PART IV **ELECTRICITY AND MAGNETISM**

CHAPTER 21

COPPER IS A CONDUCTOR, A MATERIAL THAT HAS SPECIFIC PROPERTIES WE FIND USEFUL BECAUSE THESE PROPERTIES MAKE IT POSSIBLE TO TRANSPORT ELECTRICITY. *(A. B. Joyce/Science Source.)*

The Electric Field I: Discrete Charge Distributions

21-1 Charge
21-2 Conductors and Insulators
21-3 Coulomb's Law
21-4 The Electric Field
21-5 Electric Field Lines
21-6 Action of the Electric Field on Charges

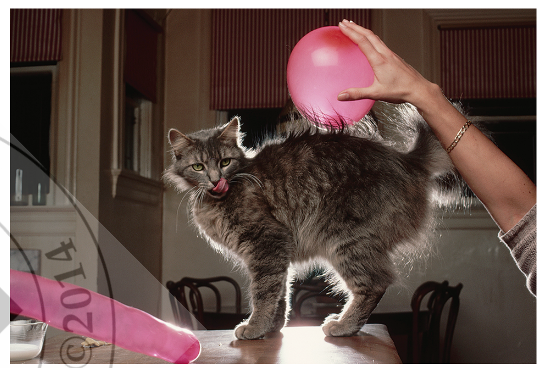
? What is the total charge of all the electrons in a penny? (See Example 21-1.)

While just a century ago we had nothing more than a few electric lights, we are now extremely dependent on electricity in our daily lives. Yet, although the use of electricity has only recently become widespread, the study of electricity has a history reaching long before the first electric lamp glowed. Observations of electrical attraction can be traced back to the ancient Greeks, who noticed that after amber was rubbed, it attracted small objects such as straw or feathers. Indeed, the word *electric* comes from the Greek word for amber, *elektron*.

Today, the study and use of electricity continue. Electrical engineers improve existing electrical technologies, increasing performance and efficiency in devices such as hybrid cars and electric power plants. Electrostatic paints are used in the auto industry for engine parts and for car frames and bodies. This painting process creates a more durable coat than does liquid paint, and is easier on the environment because no solvents are used.

In this chapter, we begin our study of electricity with electrostatics, the study of charges at rest. After introducing the concept of charge, we briefly look at conductors and insulators and how conductors can be given a net charge. We then study Coulomb's law, which describes the force exerted by one charge on another. Next, we introduce the electric field and show how it can be visualized by electric field lines that indicate the magnitude and direction of the field, just as we visualized the velocity field of a flowing fluid using streamlines (Chapter 13). Finally, we discuss the behavior of point charges and dipoles in electric fields.

21-1 CHARGE



A cat and a balloon. *(Roger Ressmeyer/CORBIS.)*

Suppose we rub a hard-rubber rod with fur and then suspend the rod from a string so that it is free to rotate. Now we bring a second hard-rubber rod that has been rubbed with fur near it. The rods repel each other (Figure 21-1*a*). Two glass rods that have been rubbed with silk (Figure 21-1*b*) also repel each other. But, when we place a hard-rubber rod rubbed with fur near a glass rod rubbed with silk (Figure 21-1*c*) they attract each other.

Rubbing a rod causes the rod to become electrically charged. If we repeat the experiment with various materials, we find that all charged objects fall into one of just two groups—those like the hard-rubber rod rubbed with fur and those like the glass rod rubbed with silk. Objects from the same group repel each other, while objects from different groups attract each other. Benjamin Franklin explained this by proposing a model in which every object has a *normal* amount of electricity that can be transferred from one object to the other when two objects are in close contact, as when they are rubbed together. One object would have an excess charge and the other object would have a deficiency of charge, and the excess charge equals the deficiency of charge. Franklin described the resulting charges as positive (plus sign) or negative (minus sign). He also chose positive to be the charge acquired by a glass rod when it is rubbed with a piece of silk. The piece of silk then acquires a negative charge of equal magnitude during the procedure. Based on Franklin's convention, if hard rubber and fur are rubbed together, the hard rubber acquires a negative charge and the fur acquires a positive charge. Two objects that have the same sign (both + or both −) repel each other, and two objects that have oppositely

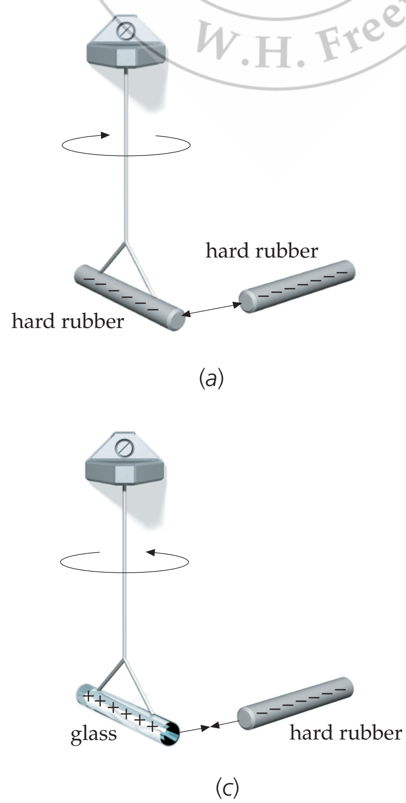


FIGURE 21-1 (*a*) Two hard-rubber rods that have been rubbed with fur repel each other. (*b*) Two glass rods that have been rubbed with silk repel each other. (*c*) A hard-rubber rod that has been rubbed with fur attracts a glass rod that has been rubbed with silk.

signed charges attract each other (Figure 21-1). An object that is neither positively nor negatively charged is said to be *electrically neutral.*

Today, we know that when glass is rubbed with silk, electrons are transferred from the glass to the silk. Because the silk is negatively charged (according to Franklin's convention, which we still use) electrons are said to have a negative charge. Table 21-1 is a short version of the **triboelectric series.** (In Greek *tribos* means "a rubbing.") The farther down the series a material is, the greater its affinity for electrons. If two of the materials are brought in contact, electrons are transferred from the material higher in the table to the one farther down the table. For example, if Teflon is rubbed with nylon, electrons are transferred from the nylon to the Teflon.

Table 21-1 The Triboelectric Series

+ Positive End of Series
Asbestos
Glass
Nylon
Wool
Lead
Silk
Aluminum
Paper
Cotton
Steel
Hard rubber
Nickel and copper
Brass and silver
Synthetic rubber
Orlon
Saran
Polyethylene
Teflon
Silicone rubber
− Negative End of Series

CHARGE QUANTIZATION

Matter consists of atoms that are neutral. Each atom has a tiny but massive nucleus that is composed of protons and neutrons. Protons are positively charged, whereas neutrons are neutral. The number of protons that an atom of a particular element has is the atomic number Z of that element. Surrounding the nucleus is an equal number of negatively charged electrons, leaving the atom with zero net charge. An electron is about 2000 times less massive than a proton, yet the charges of these two particles are exactly equal in magnitude. The charge of the proton is e and that of the electron is $-e$, where e is called the **fundamental unit of charge.** The charge of an electron or proton is an intrinsic property of the particle, just as mass and spin are intrinsic properties of these particles.

All observable charges occur in integral amounts of the fundamental unit of charge e; that is, *charge is quantized*. Any observable charge Q occurring in nature can be written $Q = \pm Ne$, where N is an integer.* For ordinary objects, however, N is usually very large and charge appears to be continuous, just as air appears to be continuous even though air consists of many discrete particles (molecules, atoms, and ions). To give an everyday example of N, charging a plastic rod by rubbing it with a piece of fur typically transfers 10^{10} or more electrons to the rod.

CHARGE CONSERVATION

When objects are rubbed together, one object is left with an excess of electrons and is therefore negatively charged; the other object is left with a deficit of electrons and is therefore positively charged. The net charge of the two objects remains constant; that is, *charge is conserved*. The **law of conservation of charge** is a fundamental law of nature. In certain interactions among elementary particles, charged particles such as electrons are created or annihilated. However, during these processes, equal amounts of positive and negative charge are produced or destroyed, so the net charge of the universe is unchanged.

The SI unit of charge is the coulomb, which is defined in terms of the unit of electric current, the ampere (A).† The **coulomb** (C) is the amount of charge flowing through a cross section of wire in one second when the current in the wire is one ampere. (The cross section of a solid object is the intersection of the object and a plane. Here we consider a plane that cuts across the wire.) The fundamental unit of electric charge e is related to the coulomb by

$$e = 1.602177 \times 10^{-19}\ \mathrm{C} \approx 1.60 \times 10^{-19}\ \mathrm{C} \qquad 21\text{-}1$$

FUNDAMENTAL UNIT OF CHARGE

* In the standard model of elementary particles, protons, neutrons, and some other elementary particles are made up of more fundamental particles called *quarks* that have charges of $\pm\frac{1}{3}e$ or $\pm\frac{2}{3}e$. Only combinations that result in a net charge of $\pm Ne$, where N is an integer, are observed.

† The ampere (A) is the unit of current used in everyday electrical work.

PRACTICE PROBLEM 21-1
A charge of magnitude 50 nC (1.0 nC = 10^{-9} C) can be produced in the laboratory by simply rubbing two objects together. How many electrons must be transferred to produce this charge?

Example 21-1 How Many in a Penny?

A copper penny* ($Z = 29$) has a mass of 3.10 grams. What is the total charge of all the electrons in the penny?

