the region $x > +2.0$ m, and between the sheets for the following situations. (*a*) When each sheet has a uniform surface charge density equal to $+3.0\ \mu$C/m^2 and (*b*) when sheet A has a uniform surface charge density equal to $+3.0\ \mu$C/m^2 and sheet B has a uniform surface charge density equal to $-3.0\ \mu$C/m^2. (c) Sketch the electric field line pattern for each case.

15 • A charge of 2.75 μC is uniformly distributed on a ring of radius 8.5 cm. Find the electric field strength on the axis at distances of (*a*) 1.2 cm, (*b*) 3.6 cm, and (*c*) 4.0 m from the center of the ring. (*d*) Find the field strength at 4.0 m using the approximation that the ring is a point charge at the origin, and compare your results for Parts (*c*) and (*d*). Is your approximate result a good one? Explain your answer.

16 • A nonconducting disk of radius R lies in the $z = 0$ plane with its center at the origin. The disk has a uniform surface charge density σ. Find the value of z for which $E_z = \sigma/(4\epsilon_0)$. Note that at this distance, the magnitude of the electric field strength is half the electric field strength at points on the x axis that are very close to the disk.

17 • A ring that has radius a lies in the $z = 0$ plane with its center at the origin. The ring is uniformly charged and has a total charge Q. Find E_z on the z axis at (*a*) $z = 0.2a$, (*b*) $z = 0.5a$, (*c*) $z = 0.7a$, (*d*) $z = a$, and (*e*) $z = 2a$. (*f*) Use your results to plot E_z versus z for both positive and negative values of z. (Assume that these distances are exact.) SSM

18 • A nonconducting disk of radius a lies in the $z = 0$ plane with its center at the origin. The disk is uniformly charged and has a total charge Q. Find E_z on the z axis at (*a*) $z = 0.2a$, (*b*) $z = 0.5a$, (*c*) $z = 0.7a$, (*d*) $z = a$, and (*e*) $z = 2a$. (*f*) Use your results to plot E_z versus z for both positive and negative values of z. (Assume that these distances are exact.)

19 •• **SPREADSHEET** (*a*) Using a **spreadsheet** program or graphing calculator, make a graph of the electric field strength on the axis of a disk that has a radius $a = 30.0$ cm and a surface charge density $\sigma = 0.500$ nC/m^2. (*b*) Compare your results to the results based on the approximation $E = 2\pi k\sigma$ (the formula for the electric field strength of a uniformly charged infinite sheet). At what distance does the solution based on approximation differ from the exact solution by 10.0 percent?

20 •• (*a*) Show that the electric field strength E on the axis of a ring charge of radius a has maximum values at $z = \pm a/\sqrt{2}$. (*b*) Sketch the field strength E versus z for both positive and negative values of z. (*c*) Determine the maximum value of E.

21 •• A line charge that has a uniform linear charge density λ lies along the x axis from $x = x_1$ to $x = x_2$ where $x_1 < x_2$. Show that the x component of the electric field at a point on the y axis is given by $E_x = \frac{k\lambda}{y}(\cos\theta_2 - \cos\theta_1)$ where $\theta_1 = \tan^{-1}(x_1/y)$, $\theta_2 = \tan^{-1}(x_2/y)$, and $y \neq 0$.

22 •• A ring of radius a has a charge distribution on it that varies as $\lambda(\theta) = \lambda_0 \sin\theta$, as shown in Figure 22-39. (*a*) What is the direction of the electric field at the center of the ring? (*b*) What is the magnitude of the field at the center of the ring?

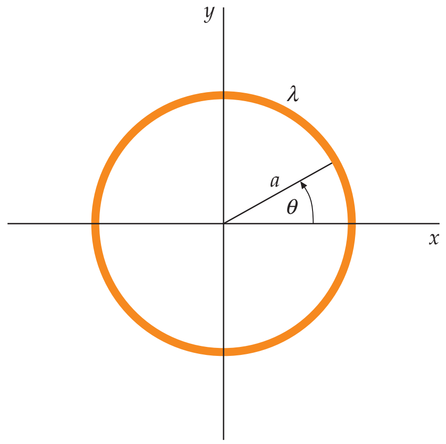


FIGURE 22-39 Problem 22

23 •• A line of charge that has uniform linear charge density λ lies on the x axis from $x = 0$ to $x = a$. Show that the y component of the electric field at a point on the y axis is given by $E_y = \dfrac{k\lambda}{y}\dfrac{a}{\sqrt{y^2 + a^2}}, y \neq 0$.

24 ••• Calculate the electric field a distance z from a uniformly charged infinite flat nonconducting sheet by modeling the sheet as a continuum of infinite straight lines of charge.