PICTURE The electrons have a total charge given by the number of electrons in the penny, N_e, multiplied by the charge of an electron, $-e$. The number of electrons in a copper atom is 29 (the atomic number of copper). So, the total charge of the electrons is 29 electrons multiplied by the number of copper atoms N_{at} in a penny. To find N_{at}, we use the fact that one mole of any substance has Avogadro's number ($N_A = 6.02 \times 10^{23}$) of particles (molecules, atoms, or ions), and the number of grams in one mole of copper is the molar mass M, which is 63.5 g/mol for copper.

SOLVE

1. The total charge Q is the number of electrons multiplied by the charge:
$$Q = N_e(-e)$$

2. The number of electrons is Z multiplied by the number of copper atoms N_{at}:
$$N_e = ZN_{at}$$

3. Compute the number of copper atoms in 3.10 g of copper:
$$N_{at} = (3.10\text{ g})\frac{6.02 \times 10^{23}\text{ atoms/mol}}{63.5\text{ g/mol}} = 2.94 \times 10^{22}\text{ atoms}$$

4. Compute the number of electrons N_e:
$$N_e = ZN_{at} = (29\text{ electrons/atom})(2.94 \times 10^{22}\text{ atoms})$$
$$= 8.53 \times 10^{23}\text{ electrons}$$

5. Use this value of N_e to find the total charge:
$$Q = N_e \times (-e) = (8.53 \times 10^{23}\text{ electrons})(-1.60 \times 10^{-19}\text{ C/electron})$$
$$= \boxed{-1.37 \times 10^{5}\text{ C}}$$

CHECK There are $29 \times (6.02 \times 10^{23})$ electrons in 63.5 g of copper, so in 3.10 g of copper there are $(3.10/63.5) \times 29 \times (6.02 \times 10^{23}) = 8.53 \times 10^{23}$ electrons—in agreement with our step-4 result.

PRACTICE PROBLEM 21-2 If one million electrons were given to each person in the United States (about 300 million people), what percentage of the number of electrons in a penny would this represent?

* The penny was composed of 100 percent copper from 1793 to 1837. In 1982, the composition changed from 95 percent copper and 5 percent zinc to 2.5 percent copper and 97.5 percent zinc.

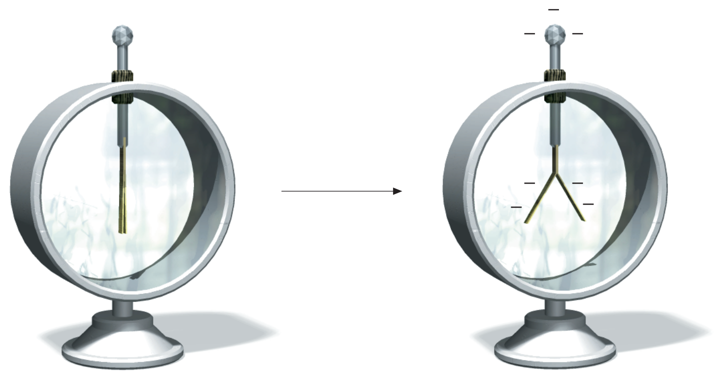


FIGURE 21-2 An electroscope. Two gold leaves are attached to a conducting post that has a conducting ball on top. The ball, post, and leaves are insulated from the container. When uncharged, the leaves hang together vertically. When the ball is touched by a negatively charged plastic rod, some of the negative charge from the rod is transferred to the ball and moves to the gold leaves, which then spread apart because of electrical repulsion between their negative charges. (Touching the ball with a positively charged glass rod would also cause the leaves to spread apart. In this case, the positively charged glass rod would remove electrons from the metal ball, leaving a net positive charge on the ball, rod, and leaves.)

21-2 CONDUCTORS AND INSULATORS

In many materials, such as copper and other metals, some of the electrons are free to move about the entire material. Such materials are called **conductors.** In other materials, such as wood or glass, all the electrons are bound to nearby atoms and none can move freely. These materials are called **insulators.**

In a single atom of copper, 29 electrons are bound to the nucleus by the electrostatic attraction between the negatively charged electrons and the positively charged nucleus. The outer (valence) electrons are more weakly bound to a nucleus than the inner (core) electrons. When a large number of copper atoms are combined in a piece of metallic copper, the strength of the attractions of electrons to a nucleus of an atom is reduced by their interactions with the electrons and nuclei of neighboring atoms. One or more of the valence electrons in each atom is no longer bound to the atom but is free to move about the whole piece of metal, much as an air molecule is free to move about in a room. The number of these free electrons depends on the particular metal, but it is typically about one per atom. (The free electrons are also referred to as conduction electrons or delocalized electrons.) An atom that has an electron removed or added, resulting in a net charge on the atom, is called an **ion.** In metallic copper, the copper ions are arranged in a regular array called a *lattice.* A conductor is neutral if for each lattice ion having a positive charge $+e$ there is a free electron having a negative charge $-e$. The net charge of the conductor can be changed by adding or removing electrons. A conductor that has a negative net charge has a surplus of free electrons, while a conductor that has a positive net charge has a deficit of free electrons.

CONCEPT CHECK 21-1

Two identical conducting spheres, one that has an initial charge $+Q$ and the other is initially uncharged, are brought into contact. (*a*) What is the new charge on each sphere? (*b*) While the spheres are in contact, a positively charged rod is moved close to one sphere, causing a redistribution of the charges on the two spheres so the charge on the sphere closest to the rod has a charge of $-Q$. What is the charge on the other sphere?

CHARGING BY INDUCTION

The conservation of charge is illustrated by a simple method of charging a conductor called **charging by induction,** as shown in Figure 21-3. Two uncharged metal spheres touch each other. When a positively charged rod (Figure 21-3*a*) is brought near one of the spheres, conduction electrons flow from one sphere to the other, toward the positively charged rod. The positively charged rod in Figure 21-3*a* attracts the negatively charged electrons, and the sphere nearest the rod acquires electrons from the sphere farther away. This leaves the near

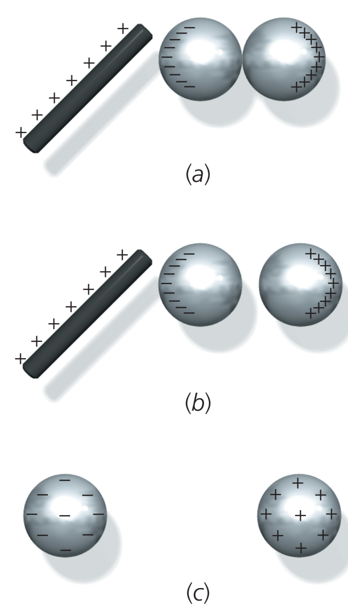


FIGURE 21-3 Charging by induction. (*a*) Neutral conductors in contact become oppositely charged when a charged rod attracts electrons to the left sphere. (*b*) If the spheres are separated before the rod is removed, they will retain their equal and opposite charges. (*c*) When the rod is removed and the spheres are far apart, the distribution of charge on each sphere approaches uniformity.

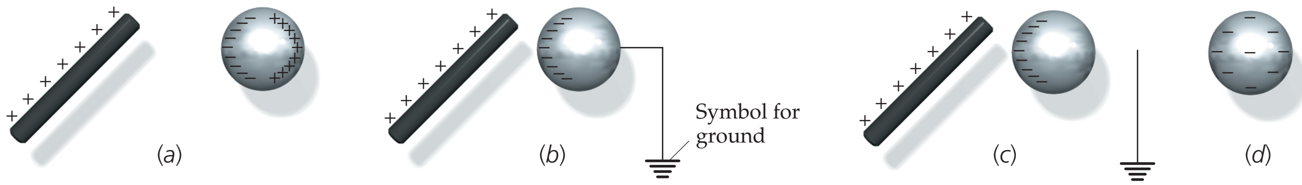


FIGURE 21-4 Induction via grounding. (*a*) The free charge on the single neutral conducting sphere is polarized by the positively charged rod, which attracts negative charges on the sphere. (*b*) When the conductor is grounded by connecting it with a wire to a very large conductor, such as Earth, electrons from the ground neutralize the positive charge on the far face. The conductor is then negatively charged. (*c*) The negative charge remains if the connection to the ground is broken before the rod is removed. (*d*) After the rod is removed, the sphere has a uniform negative charge.

sphere with a net negative charge and the far sphere with an equal net positive charge. A conductor that has *separated* equal and opposite charges is said to be **polarized.** If the spheres are separated before the rod is removed, they will be left with equal amounts of opposite charges (Figure 21-3*b*). A similar result would be obtained with a negatively charged rod, which would drive electrons from the near sphere to the far sphere.

For many purposes, Earth itself can be modeled as an infinitely large conductor that has an infinite supply of charged particles. If a conductor is electrically connected to Earth it is said to be **grounded.** Grounding a metal sphere is indicated schematically in Figure 21-4*b* by a connecting wire ending in parallel horizontal lines. Figure 21-4 demonstrates how we can induce a charge in a single conductor by transferring charge from Earth through a ground wire and then breaking the connection to the ground. (In practice, a person standing on the floor and touching the sphere with his hand provides an adequate ground for electrostatic demonstrations such the one described here.)

CONCEPT CHECK 21-2

Two identical conducting spheres are charged by induction and then separated by a large distance; sphere 1 has charge $+Q$ and sphere 2 has charge $-Q$. A third identical sphere is initially uncharged. If sphere 3 is touched to sphere 1 and separated, then touched to sphere 2 and separated, what is the final charge on each of the three spheres?

The lightning rod on this building is grounded so that it can conduct electrons from the ground to the positively charged clouds, thus neutralizing them. *(Robert Jakatics/Shutterstock.)*

These fashionable ladies are wearing hats with metal chains that drag along the ground, which were supposed to protect them from lightning. *(Jacques Boyer/Roger-Viollet/The Image Works.)*

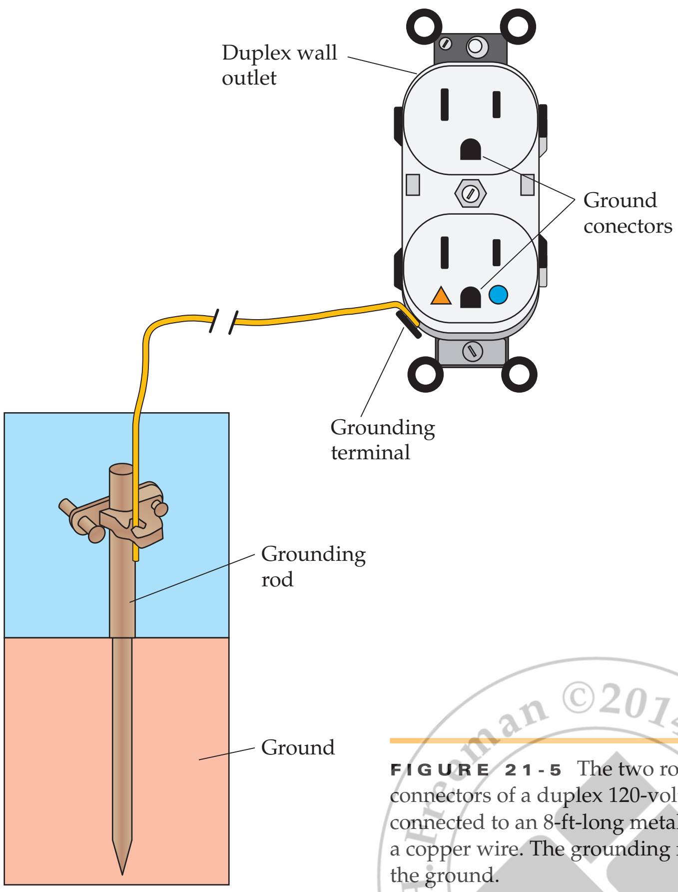
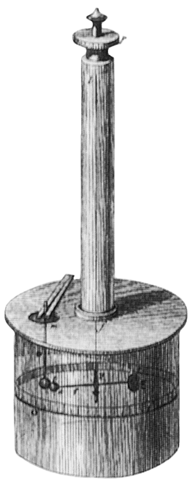


FIGURE 21-5 The two round ground connectors of a duplex 120-volt wall outlet are connected to an 8-ft-long metal grounding rod by a copper wire. The grounding rod is driven into the ground.

21-3 COULOMB'S LAW

Charles Coulomb (1736–1806) studied the force exerted by one charge on another using a torsion balance of his own invention.* In Coulomb's experiment, the charged spheres were much smaller than the distance between them so that the charges could be treated as point charges. Coulomb used the method of charging by induction to produce equally charged spheres and to vary the amount of charge on the spheres. For example, beginning with charge q_0 on each sphere, he could reduce the charge to $\frac{1}{2}q_0$ by temporarily grounding one sphere to discharge it, disconnecting it from ground, and then placing the two spheres in contact. The results of the experiments of Coulomb and others are summarized in **Coulomb's law:**



Coulomb's torsion balance. *(Bundy Library, Norwalk, CT.)*

> The force exerted by one point charge on another acts along the line between the charges. It varies inversely as the square of the distance separating the charges and is proportional to the product of the charges. The force is repulsive if the charges have the same sign and attractive if the charges have opposite signs.
>
> COULOMB'S LAW

* Coulomb's experimental apparatus was essentially the same as that described for the Cavendish experiment in Chapter 11, with the masses replaced by small charged spheres. For the magnitudes of charges easily transferred by rubbing, the gravitational attraction of the spheres is completely negligible compared with their electric attraction or repulsion.

The *magnitude* of the electric force exerted by a point charge q_1 on another point charge q_2 a distance r away is thus given by

$$F = \frac{k|q_1 q_2|}{r^2} \tag{21-2}$$

COULOMB'S LAW FOR THE MAGNITUDE OF THE FORCE EXERTED BY q_1 ON q_2

where k is an experimentally determined positive constant called the **Coulomb constant,** which has the value

$$k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2} \tag{21-3}$$

If q_1 is at position $\vec{r}_1$ and q_2 is at $\vec{r}_2$ (Figure 21-6), the force $\vec{F}_{12}$ exerted by q_1 on q_2 is

$$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12} \tag{21-4}$$

COULOMB'S LAW (VECTOR FORM)

where $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ is the vector pointing from q_1 to q_2, and $\hat{r}_{12} = \vec{r}_{12}/r_{12}$ is a unit vector in the same direction.

In accord with Newton's third law, the electrostatic force $\vec{F}_{21}$ exerted by q_2 on q_1 is the negative of $\vec{F}_{12}$. Note the similarity between Coulomb's law and Newton's law of gravity. (See Equation 11-3.) Both are inverse-square laws. But the gravitational force between two particles is proportional to the masses of the particles and is always attractive, whereas the electric force is proportional to the charges of the particles and is repulsive if the charges have the same sign and attractive if they have opposite signs.

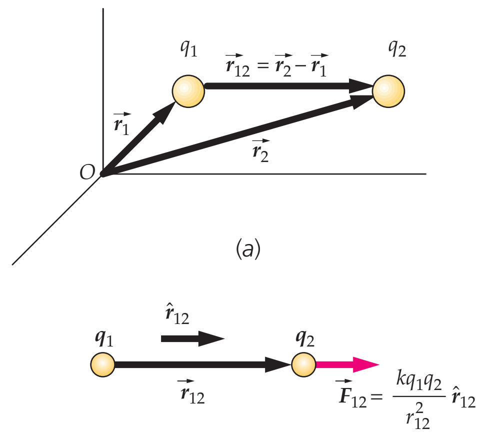


FIGURE 21-6 (*a*) Charge q_1 at position $\vec{r}_1$ and charge q_2 at $\vec{r}_2$ relative to the origin *O*. (*b*) The force $\vec{F}_{12}$ exerted by q_1 on q_2 is in the direction of the vector $\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$ if both charges have the same sign, and in the opposite direction if they have opposite signs. The unit vector $\hat{r}_{12} = \vec{r}_{12}/r_{12}$ is in the direction from q_1 to q_2.

! Equation 21-4 gives the correct direction for the force, whether or not the two charges are both positive, both negative, or one positive and one negative.

Example 21-2 Electric Force in Hydrogen

In a hydrogen atom, the electron is separated from the proton by an average distance of about 5.3×10^{-11} m. Calculate the magnitude of the electrostatic force of attraction exerted by the proton on the electron.

PICTURE Assign the proton as q_1 and the electron as q_2. Use Coulomb's law to determine the magnitude of the electrostatic force of attraction exerted by the proton on the electron.

SOLVE

1. Sketch the electron and the proton and label the sketch with the suitable symbols (Figure 21-7):

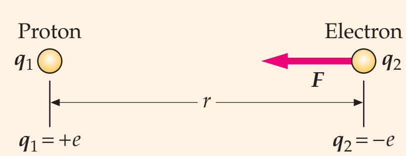


FIGURE 21-7

2. Use the given information and Equation 21-2 (Coulomb's law) to calculate the electrostatic force:

$$F = \frac{k|q_1q_2|}{r^2} = \frac{ke^2}{r^2} = \frac{(8.99 \times 10^9\ \mathrm{N\cdot m^2/C^2})(1.60 \times 10^{-19}\ \mathrm{C})^2}{(5.3 \times 10^{-11}\ \mathrm{m})^2}$$
$$= \boxed{8.2 \times 10^{-8}\ \mathrm{N}}$$

CHECK The order of magnitude is plausible. The powers of ten in the numerator combined are $10^9 \times 10^{-38} = 10^{-29}$, the power of ten in the denominator is 10^{-22}, and $10^{-29}/10^{-22} = 10^{-7}$. In comparison, $8.2 \times 10^{-8} \approx 10^{-7}$.

TAKING IT FURTHER Compared with macroscopic interactions, this is a very small force. However, because the mass of the electron is only about 10^{-30} kg, this force produces an acceleration of $F/m \approx 9 \times 10^{22}\ \mathrm{m/s^2}$. The proton is almost 2000 times more massive than the electron, so the acceleration of the proton is about $4 \times 10^{19}\ \mathrm{m/s^2}$. To put these accelerations in perspective, the acceleration due to gravity g is a mere $10^1\ \mathrm{m/s^2}$.

PRACTICE PROBLEM 21-3 Two point charges of 0.0500 μC each are separated by 10.0 cm. Find the magnitude of the force exerted by one point charge on the other.

Because the electrical force and the gravitational force between any two particles both vary inversely with the square of the distance between the particles, the ratio of these forces is independent of that distance. We can therefore compare the relative strengths of the electrical and gravitational forces for elementary particles such as the electron and proton.

Example 21-3 Ratio of Electric and Gravitational Forces

Compute the ratio of the electric force to the gravitational force exerted by a proton on an electron in a hydrogen atom.

PICTURE Use Coulomb's law and $q_1 = e$ and $q_2 = -e$ to find the electric force. Use Newton's law of gravity, the mass of the proton, $m_p = 1.67 \times 10^{-27}$ kg, and the mass of the electron, $m_e = 9.11 \times 10^{-31}$ kg, to find the gravitational force.