25 •• Calculate the electric field a distance z from a uniformly charged infinite flat nonconducting sheet by modeling the sheet as a continuum of infinite circular rings of charge. SSM

26 ••• A thin hemispherical shell of radius R has a uniform surface charge σ. Find the electric field at the center of the base of the hemispherical shell.

GAUSS'S LAW

27 • A square that has 10-cm-long edges is centered on the x axis in a region where there exists a uniform electric field given by $\vec{E} = (2.00\ \text{kN/C})\hat{i}$. (*a*) What is the electric flux of this electric field through the surface of a square if the normal to the surface is in the $+x$ direction? (*b*) What is the electric flux through the same square surface if the normal to the surface makes a 60° angle with the y axis and an angle of 90° with the z axis?

28 • A single point charge ($q = +2.00\ \mu\text{C}$) is fixed at the origin. An imaginary spherical surface of radius 3.00 m is centered on the x axis at $x = 5.00$ m. (*a*) Sketch electric field lines for this charge (in two dimensions) assuming twelve equally spaced field lines in the xy plane leave the charge location, with one of the lines in the $+x$ direction. Do any lines enter the spherical surface? If so, how many? (*b*) Do any lines leave the spherical surface? If so, how many? (*c*) Counting the lines that enter as negative and the ones that leave as positive, what is the net number of field lines that penetrate the spherical surface? (*d*) What is the net electric flux through this spherical surface?

29 • An electric field is given by $\vec{E} = \text{sign}(x) \cdot (300\ \text{N/C})\hat{i}$, where sign (x) equals -1 if $x < 0$, 0 if $x = 0$, and $+1$ if $x > 0$. A cylinder of length 20 cm and radius 4.0 cm has its center at the origin and its axis along the x axis such that one end is at $x = +10$ cm and the other is at $x = -10$ cm. (*a*) What is the electric flux through each end? (*b*) What is the electric flux through the curved surface of the cylinder? (*c*) What is the electric flux through the entire closed surface? (*d*) What is the net charge inside the cylinder? SSM

30 • Careful measurement of the electric field at the surface of a black box indicates that the net outward electric flux through the surface of the box is 6.0 kN·m²/C. (*a*) What is the net charge inside the box? (*b*) If the net outward electric flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Explain your answer.

31 • A point charge ($q = +2.00\ \mu\text{C}$) is at the center of an imaginary sphere that has a radius equal to 0.500 m. (*a*) Find the surface area of the sphere. (*b*) Find the magnitude of the electric field at all points on the surface of the sphere. (*c*) What is the flux of the electric field through the surface of the sphere? (*d*) Would your answer to Part (*c*) change if the point charge were moved so that it was inside the sphere but not at its center? (*e*) What is the flux of the electric field through the surface of an imaginary cube that has 1.00-m-long edges and encloses the sphere?

32 • What is the electric flux through one side of a cube that has a single point charge of $-3.00\ \mu\text{C}$ placed at its center? *Hint: You do not need to integrate any equations to get the answer.*

33 • A single point charge is placed at the center of an imaginary cube that has 20-cm-long edges. The electric flux out of one of the cube's sides is -1.50 kN·m²/C. How much charge is at the center? SSM

34 •• Because the formulas for Newton's law of gravity and for Coulomb's law have the same inverse-square dependence on distance, a formula analogous to the formula for Gauss's law can be found for gravity. The gravitational field $\vec{g}$ at a location is the force per unit mass on a test mass m_0 placed at that location. (Then, for a point mass m at the origin, the gravitational field g at some position $\hat{r}$ is $\vec{g} = -(Gm/r^2)\hat{r}$.) Compute the flux of the gravitational field through a spherical surface of radius R centered at the origin, and verify that the gravitational analog of Gauss's law is $\phi_{\text{net}} = -4\pi G m_{\text{inside}}$.

35 •• An imaginary right circular cone (Figure 22-40) that has a base angle θ and a base radius R is in charge free region that has a uniform electric field $\vec{E}$ (field lines are vertical and parallel to the cone's axis). What is the ratio of the number of field lines per unit area penetrating the base to the number of field lines per unit area penetrating the conical surface of the cone? Use Gauss's law in your answer. (The field lines in the figure are only a representative sample.)

36 •• In the atmosphere and at an altitude of 250 m, you measure the electric field to be 150 N/C directed downward, and you measure the electric field to be 170 N/C directed downward at an altitude of 400 m. Calculate the volume charge density of the atmosphere in the region between altitudes of 250 m and 400 m, assuming it to be uniform. (You may neglect the curvature of Earth. Why?)

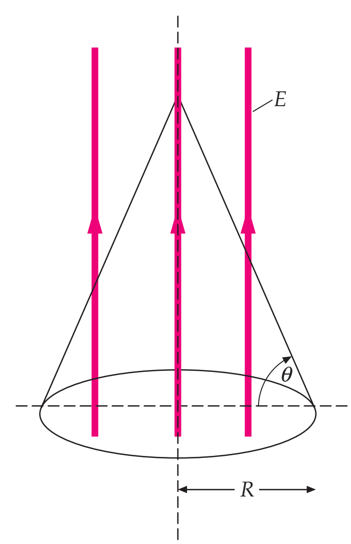


FIGURE 22-40
Problem 35

GAUSS'S LAW APPLICATIONS IN SPHERICAL SYMMETRY SITUATIONS

37 • A thin nonconducting spherical shell of radius R_1 has a total charge q_1 that is uniformly distributed on its surface. A second, larger thin nonconducting spherical shell of radius R_2 that is coaxial with the first has a charge q_2 that is uniformly distributed on its surface. (*a*) Use Gauss's law to obtain expressions for the electric field in each of the three regions: $r < R_1$, $R_1 < r < R_2$, and $r > R_2$. (*b*) What should the ratio of the charges q_1/q_2 and the relative signs for q_1 and q_2 be for the electric field to be zero throughout the region $r > R_2$? (*c*) Sketch the electric field lines for the situation in Part (*b*) when q_1 is positive.

38 • A nonconducting thin spherical shell of radius 6.00 cm has a uniform surface charge density of 9.00 nC/m^2. (*a*) What is the total charge on the shell? Find the electric field at the following distances from the sphere's center: (*b*) 2.00 cm, (*c*) 5.90 cm, (*d*) 6.10 cm, and (*e*) 10.0 cm.

39 •• A nonconducting sphere of radius 6.00 cm has a uniform volume charge density of 450 nC/m^3. (*a*) What is the total charge on the sphere? Find the electric field at the following distances from the sphere's center: (*b*) 2.00 cm, (*c*) 5.90 cm, (*d*) 6.10 cm, and (*e*) 10.0 cm. SSM

40 •• Consider the solid conducting sphere and the concentric conducting spherical shell in Figure 22-41. The spherical shell has a charge $-7Q$. The solid sphere has a charge $+2Q$. (*a*) How much charge is on the outer surface and how much charge is on the inner surface of the spherical shell? (*b*) Suppose a metal wire is now connected between the solid sphere and the shell. After electrostatic equilibrium is reestablished, how much charge is on the solid sphere and on each surface of the spherical shell? Does the electric field at the surface of the solid sphere change when the wire is connected? If so, in what way? (*c*) Suppose we return to the conditions in Part (*a*), with $+2Q$ on the solid sphere and $-7Q$ on the spherical shell. We next connect the solid sphere to ground with a metal wire, and then disconnect it. Then how much total charge is on the solid sphere and on each surface of the spherical shell?

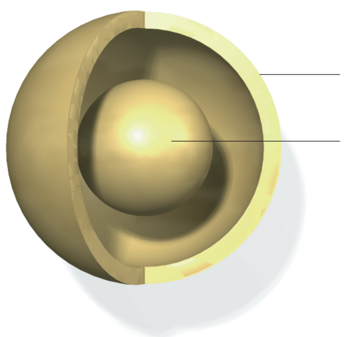


FIGURE 22-41
Problem 40

41 •• A nonconducting solid sphere of radius 10.0 cm has a uniform volume charge density. The magnitude of the electric field at 20.0 cm from the sphere's center is 1.88×10^3 N/C. (*a*) What is the sphere's volume charge density? (*b*) Find the magnitude of the electric field at a distance of 5.00 cm from the sphere's center.

42 •• A nonconducting solid sphere of radius R has a volume charge density that is proportional to the distance from the center. That is, $\rho = Ar$ for $r \leq R$, where A is a constant. (*a*) Find the total charge on the sphere. (*b*) Find the expressions for the electric field inside the sphere ($r < R$) and outside the sphere ($r > R$). (*c*) Sketch the magnitude of the electric field as a function of the distance r from the sphere's center.

43 •• A sphere of radius R has volume charge density $\rho = B/r$ for $r < R$, where B is a constant and $\rho = 0$ for $r > R$. (*a*) Find the total charge on the sphere. (*b*) Find the expressions for the electric field inside and outside the charge distribution. (*c*) Sketch the magnitude of the electric field as a function of the distance r from the sphere's center. SSM

44 •• A sphere of radius R has volume charge density $\rho = C/r^2$ for $r < R$, where C is a constant and $\rho = 0$ for $r > R$. (*a*) Find the total charge on the sphere. (*b*) Find the expressions for the electric field inside and outside the charge distribution. (*c*) Sketch the magnitude of the electric field as a function of the distance r from the sphere's center.