SOLVE

1. Express the magnitudes of the electric force F_e and the gravitational force F_g in terms of the charges, masses, separation distance r, and electrical and gravitational constants:

$$F_e = \frac{ke^2}{r^2} \qquad F_g = \frac{Gm_pm_e}{r^2}$$

2. Determine the ratio. Note that the separation distance r cancels:

$$\frac{F_e}{F_g} = \frac{ke^2}{Gm_pm_e}$$

3. Substitute numerical values:

$$\frac{F_e}{F_g} = \frac{(8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})(1.60 \times 10^{-19}\ \mathrm{C})^2}{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(1.67 \times 10^{-27}\ \mathrm{kg})(9.11 \times 10^{-31}\ \mathrm{kg})}$$

$$= \boxed{2.27 \times 10^{39}}$$

CHECK In the numerator of the fraction in step 3, the coulomb units cancel out. In the denominator of the fraction, the kilogram units cancel out. The result is that both numerator and denominator have units of $\mathrm{N \cdot m^2}$. The fraction has no units, as expected for a ratio of two forces.

TAKING IT FURTHER The fact that the ratio (step 3) is so large reveals why the effects of gravity are not considered when discussing atomic or molecular interactions.

Although the gravitational force is incredibly weak compared with the electric force and plays essentially no role at the atomic level, it is the dominant force between large objects such as planets and stars. Because large objects contain almost equal numbers of positive and negative charges, the attractive and repulsive electrical forces cancel. The net force between astronomical objects is therefore essentially the force of gravitational attraction alone.

FORCE EXERTED BY A SYSTEM OF CHARGES

In a system of charges, each charge exerts a force, given by Equation 21-4, on every other charge. The net force on any charge is the vector sum of the individual forces exerted on that charge by all the other charges in the system. This result follows from the *principle of superposition* of forces.

See **Math Tutorial** *for more information on* **Trigonometry**

Example 21-4 Electric Force on a Charge

Three point charges lie on the x axis; q_1 is at the origin, q_2 is at $x = 2.0$ m, and q_0 is at position x ($x > 2.0$ m). (*a*) Find the total electric force on q_0 due to q_1 and q_2 if $q_1 = +25$ nC, $q_2 = -10$ nC, $q_0 = +20$ nC, and $x = 3.5$ m. (*b*) Find an expression for the total electric force on q_0 due to q_1 and q_2 throughout the region $2.0 \text{ m} < x < \infty$.

PICTURE The total electric force on q_0 is the vector sum of the force $\vec{F}_{10}$ exerted by q_1 and the force $\vec{F}_{20}$ exerted by q_2. The individual forces are found using Coulomb's law and the principle of superposition. Note that $\hat{r}_{10} = \hat{r}_{20} = \hat{i}$ because both $\hat{r}_{10}$ and $\hat{r}_{20}$ are in the $+x$ direction.

SOLVE

(*a*) 1. Draw a sketch of the system of charges (Figure 21-8*a*). Identify the distances r_{10} and r_{20} on the graph:

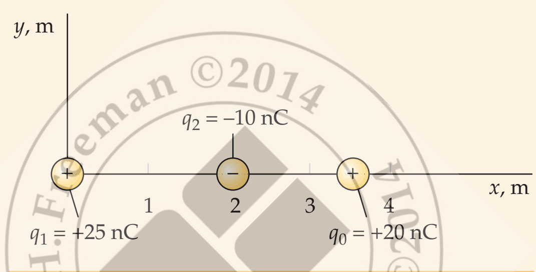


FIGURE 21-8*a*

2. Find the force exerted by q_1 on q_0. These charges have the same sign, so they repel. The force is in the $+x$ direction:

$$F_{10} = \frac{k|q_1 q_0|}{r_{10}^2}$$

$$\vec{F}_{10} = +F_{10}\hat{i} = +\frac{k|q_1 q_0|}{r_{10}^2}\hat{i} = \frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(25 \times 10^{-9}\text{ C})(20 \times 10^{-9}\text{ C})}{(3.5\text{ m})^2}\hat{i}$$

$$= (0.37 \times 10^{-6}\text{ N})\hat{i}$$

3. Find the force exerted by q_2 on q_0. These charges have opposite signs, so they attract. The force is in the $-x$ direction:

$$F_{20} = \frac{k|q_2 q_0|}{r_{20}^2}$$

$$\vec{F}_{20} = -F_{20}\hat{i} = -\frac{k|q_2 q_0|}{r_{20}^2}\hat{i} = -\frac{(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(10 \times 10^{-9}\text{ C})(20 \times 10^{-9}\text{ C})}{(1.5\text{ m})^2}\hat{i}$$

$$= -(0.80 \times 10^{-6}\text{ N})\hat{i}$$

4. Combine your results to obtain the net force.

$$\vec{F}_{\text{net}} = \vec{F}_{10} + \vec{F}_{20} = \boxed{-(0.43 \times 10^{-6}\text{ N})\hat{i}}$$

(*b*) 1. Draw a sketch of the system of charges. Label the distances r_{10} and r_{20} (Figure 21-8*b*):

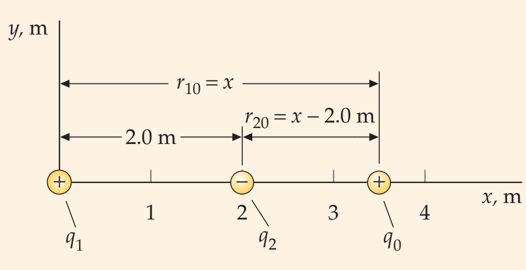


FIGURE 21-8*b*

2. Find an expression for the force on q_0 due to q_1. $$\vec{F}_{10} = \frac{k|q_1q_0|}{x^2}\hat{i}$$

3. Find an expression for the force on q_0 due to q_2. $$\vec{F}_{20} = -\frac{k|q_2q_0|}{(x - 2.0\text{ m})^2}\hat{i}$$

4. Combine your results to obtain an expression for the net force. $$\vec{F}_{\text{net}} = \vec{F}_{10} + \vec{F}_{20} = \left(\frac{k|q_1q_0|}{x^2} - \frac{k|q_2q_0|}{(x - 2.0\text{ m})^2}\right)\hat{i}$$

CHECK In steps 2, 3, and 4 of Part (*b*) , both forces approach zero as $x \to \infty$, as expected. In addition, the magnitude of the step-3 result approaches infinity as $x \to 2.0$ m, also as expected.

TAKING IT FURTHER The charge q_2 is located between charges q_1 and q_0, so you might think that the presence of q_2 will affect the force $\vec{F}_{10}$ exerted by q_1 on q_0. However, this is not the case. That is, the presence of q_2 does not effect the force $\vec{F}_{10}$ exerted by q_1 on q_0. (That this is so is called the principle of superposition.) Figure 21-9 shows the x component of the force on q_0 as a function of the position x of q_0 throughout the region $2.0\text{ m} < x < \infty$. Near q_2 the force due to q_2 dominates, and because opposite charges attract the force on q_2 is in the $-x$ direction. For $x \gg 2.0$ m the force is in the $+x$ direction. This is because for large x the distance between q_1 and q_2 is negligible so the force due to the two charges is almost the same as that for a single charge of +15 nC.

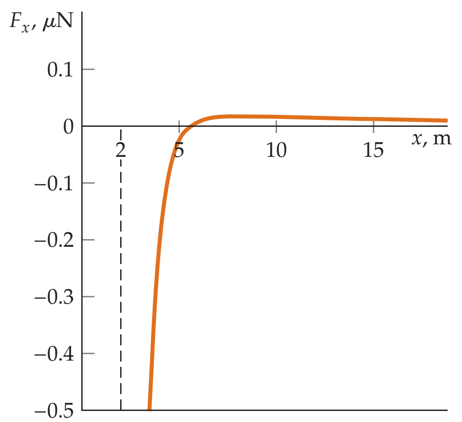


FIGURE 21-9

PRACTICE PROBLEM 21-4 If q_0 is at $x = 1.0$ m, find the total electric force acting on q_0.

For the charges in a system to remain stationary, there must be forces, other than the electric forces the charges exert on each other, acting on the charges so that the net force on each charge is zero. In the preceding example, and those that follow throughout the book, we assume that there are such forces so that all the charges remain stationary.

Example 21-5 Summing Forces in Two Dimensions

Charge $q_1 = +25$ nC is at the origin, charge $q_2 = -15$ nC is on the x axis at $x = 2.0$ m, and charge $q_0 = +20$ nC is at the point $x = 2.0$ m, $y = 2.0$ m as shown in Figure 21-10. Find the magnitude and direction of the resultant electric force on q_0.

PICTURE The resultant electric force is the vector sum of the individual forces exerted by each charge on q_0. We compute each force from Coulomb's law and write it in terms of its rectangular components.