45 ••• A nonconducting spherical shell of inner radius R_1 and outer radius R_2 has a uniform volume charge density ρ. (*a*) Find the total charge on the shell. (*b*) Find expressions for the electric field everywhere.

GAUSS'S LAW APPLICATIONS IN CYLINDRICAL SYMMETRY SITUATIONS

46 • **CONTEXT-RICH, ENGINEERING APPLICATION** For your senior project, you are designing a Geiger tube for detecting radiation in the nuclear physics laboratory. This instrument will consist of a long metal cylindrical tube that has a long straight metal wire running down its central axis. The diameter of the wire will be 0.500 mm and the inside diameter of the tube will be 4.00 cm. The tube is to be filled with a dilute gas in which an electrical discharge (breakdown of the gas) occurs when the electric field reaches 5.50×10^6 N/C. Determine the maximum linear charge density on the wire if breakdown of the gas is not to happen. Assume that the tube and the wire are infinitely long.

47 ••• In Problem 46, suppose ionizing radiation produces an ion and an electron at a distance of 1.50 cm from the long axis of the central wire of the Geiger tube. Suppose that the central wire is positively charged and has a linear charge density equal to 76.5 pC/m. (*a*) In this case, what will be the electron's speed as it impacts the wire? (*b*) How will the electron's speed compare to the ion's final speed when it impacts the outside cylinder? Explain your answer.

48 •• Show that the electric field due to an infinitely long, uniformly charged thin cylindrical shell of radius a having a surface charge density σ is given by the following expressions: $E = 0$ for $0 \leq R < a$ and $E_R = \sigma a/(\epsilon_0 R)$ for $R > a$.

49 • A thin cylindrical shell of length 200 m and radius 6.00 cm has a uniform surface charge density of 9.00 nC/m^2. (*a*) What is the total charge on the shell? Find the electric field at the following radial distances from the long axis of the cylinder: (*b*) 2.00 cm, (*c*) 5.90 cm, (*d*) 6.10 cm, and (*e*) 10.0 cm. (Use the results of Problem 48.)

50 •• An infinitely long nonconducting solid cylinder of radius a has a uniform volume charge density of ρ_0. Show that the electric field is given by the following expressions: $E_R = \rho_0 R/(2\epsilon_0)$ for $0 \leq R < a$ and $E_R = \rho_0 a^2/(2\epsilon_0 R)$ for $R > a$, where R is the distance from the long axis of the cylinder.

51 •• A solid cylinder of length 200 m and radius 6.00 cm has a uniform volume charge density of 300 nC/m^3. (*a*) What is the total charge of the cylinder? Use the formulas given in Problem 50 to calculate the electric field at a point equidistant from the ends at the following radial distances from the cylindrical axis: (*b*) 2.00 cm, (*c*) 5.90 cm, (*d*) 6.10 cm, and (*e*) 10.0 cm. SSM

52 •• Consider two infinitely long, coaxial thin cylindrical shells. The inner shell has a radius a_1 and has a uniform surface charge density of σ_1, and the outer shell has a radius a_2 and has a uniform surface charge density of σ_2. (*a*) Use Gauss's law to find expressions for the electric field in the three regions: $0 \le R < a_1$, $a_1 < R < a_2$, and $R > a_2$, where R is the distance from the axis. (*b*) What is the ratio of the surface charge densities σ_2/σ_1 and their relative signs if the electric field is to be zero everywhere outside the largest cylinder? (*c*) For the case in Part (*b*), what would be the electric field between the shells? (*d*) Sketch the electric field lines for the situation in Part (*b*) if σ_1 is positive.

53 •• Figure 22-42 shows a portion of an infinitely long, concentric cable in cross section. The inner conductor has a linear charge density of 6.00 nC/m and the outer conductor has no net charge. (*a*) Find the electric field for all values of R, where R is the perpendicular distance from the common axis of the cylindrical system. (*b*) What are the surface charge densities on the inside and the outside surfaces of the outer conductor?

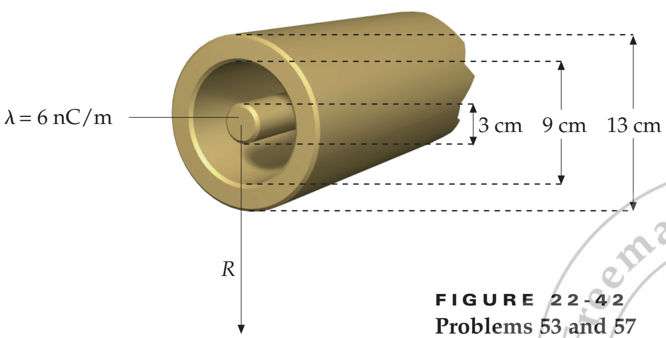


FIGURE 22-42
Problems 53 and 57

54 •• An infinitely long nonconducting solid cylinder of radius a has a nonuniform volume charge density. This density varies linearly with R, the perpendicular distance from its axis, according to $\rho(R) = \beta R$, where β is a constant. (*a*) Show that the linear charge density of the cylinder is given by $\lambda = 2\pi\beta a^3/3$. (*b*) Find expressions for the electric field for $R < a$ and $R > a$.

55 •• An infinitely long nonconducting solid cylinder of radius a has a nonuniform volume charge density. This density varies with R, the perpendicular distance from its axis, according to $\rho(R) = bR^2$, where b is a constant. (*a*) Show that the linear charge density of the cylinder is given by $\lambda = \pi b a^4/2$. (*b*) Find expressions for the electric field for $R < a$ and $R > a$. SSM

56 ••• An infinitely long, nonconducting cylindrical shell of inner radius a_1 and outer radius a_2 has a uniform volume charge density ρ. Find expressions for the electric field everywhere.

57 ••• The inner cylinder of Figure 22-42 is made of nonconducting material and has a volume charge distribution given by $\rho(R) = C/R$, where $C = 200\ \text{nC/m}^2$. The outer cylinder is metallic, and both cylinders are infinitely long. (*a*) Find the charge per unit length (that is, the linear charge density) on the inner cylinder. (*b*) Calculate the electric field for all values of R. SSM

ELECTRIC CHARGE AND FIELD AT CONDUCTOR SURFACES

58 • An uncharged penny is in a region that has a uniform electric field of magnitude 1.60 kN/C directed perpendicular to its faces. (*a*) Find the charge density on each face of the penny, assuming the faces are planes. (*b*) If the radius of the penny is 1.00 cm, find the total charge on one face.

59 • A thin metal slab has a net charge of zero and has square faces that have 12-cm-long sides. It is in a region that has a uniform electric field that is perpendicular to its faces. The total charge induced on one of the faces is 1.2 nC. What is the magnitude of the electric field?

60 • A charge of −6.00 nC is uniformly distributed on a thin square sheet of nonconducting material of edge length 20.0 cm. (*a*) What is the surface charge density of the sheet? (*b*) What are the magnitude and direction of the electric field next to the sheet and proximate to the center of the sheet?

61 • A conducting spherical shell that has zero net charge has an inner radius R_1 and an outer radius R_2. A positive point charge q is placed at the center of the shell. (*a*) Use Gauss's law and the properties of conductors in electrostatic equilibrium to find the electric field in the three regions: $0 \le r < R_1$, $R_1 < r < R_2$, and $r > R_2$, where r is the distance from the center. (*b*) Draw the electric field lines in all three regions. (*c*) Find the charge density on the inner surface ($r = R_1$) and on the outer surface ($r = R_2$) of the shell.

62 •• The electric field just above the surface of Earth has been measured to typically be 150 N/C pointing downward. (*a*) What is the sign of the net charge on Earth's surface under typical conditions? (*b*) What is the total charge on Earth's surface implied by this measurement?

63 •• A positive point charge of 2.5 μC is at the center of a conducting spherical shell that has a net charge of zero, an inner radius equal to 60 cm, and an outer radius equal to 90 cm. (*a*) Find the charge densities on the inner and outer surfaces of the shell and the total charge on each surface. (*b*) Find the electric field everywhere. (*c*) Repeat Part (*a*) and Part (*b*) with a net charge of +3.5 μC placed on the shell. SSM

64 •• If the magnitude of an electric field in air is as great as 3.0×10^6 N/C, the air becomes ionized and begins to conduct electricity. This phenomenon is called *dielectric breakdown*. A charge of 18 μC is to be placed on a conducting sphere. What is the minimum radius of a sphere that can hold this charge without breakdown?

65 •• A thin square conducting sheet that has 5.00-m-long edges has a net charge of 80.0 μC. The square is in the $x = 0$ plane and is centered at the origin. (Assume the charge on each surface is uniformly distributed.) (*a*) Find the charge density on each side of the sheet and find the electric field on the x axis in the region $|x| \ll 5.00$ m. (*b*) A thin but infinite nonconducting sheet that has a uniform charge density of $2.00\ \mu\text{C/m}^2$ is now placed in the $x = -2.50$ m plane. Find the electric field on the x axis on each side of the square sheet in the region $|x| \ll 2.50$ m. Find the charge density on each surface of the square sheet. SSM

GENERAL PROBLEMS

66 •• Consider the concentric metal sphere and spherical shells that are shown in Figure 22-43. The innermost is a solid sphere that has a radius R_1. A spherical shell surrounds the sphere and has an