SOLVE

1. Draw the coordinate axes showing the positions of the three charges. Show the resultant electric force $\vec{F}$ on charge q_0 as the vector sum of the forces $\vec{F}_{10}$ due to q_1 and $\vec{F}_{20}$ due to q_2 (Figure 21-10*a*):

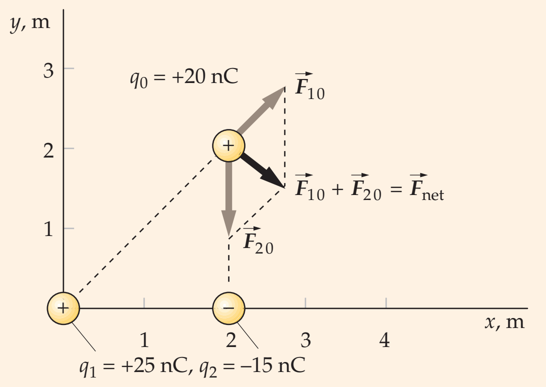


FIGURE 21-10*a*

2. The resultant force $\vec{F}$ on q_0 is the sum of the individual forces:

$$\vec{F} = \vec{F}_{10} + \vec{F}_{20}$$
$$\text{so} \quad \Sigma F_x = F_{10x} + F_{20x} \quad \text{and} \quad \Sigma F_y = F_{10y} + F_{20y}$$

3. The force $\vec{F}_{10}$ is directed away from the origin along the line from q_1 to q_0. Use $r_{10} = 2.0\sqrt{2}$ m for the distance between q_1 and q_0 to calculate its magnitude:

$$F_{10} = \frac{k|q_1 q_0|}{r_{10}^2} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(25 \times 10^{-9}\ \text{C})(20 \times 10^{-9}\ \text{C})}{(2.0\sqrt{2}\ \text{m})^2}$$
$$= 5.62 \times 10^{-7}\ \text{N}$$

4. Because $\vec{F}_{10}$ makes an angle of 45° with the x and y axes, its x and y components are equal to each other:

$$F_{10x} = F_{10y} = F_{10}\cos 45° = (5.62 \times 10^{-7}\text{N})\cos 45°$$
$$= 3.97 \times 10^{-7}\ \text{N}$$

5. The force $\vec{F}_{20}$ exerted by q_2 on q_0 is attractive and in the $-y$ direction as shown in Figure 21-10a:

$$\vec{F}_{20} = -\frac{k|q_2 q_0|}{r_{20}^2}\hat{j} = -\frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(15 \times 10^{-9}\ \text{C})(20 \times 10^{-9}\ \text{C})}{(2.0\ \text{m})^2}\hat{j}$$
$$= -(6.74 \times 10^{-7}\text{N})\hat{j}$$

6. Calculate the components of the resultant force:

$$F_x = F_{10x} + F_{20x} = (3.97 \times 10^{-7}\ \text{N}) + 0 = 3.97 \times 10^{-7}\ \text{N}$$
$$F_y = F_{10y} + F_{20y} = (3.97 \times 10^{-7}\ \text{N}) + (-6.74 \times 10^{-7}\ \text{N})$$
$$F_y = -2.77 \times 10^{-7}\ \text{N}$$

7. Draw the resultant force (Figure 21-10b) along with its two components:

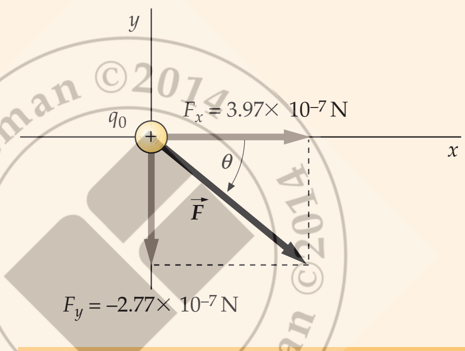


FIGURE 21-10b

8. The magnitude of the resultant force is found from its components:

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(3.97 \times 10^{-7}\ \text{N})^2 + (-2.77 \times 10^{-7}\ \text{N})^2}$$
$$= 4.84 \times 10^{-7}\ \text{N} = \boxed{4.8 \times 10^{-7}\ \text{N}}$$

9. The resultant force points to the right and downward as shown in Figure 21-10b, making an angle θ with the x axis given by:

$$\tan\theta = \frac{F_y}{F_x} = \frac{-2.77}{3.97} = -0.698$$
$$\theta = \tan^{-1}(-0.698) = -34.9° = \boxed{-35°}$$

CHECK We expect the two forces to be approximately equal in magnitude because even though q_1 is a bit larger than $|q_2|$, q_2 is a bit closer to q_0 than is q_1. Comparing the results of steps 3 and 5 shows agreement with this expectation.

PRACTICE PROBLEM 21-5 Express $\hat{r}_{10}$ in Example 21-5 in terms of $\hat{i}$ and $\hat{j}$.

PRACTICE PROBLEM 21-6 In Example 21-5, is the x component of the force $\vec{F}_{10} = (kq_1q_0/r_{10}^2)\hat{r}_{10}$ equal to kq_1q_0/x_{10}^2 (where x_{10} is the x component of $\hat{r}_{10}$)?

21-4 THE ELECTRIC FIELD

The electric force exerted by one charge on another is an example of an action-at-a-distance force, similar to the gravitational force exerted by one mass on another. The idea of action at a distance presents a difficult conceptual challenge. What is the mechanism by which one particle can exert a force on another across the empty space between the particles? Suppose that a charged particle at some point is suddenly moved. Does the force exerted on the second particle some distance r away change instantaneously? To address the challenge of action at a distance, the concept of the **electric field** is introduced. One charge produces an electric field $\vec{E}$ everywhere in space, and this field exerts the force on the second charge. Thus, it is the *field* $\vec{E}$ at the location of the second charge that exerts the force on it, not the first charge itself (which is some distance away). Changes in the field propagate through space at the speed of light, c. Thus, if a charge is suddenly moved, the force it exerts on a second charge a distance r away does not change until a time r/c later.

Figure 21-11*a* shows a set of point charges q_1, q_2, and q_3 arbitrarily arranged in space. These charges produce an electric field $\vec{E}$ everywhere in space. If we place a small positive test charge q_0 at some point near the three charges, there will be a force exerted on q_0 due to the other charges. The net force on q_0 is the vector sum of the individual forces exerted on q_0 by the other charges in the system. Because each of these forces is proportional to q_0, the net force will be proportional to q_0. The electric field $\vec{E}$ at a point is this force divided by q_0:*

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad (q_0 \text{ is small}) \qquad 21\text{-}5$$

DEFINITION—ELECTRIC FIELD

The SI unit of the electric field is the newton per coulomb (N/C). In addition, the test charge q_0 will exert a force on each of the other point charges (Figure 21-11*b*). Because these forces on the other charges might cause some of the other charges to move, the charge q_0 must be so small that the forces it exerts on the other charges are negligible. Thus, the electric field at the location in question is actually defined by Equation 21-5, but in the limit that q_0 approaches zero. Table 21-2 lists the magnitudes of some of the electric fields found in nature.

The electric field describes the condition in space set up by the system of point charges. By moving a test charge q_0 from point to point, we can find $\vec{E}$ at all points in space (except at any point occupied by a charge q). The electric field $\vec{E}$ is thus a vector function of position. The force exerted on a test charge q_0 at any point is related to the electric field at that point by

$$\vec{F} = q_0\vec{E} \qquad 21\text{-}6$$

PRACTICE PROBLEM 21-7

When a 5.0-nC test charge is placed at a certain point, it experiences a force of 2.0×10^{-4} N in the direction of increasing x. What is the electric field $\vec{E}$ at that point?

PRACTICE PROBLEM 21-8

What is the force on an electron placed at a point where the electric field is $\vec{E} = (4.0 \times 10^4 \text{ N/C})\hat{i}$?

The electric field due to a single point charge can be calculated from Coulomb's law. Consider a small, positive test charge q_0 at some point P a distance r_{iP} away from a charge q_i. The force on q_0 is

$$\vec{F}_{i0} = \frac{kq_iq_0}{r_{iP}^2}\hat{r}_{iP}$$

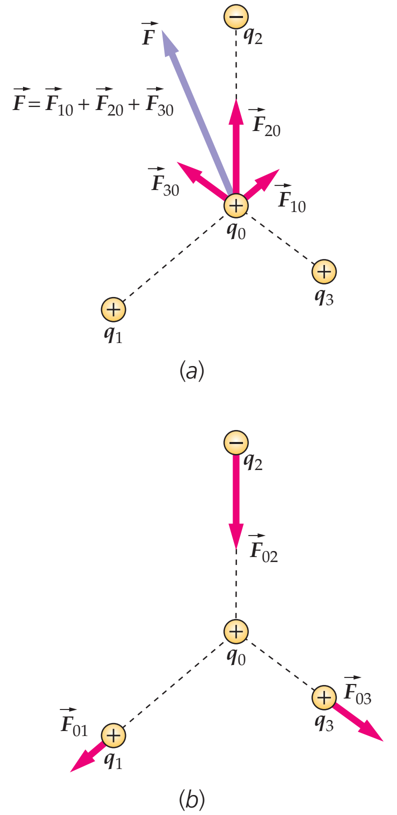


FIGURE 21-11 (*a*) A small test charge q_0 in the vicinity of a system of charges $q_1, q_2, q_3, \ldots$, experiences a resultant electric force $\vec{F}$ that is proportional to q_0. The ratio $\vec{F}/q_0$ is the electric field at that point. (*b*) The test charge q_0 also exerts a force on each of the surrounding charges, and each of these forces is proportional to q_0.

Table 21-2 Some Electric Fields in Nature

	E, N/C
In household wires	10^{-2}
In radio waves	10^{-1}
In the atmosphere	10^{2}
In sunlight	10^{3}
Under a thundercloud	10^{4}
In a lightning bolt	10^{4}
In an X-ray tube	10^{6}
At the electron in a hydrogen atom	5×10^{11}
At the surface of a uranium nucleus	2×10^{21}

* This definition is similar to that for the gravitational field of Earth, which was defined in Section 4-3 as the force per unit mass exerted by Earth on an object.