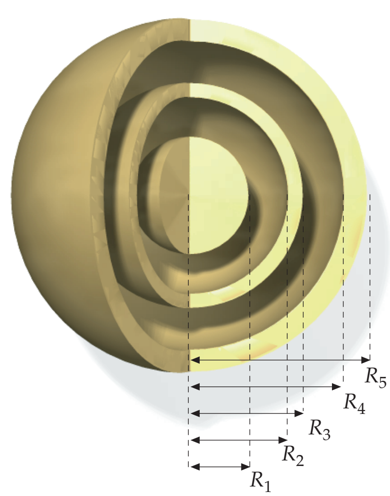


FIGURE 22-43
Problem 66

inner radius R_2 and an outer radius R_3. The sphere and the shell are both surrounded by a second spherical shell that has an inner radius R_4 and an outer radius R_5. None of the three objects initially have a net charge. Then, a negative charge $-Q_0$ is placed on the inner sphere and a positive charge $+Q_0$ is placed on the outermost shell. (*a*) After the charges have reached equilibrium, what will be the direction of the electric field between the inner sphere and the middle shell? (*b*) What will be the charge on the inner surface of the middle shell? (*c*) What will be the charge on the outer surface of the middle shell? (*d*) What will be the charge on the inner surface of the outermost shell? (*e*) What will be the charge on the outer surface of the outermost shell? (*f*) Plot E as a function of r for all values of r.

67 •• A large, flat, nonconducting, nonuniformly charged surface lies in the $x = 0$ plane. At the origin, the surface charge density is $+3.10\,\mu\text{C/m}^2$. A small distance away from the surface on the positive x axis, the x component of the electric field is 4.65×10^5 N/C. What is E_x a small distance away from the surface on the negative x axis? SSM

68 •• An infinitely long line charge that has a uniform linear charge density equal to $-1.50\,\mu\text{C/m}$ lies parallel to the y axis at $x = -2.00$ m. A positive point charge that has a magnitude equal to $1.30\,\mu\text{C}$ is located at $x = 1.00$ m, $y = 2.00$ m. Find the electric field at $x = 2.00$ m, $y = 1.50$ m.

69 •• A thin, nonconducting, uniformly charged spherical shell of radius R (Figure 22-44*a*) has a total positive charge of Q. A small circular plug is removed from the surface. (*a*) What are the magnitude and direction of the electric field at the center of the hole? (*b*) The plug is now put back in the hole (Figure 22-44*b*). Using the result of Part (*a*), find the electric force acting on the plug. (*c*) Using the magnitude of the force, calculate the "electrostatic pressure" (force/unit area) that tends to expand the sphere. SSM

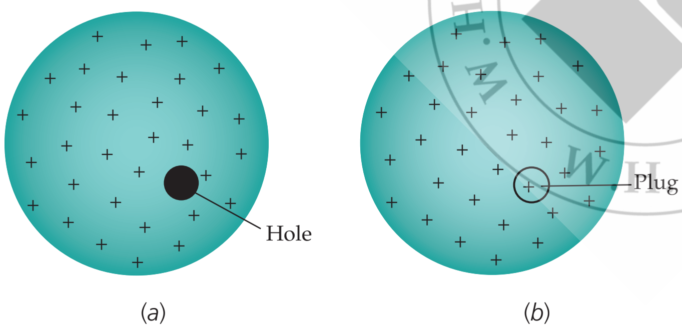


FIGURE 22-44 Problem 69

70 •• An infinite thin sheet in the $y = 0$ plane has a uniform surface charge density $\sigma_1 = +65\,\text{nC/m}^2$. A second infinite thin sheet has a uniform charge density $\sigma_2 = +45\,\text{nC/m}^2$ and intersects the $y = 0$ plane at the z axis and makes an angle of 30° with the xz plane, as shown in Figure 22-45. Find the electric field at (*a*) $x = 6.0$ m, $y = 2.0$ m and (*b*) $x = 6.0$ m, $y = 5.0$ m.

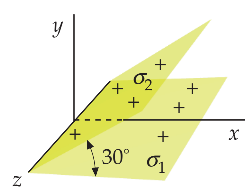


FIGURE 22-45 Problem 70

71 ••• Two identical square parallel metal plates each have an area of 500 cm². They are separated by 1.50 cm. They are both initially uncharged. Now a charge of +1.50 nC is transferred from the plate on the left to the plate on the right and the charges then establish electrostatic equilibrium. (Neglect edge effects.) (*a*) What is the electric field between the plates at a distance of 0.25 cm from the plate on the right? (*b*) What is the electric field between the plates a distance of 1.00 cm from the plate on the left? (*c*) What is the electric field just to the left of the plate on the left? (*d*) What is the electric field just to the right of the plate on the right? SSM

72 •• Two infinite nonconducting uniformly charged planes lie parallel to each other and to the yz plane. One is at $x = -2.00$ m and has a surface charge density of $-3.50\,\mu\text{C/m}^2$. The other is at $x = 2.00$ m and has a surface charge density of $6.00\,\mu\text{C/m}^2$. Find the electric field in the region: (*a*) $x < -2.00$ m (*b*) $-2.00\text{ m} < x < 2.00$ m, and (*c*) $x > 2.00$ m.

73 ••• A quantum-mechanical treatment of the hydrogen atom shows that the electron in the atom can be treated as a smeared-out distribution of negative charge of the form $\rho(r) = -\rho_0 e^{-2r/a}$. Here r represents the distance from the center of the nucleus and a represents the *first Bohr radius*, which has a numerical value of 0.0529 nm. Recall that the nucleus of a hydrogen atom consists of just one proton and treat this proton as a positive point charge. (*a*) Calculate ρ_0, using the fact that the atom is neutral. (*b*) Calculate the electric field at any distance r from the nucleus. SSM

74 •• A uniformly charged ring has a radius a, lies in a horizontal plane, and has a negative charge given by $-Q$. A small particle of mass m has a positive charge given by q. The small particle is located on the axis of the ring. (*a*) What is the minimum value of q/m such that the particle will be in equilibrium under the action of gravity and the electrostatic force? (*b*) If q/m is twice the value calculated in Part (*a*) , where will the particle be when it is in equilibrium? Express your answer in terms of a.

75 •• A long, thin, nonconducting plastic rod is bent into a circular loop that has a radius a. Between the ends of the rod a short gap of length ℓ, where $\ell \ll a$, remains. A positive charge of magnitude Q is evenly distributed on the loop. (*a*) What is the direction of the electric field at the center of the loop? Explain your answer. (*b*) What is the magnitude of the electric field at the center of the loop?

76 •• A nonconducting solid sphere that is 1.20 m in diameter and has its center on the x axis at $x = 4.00$ m has a uniform volume charge of density of $+5.00\,\mu\text{C/m}^3$. Concentric with the sphere is a thin nonconducting spherical shell that has a diameter of 2.40 m and a uniform surface charge density of $-1.50\,\mu\text{C/m}^2$. Calculate the magnitude and direction of the electric field at (*a*) $x = 4.50$ m, $y = 0$, (*b*) $x = 4.00$ m, $y = 1.10$ m, and (*c*) $x = 2.00$ m, $y = 3.00$ m.

77 •• An infinite nonconducting plane sheet of charge that has a surface charge density $+3.00\,\mu\text{C/m}^2$ lies in the $y = -0.600$ m plane. A second infinite nonconducting plane sheet of charge that has a surface charge density of $-2.00\,\mu\text{C/m}^2$ lies in the $x = 1.00$ m plane. Lastly, a nonconducting thin spherical shell that has a radius of 1.00 m and its center in the $z = 0$ plane at the intersection of the two charged planes has a surface charge density of $-3.00\,\mu\text{C/m}^2$. Find the magnitude and direction of the electric field on the x axis at (*a*) $x = 0.400$ m and (*b*) $x = 2.50$ m.

78 •• An infinite nonconducting plane sheet lies in the $x = 2.00$ m plane and has a uniform surface charge density of $+2.00\,\mu\text{C/m}^2$. An infinite nonconducting line charge of uniform linear charge density $4.00\,\mu\text{C/m}$ passes through the origin at an angle of 45.0° with the x axis in the xy plane. A solid nonconducting sphere of volume charge density $-6.00\,\mu\text{C/m}^3$ and radius 0.800 m is centered on the x axis at $x = 1.00$ m. Calculate the magnitude and direction of the electric field in the $z = 0$ plane at $x = 1.50$ m, $y = 0.50$ m.

79 •• A uniformly charged, infinitely long line of negative charge has a linear charge density of $-\lambda$ and is located on the z axis. A small positively charged particle that has a mass m and a charge q is in a circular orbit of radius R in the xy plane centered on the line of charge. (*a*) Derive an expression for the speed of the particle. (*b*) Obtain an expression for the period of the particle's orbit. SSM

80 •• A stationary ring of radius a lies in the yz plane and has a uniform positive charge Q. A small particle that has mass m and a negative charge $-q$ is located at the center of the ring. (a) Show that if $x \ll a$, the electric field along the axis of the ring is proportional to x. (b) Find the force on the particle as a function of x. (c) Show that if the particle is given a small displacement in the $+x$ direction, it will perform simple harmonic motion. (d) What is the frequency of that motion?