The electric field at point P due to charge q_i (Figure 21-12) is thus

$$\vec{E}_{iP} = \frac{kq_i}{r_{iP}^2}\hat{r}_{iP} \qquad 21\text{-}7$$

COULOMB'S LAW FOR $\vec{E}$

where $\hat{r}_{iP}$ is the unit vector pointing from the **source point** i to the **field point** P.

The resultant electric field at P due to a distribution of point charges is found by summing the fields due to each charge separately:

$$\vec{E}_P = \sum_i \vec{E}_{iP} \qquad 21\text{-}8$$

ELECTRIC FIELD $\vec{E}$ DUE TO A SYSTEM OF POINT CHARGES

That is, electric fields follow the principle of superposition.

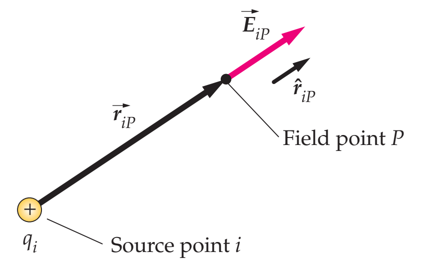


FIGURE 21-12 The electric field $\vec{E}$ at a field point P due to charge q_i at a source point i.

! Even though the expression for the electric field (Equation 21-7) does depend on the location of point P, it does *not* depend on the test charge q_0. That is, q_0 itself does not appear in Equation 21-7.

PROBLEM-SOLVING STRATEGY

Calculating the Resultant Electric Field

PICTURE To calculate the resultant electric field $\vec{E}_P$ at field point P due to a specified distribution of point charges, draw the charge configuration. Include coordinate axes and the field point on the drawing.

SOLVE

1. On the drawing label the distance r_{iP} from each charge to point P. Include an electric field vector $\vec{E}_{iP}$ for the electric field at P due to each point charge.
2. If the field point P and the point charges are not on a single line, then label the angle each individual electric field vector $\vec{E}_{iP}$ makes with one of the coordinate axes.
3. Calculate the component of each individual field vector $\vec{E}_{iP}$ along each axis and use these to calculate the components of the resultant electric field $\vec{E}_P$.

Example 21-6 Direction of Electric Field — *Conceptual*

A positive point charge $q_1 = +q$ and a negative point charge of $q_2 = -2q$ are located on the x axis at $x = a$ and $x = -a$, respectively, as shown in Figure 21-13. Consider the following regions on the x axis: region I ($x < -a$), region II ($-a < x < +a$), and region III ($x > a$). In which region, or regions, is there a point at which the resultant electric field is equal to zero?

PICTURE Let $\vec{E}_1$ and $\vec{E}_2$ be the electric fields due to q_1 and q_2, respectively. Because q_1 is positive, $\vec{E}_1$ points away from q_1 everywhere, and because q_2 is negative, $\vec{E}_2$ points toward from q_2 everywhere. The resultant electric field $\vec{E}$ is equal to the sum of the electric fields of the two charges ($\vec{E} = \vec{E}_1 + \vec{E}_2$). The resultant field is zero if $\vec{E}_1$ and $\vec{E}_2$ are equal in magnitude and oppositely directed. The magnitude of the electric field due to a point charge approaches infinity at points close to a point charge. In addition, at points far from the charge configuration, the electric field approaches the electric field of a point charge equal to $q_1 + q_2$ that is located at the center of charge. The electric field far from the charge configuration is that of a negative point charge because $q_1 + q_2$ is negative.

SOLVE

1. Sketch a figure showing the two charges, the x axis, and the electric fields due the charges at points on the x axis in each of regions I, II, and III. Label these points P_I, P_{II}, and P_{III}, respectively (Figure 21-13):

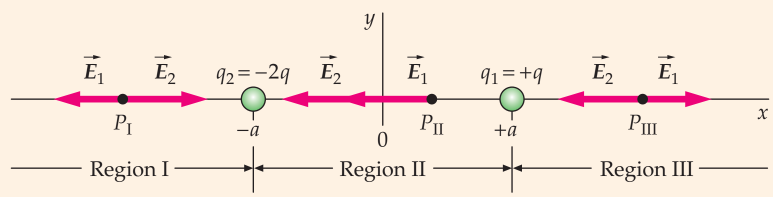


FIGURE 21-13

2. Check to see if the two electric field vectors can be equal in magnitude and opposite in direction anywhere in region I:

Throughout region I, the two electric field vectors are oppositely directed. However, each point in the region is closer to q_2 $(= -2q)$ than q_1 $(= +q)$, so E_2 is greater than E_1 at each point in the region. Thus, in region I there are no points where the electric field is equal to zero.

3. Check to see if the two electric field vectors can be equal in magnitude and opposite in direction anywhere in region II:

Throughout region II, the two electric field vectors are in the same direction at each point on the x axis. Thus, in region II there are no points where the electric field is equal to zero.

4. Check to see if the two electric field vectors can be equal in magnitude and opposite in direction anywhere in region III:

Throughout region III, the two electric field vectors are oppositely directed. At points very close to $x = a$, E_1 is greater than E_2 (because at points close to a point charge the magnitude of the electric field due to that charge approaches infinity). However, at points where $x \gg a$, E_2 is greater than E_1 (because at large distances from the two charges the field direction is determined by the sign of $q_1 + q_2$). Thus, there must be a point somewhere in region III where E_1 is equal to E_2. At that point the net electric field is zero.

CHECK The resultant electric field is zero at a point in region III, the region in which $\vec{E}_1$ and $\vec{E}_2$ are oppositely directed AND in which all points are farther from q_2, the charge with the larger magnitude, than from q_1. This result is as one would expect.

Example 21-7 Electric Field on a Line through Two Positive Point Charges

A positive point charge $q_1 = +8.0$ nC is on the x axis at $x = x_1 = -1.0$ m, and a second positive point charge $q_2 = +12$ nC is on the x axis at $x = x_2 = 3.0$ m. Find the net electric field (a) at point A on the x axis at $x = 6.0$ m, and (b) at point B on the x axis at $x = 2.0$ m.

PICTURE Let $\vec{E}_1$ and $\vec{E}_2$ be the electric fields due to q_1 and q_2, respectively. Because q_1 is positive, $\vec{E}_1$ points away from q_1 everywhere, and because q_2 is positive, $\vec{E}_2$ points away from q_2 everywhere. We calculate the resultant field using $\vec{E} = \vec{E}_1 + \vec{E}_2$.

SOLVE

(a) 1. Draw the charge configuration and place the field point A on the x axis at the appropriate place. Draw vectors representing the electric field at A due to each point charge. Repeat this procedure for field point B (Figure 21-14):

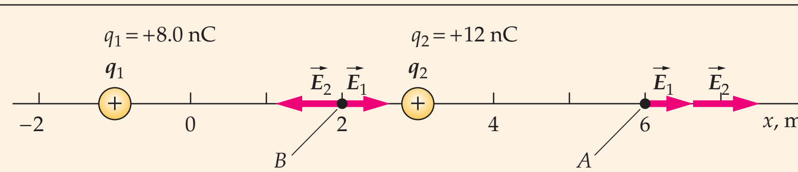


FIGURE 21-14 Because q_1 is a positive charge, $\vec{E}_1$ points away from q_1, at both point A and point B. Because q_2 is a positive charge, $\vec{E}_2$ points away from q_2 at both point A and point B.

2. Calculate $\vec{E}$ at point A, using $r_{1A} = |x_A - x_1| = 6.0\text{ m} - (-1.0\text{ m}) = 7.0$ m and $r_{2A} = |x_A - x_2| = 6.0\text{ m} - (3.0\text{ m}) = 3.0$ m:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{kq_1}{r_{1A}^2}\hat{r}_{1A} + \frac{kq_2}{r_{2A}^2}\hat{r}_{2A} = \frac{kq_1}{(x_A - x_1)^2}\hat{i} + \frac{kq_2}{(x_A - x_2)^2}\hat{i}$$

$$= \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(8.0 \times 10^{-9}\text{ C})}{(7.0\text{ m})^2}\hat{i} + \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(12 \times 10^{-9}\text{ C})}{(3.0\text{ m})^2}\hat{i}$$

$$= (1.47\text{ N/C})\hat{i} + (12.0\text{ N/C})\hat{i} = \boxed{(13\text{ N/C})\hat{i}}$$

(*b*) Calculate $\vec{E}$ at point *B*, where
$r_{1B} = |x_B - x_1| = 2.0\text{ m} - (-1.0\text{ m}) = 3.0\text{ m}$
and $r_{2B} = |x_B - x_2| = |2.0\text{ m} - (3.0\text{ m})| = 1.0\text{ m}$:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{kq_1}{r_{1B}^2}\hat{r}_{1B} + \frac{kq_2}{r_{2B}^2}\hat{r}_{2B} = \frac{kq_1}{(x_B - x_1)^2}\hat{i} + \frac{kq_2}{(x_B - x_2)^2}(-\hat{i})$$

$$= \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(8.0 \times 10^{-9}\text{ C})}{(3.0\text{ m})^2}\hat{i} - \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(12 \times 10^{-9}\text{ C})}{(1.0\text{ m})^2}\hat{i}$$

$$= (7.99\text{ N/C})\hat{i} - (108\text{ N/C})\hat{i} = \boxed{-(100\text{ N/C})\hat{i}}$$

CHECK The Part (*b*) result is large and in the $-x$ direction. This result is expected because point *B* is close to q_2, and q_2 is a large positive charge (+12 nC) that produces electric field $\vec{E}_2$ in the $-x$ direction at *B*.