81 •• The charges Q and q of Problem 80 are $+5.00\ \mu$C and $-5.00\ \mu$C, respectively, and the radius of the ring is 8.00 cm. When the particle is given a small displacement in the x direction, it oscillates about its equilibrium position at a frequency of 3.34 Hz. (a) What is the particle's mass? (b) What is the frequency if the radius of the ring is doubled to 16.0 cm and all other parameters remain unchanged? SSM

82 •• If the radius of the ring in Problem 80 is doubled while keeping the linear charge density on the ring the same, does the frequency of oscillation of the particle change? If so, by what factor does it change?

83 ••• A uniformly charged nonconducting solid sphere of radius R has its center at the origin and has a volume charge density of ρ. (a) Show that at a point within the sphere a distance r from the center $\vec{E} = \frac{\rho}{3\epsilon_0} r\hat{r}$. ($b$) Material is removed from the sphere leaving a spherical cavity that has a radius $b = R/2$ and its center at $x = b$ on the x axis (Figure 22-46). Calculate the electric field at points 1 and 2 shown in Figure 22-46. *Hint: Model the sphere-with-cavity as two uniform spheres of equal positive and negative charge densities.*

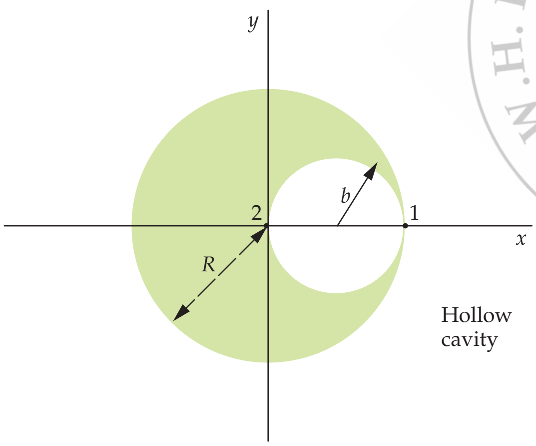


FIGURE 22-46 **Problems 83 and 85**

84 ••• Show that the electric field throughout the cavity of Problem 83b is uniform and is given by $\vec{E} = \frac{\rho}{3\epsilon_0} b\hat{i}$.

85 ••• The cavity in Problem 83b is now filled with a uniformly charged nonconducting material with a total charge of Q. Calculate the new values of the electric field at points 1 and 2 shown in Figure 22-46.

86 ••• A *small* Gaussian surface in the shape of a cube has faces parallel to the xy, xz, and yz planes (Figure 22-47) and is in a region in which the electric field is parallel to the x axis. (a) Using the differential approximation, show that the net electric flux of the electric field out of the Gaussian surface is given by $\phi_{\text{net}} \approx \frac{\partial E_x}{\partial x}\Delta V$, where ΔV is the volume enclosed by the Gaussian surface. (b) Using Gauss's law and the results of Part (a) show that $\frac{\partial E_x}{\partial x} = \frac{\rho}{\epsilon_0}$, where ρ is the volume charge density inside the cube. (This equation is the one-dimensional version of the point form of Gauss's law.)

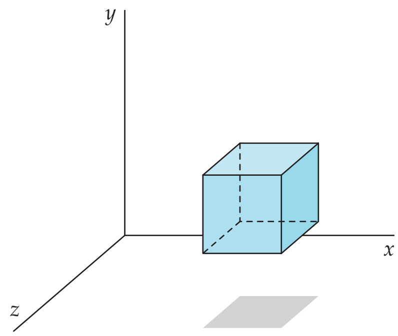


FIGURE 22-47 **Problem 86**

87 ••• Consider a simple but surprisingly accurate model for the hydrogen molecule: two positive point charges, each having charge $+e$, are placed inside a uniformly charged sphere of radius R, which has a charge equal to $-2e$. The two point charges are placed symmetrically, equidistant from the center of the sphere (Figure 22-48). Find the distance from the center, a, where the net force on either point charge is zero. SSM

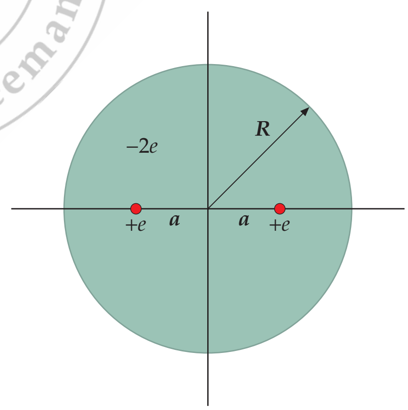


FIGURE 22-48 **Problem 87**

88 ••• An electric dipole that has a dipole moment of $\vec{p}$ is located at a perpendicular distance R from an infinitely long line charge that has a uniform linear charge density λ. Assume that the dipole moment is in the same direction as the field of the line of charge. Determine an expression for the electric force on the dipole.