TAKING IT FURTHER The resultant electric field at source points close to $q_1 = +8.0$-nC is dominated by the field $\vec{E}_1$ due to q_1. There is one point between q_1 and q_2 where the resultant electric field is zero. A test charge placed at this point would experience no electric force. A sketch of E_x versus *x* for this charge configuration is shown in Figure 21-15.

PRACTICE PROBLEM 21-9 Regarding Example 21-7, find the point on the *x* axis where the electric field is zero.

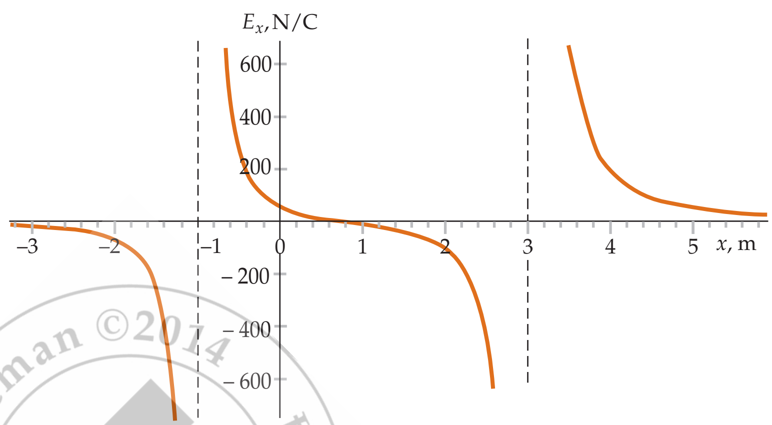


FIGURE 21-15

Example 21-8 Electric Field Due to Point Charges on the *x* Axis — *Try It Yourself*

A point charge $q_1 = +8.0$ nC is at the origin and a second point charge $q_2 = +12.0$ nC is on the *x* axis at $x = 4.0$ m. Find the electric field on the *y* axis at $y = 3.0$ m.

PICTURE As in Example 21-7, $\vec{E} = \vec{E}_1 + \vec{E}_2$. At points on the *y* axis, the electric field $\vec{E}_1$ due to charge q_1 is directed along the *y* axis, and the field $\vec{E}_2$ due to charge q_2 is in the second quadrant. To find the resultant field $\vec{E}$, we first find the *x* and *y* components of $\vec{E}$.

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps

1. Sketch the two charges and the field point. Include the coordinate axes. Draw the electric field due to each charge at the field point and label distances and angles appropriately (Figure 21-16*a*):

Answers

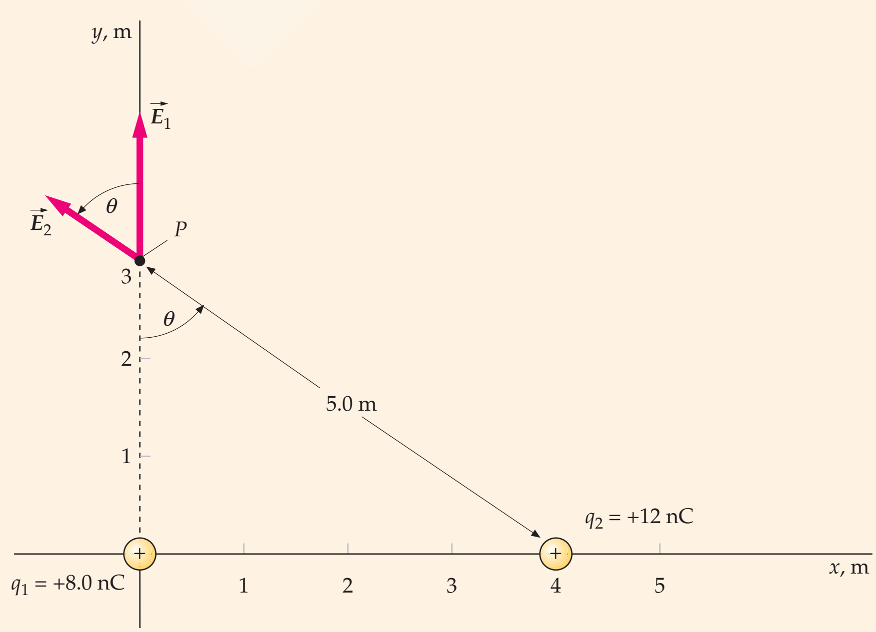


FIGURE 21-16*a*

2. Calculate the magnitude of the field $\vec{E}_1$ at (0, 3.0 m) due to q_1. Find the x and y components of $\vec{E}_1$
   $E_1 = kq_1/y^2 = 7.99\text{ N/C}$
   $E_{1x} = 0,\ E_{1y} = E_1 = 7.99\text{ N/C}$

3. Calculate the magnitude of the field $\vec{E}_2$ at $(0, y)$ due to q_2.
   $E_2 = 4.32\text{ N/C}$

4. Write the x and y components of $\vec{E}_2$ in terms of the angle θ.
   $E_{2x} = -E_2 \sin\theta;\ E_{2y} = E_2 \cos\theta$

5. Compute $\sin\theta$ and $\cos\theta$.
   $\sin\theta = 0.80;\ \cos\theta = 0.60$

6. Calculate E_{2x} and E_{2y}.
   $E_{2x} = -3.46\text{ N/C};\ E_{2y} = 2.59\text{ N/C}$

7. Sketch the components of the resultant field. Include both the vector $\vec{E}$ and the angle $\vec{E}$ makes with the x axis (Figure 21-16b):

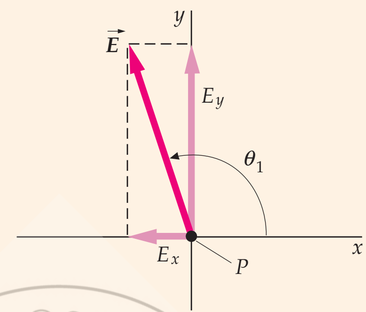


FIGURE 21-16*b*

8. Find the x and y components of the resultant field $\vec{E}$.
   $E_x = E_{1x} + E_{2x} = -3.46\text{ N/C}$
   $E_y = E_{1y} + E_{2y} = 10.6\text{ N/C}$

9. Calculate the magnitude of $\vec{E}$ from its components.
   $E = \sqrt{E_x^2 + E_y^2} = 11.2\text{ N/C} = \boxed{11\text{ N/C}}$

10. Find the angle θ_1 made by $\vec{E}$ with the x axis.
   $\theta_1 = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \boxed{108°}$

CHECK As expected, E is larger than either E_1 or E_2, but less than $E_1 + E_2$. (This result is expected because the angle between $\vec{E}_1$ and $\vec{E}_2$ is less than 90°.)

Example 21-9 Electric Field Due to Two Equal and Opposite Charges

A charge $+q$ is at $x = a$ and a second charge $-q$ is at $x = -a$ (Figure 21-17). (a) Find the electric field on the x axis at an arbitrary point $x > a$. (b) Find the limiting form of the electric field for $x \gg a$.

PICTURE We calculate the electric field at point P using the principle of superposition, $\vec{E}_P = \vec{E}_{1P} + \vec{E}_{2P}$. For $x > a$, the electric field $\vec{E}_+$ due to the positive charge is in the $+x$ direction and the electric field $\vec{E}_-$ due to the negative charge is in the $-x$ direction. The distances are $x - a$ to the positive charge and $x - (-a) = x + a$ to the negative charge.

SOLVE

(a) 1. Draw the charge configuration on a coordinate axis and label the distances from each charge to the field point (Figure 21-17):

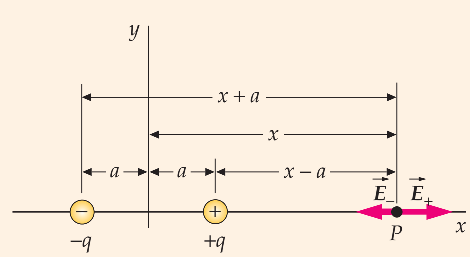


FIGURE 21-17

2. Calculate $\vec{E}$ due to the two charges for $x > a$: (*Note:* The equation on the right holds only for $x > a$.

$$\vec{E} = \vec{E}_+ + \vec{E}_- = \frac{kq}{[x-a]^2}\hat{i} + \frac{kq}{[x-(-a)]^2}\left(-\hat{i}\right)$$
$$= kq\left[\frac{1}{(x-a)^2} - \frac{1}{(x+a)^2}\right]\hat{i}$$

3. Put the terms in square brackets under a common denominator and simplify:

$$\vec{E} = kq\left[\frac{(x+a)^2-(x-a)^2}{(x+a)^2(x-a)^2}\right]\hat{i} = \boxed{kq\frac{4ax}{(x^2-a^2)^2}\hat{i} \qquad x > a}$$

(*b*) In the limit $x \gg a$, we can neglect a^2 compared with x^2 in the denominator:

$$\vec{E} = kq\frac{4ax}{(x^2-a^2)^2}\hat{i} \approx kq\frac{4ax}{x^4}\hat{i} = \boxed{\frac{4kqa}{x^3}\hat{i} \qquad x \gg a}$$

CHECK Both boxed answers approach zero as x approaches infinity, which is as expected.

TAKING IT FURTHER Figure 21-18 shows E_x versus x for all x, for $q = 1.0$ nC and $a = 1.0$ m. For $|x| \gg a$ (far from the charges), the field is given by

$$\vec{E} = \frac{4kqa}{|x|^3}\hat{i} \qquad |x| \gg a$$

Between the charges, the contribution from each charge is in the negative direction. An expression for $\vec{E}$ is

$$\vec{E} = \frac{kq}{(x-a)^2}\hat{e}_+ + \frac{k(-q)}{(x+a)^2}\hat{e}_- \qquad -a < x < a$$

where $\hat{e}_+$ is a unit vector that points away from the point $x = a$ for all values of x (except $x = a$) and $\hat{e}_-$ is a unit vector that points away from the point $x = -a$ for all values of x (except $x = -a$). (Note that $\hat{e}_+ = \frac{x-a}{|x-a|}\hat{i}$ and $\hat{e}_- = \frac{x+a}{|x+a|}\hat{i}$.)

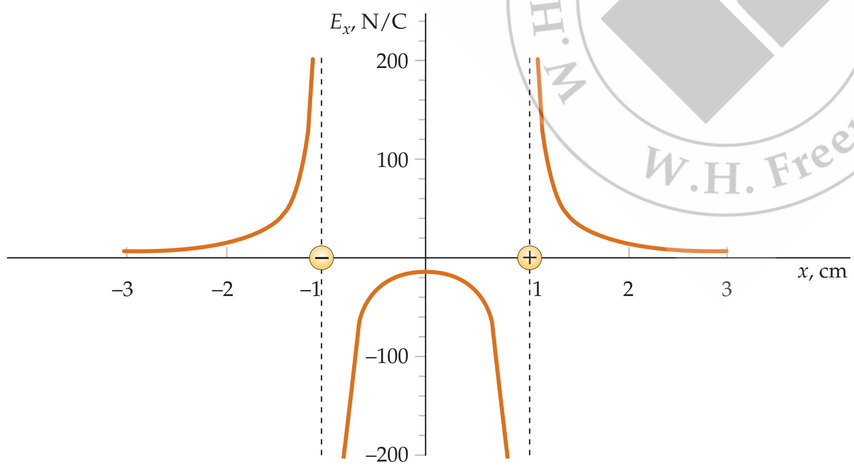


FIGURE 21-18 A plot of E_x versus x on the x axis for the charge distribution in Example 21-9.

ELECTRIC DIPOLES

A system of two equal and opposite charges q separated by a small distance L is called a **dipole.** Its strength and orientation are described by the **dipole moment** $\vec{p}$, which is a vector that points from the negative charge $-q$ toward the positive charge $+q$ and has the magnitude $q\vec{L}$ (Figure 21-19):

$$\vec{p} = q\vec{L} \qquad 21\text{-}9$$

DEFINITION—DIPOLE MOMENT

where $\vec{L}$ is the position of the positive charge relative to the negative charge.

For the system of charges in Figure 21-17, $\vec{L} = 2a\hat{i}$ and the dipole moment is

$$\vec{p} = 2aq\hat{i}$$

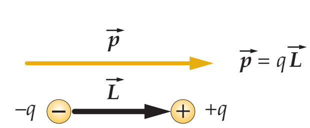


FIGURE 21-19 A dipole consists of a pair of equal and opposite charges. The dipole moment is $\vec{p} = q\vec{L}$, where q is the magnitude of one of the charges and $\vec{L}$ is the position of the negative charge relative to the positive charge.

In terms of the dipole moment $\vec{p}$, the electric field on the axis of the dipole at a point a great distance $|x|$ away is in the same direction as $\vec{p}$ and has magnitude

$$E = \frac{2kp}{|x|^3} \qquad 21\text{-}10$$

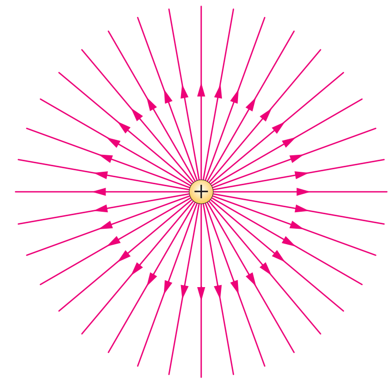
(see Example 21-9). At a point far from a dipole in any direction, the magnitude of the electric field is proportional to the magnitude of the dipole moment and decreases with the cube of the distance. If a system has a nonzero net charge, the electric field decreases as $1/r^2$ at large distances. In a system that has zero net charge, the electric field falls off more rapidly with distance. In the case of a dipole, the field falls off as $1/r^3$ in all directions.

21-5 ELECTRIC FIELD LINES

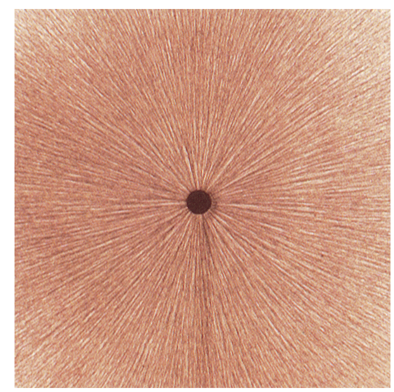
We can visualize the electric field by drawing a number of directed curved lines, called **electric field lines,** to indicate both the magnitude and the direction of the field. At any given point, the field vector $\vec{E}$ is tangent to the line through that point. (Electric field lines are also called *lines of force* because they show the direction of the electric force exerted on a positive test charge.) At points very near a positive point charge, the electric field $\vec{E}$ points directly away from the charge. Consequently, the electric field lines very near a positive charge also point directly away from the charge. Similarly, very near a negative point charge the electric field lines point directly toward the charge.

Figure 21-20 shows the electric field lines of a single positive point charge. The spacing of the lines is related to the strength of the electric field. As we move away from the charge, the field becomes weaker and the lines become farther apart. Consider an imaginary spherical surface of radius r that has its center at the charge. Its area is $4\pi r^2$. Thus, as r increases, the density of the field lines (the number of lines per unit area through a surface element normal to the field lines) decreases as $1/r^2$, the same rate of decrease as E. So, we adopt the convention of drawing a fixed number of lines from a point charge, the number being proportional to the charge q, and if we draw the lines equally spaced very near the point charge, the field strength is indicated by the density of the lines. The more closely spaced the lines, the stronger the electric field. The magnitude of the electric field is also called the **electric field strength.**

Figure 21-21 shows the electric field lines for two equal positive point charges q separated by a small distance. Near each point charge, the field is approximately that due to that charge alone. This is because the magnitude of the field of a single point charge is extremely large at points very close to the charge, and because the second charge is relatively far away. Consequently, the field lines near either charge are radial

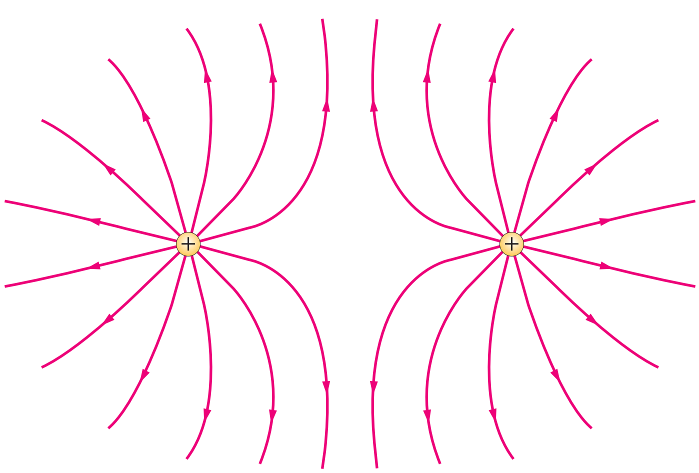


(a)

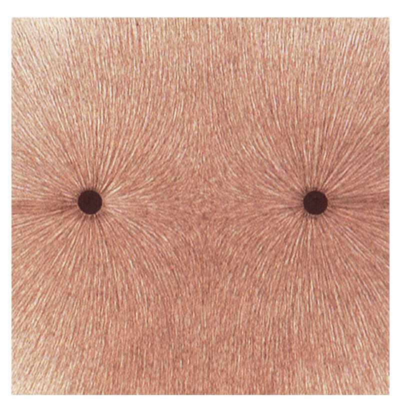


(b)

FIGURE 21-20 (*a*) Electric field lines of a single positive point charge. If the charge were negative, the arrows would be reversed. (*b*) The same electric field lines shown by bits of thread suspended in oil. The electric field of the charged object in the center induces opposite charges on the ends of each bit of thread, causing the threads to align themselves parallel to the field. *(Harold M. Waage.)*



(a)



(b)

FIGURE 21-21 (*a*) Electric field lines due to two positive point charges. The arrows would be reversed if both charges were negative. (*b*) The same electric field lines shown by bits of thread in oil. *(Harold M. Waage.)*

and equally spaced. Because the charges are of equal magnitude, we draw an equal number of lines originating from each charge. At very large distances, the details of the charge configuration are not important and the electric field lines are indistinguishable from those of a point charge of magnitude $2q$ a very large distance away. (For example, if the two charges were 1 mm apart and we look at the field lines near a point 100 km away, the field lines would look like those of a single charge of magnitude $2q$ a distance 100 km away.) So at a large distance from the charges, the field is approximately the same as that due to a point charge $2q$ and the lines are approximately equally spaced. Looking at Figure 21-21, we see that the density of field lines in the region between the two charges is small compared to the density of lines in the region just to the left and just to the right of the charges. This indicates that the magnitude of the electric field is weaker in the region between the charges than it is in the region just to the right or left of the charges, where the lines are more closely spaced. This information can also be obtained by direct calculation of the field at points in these regions.

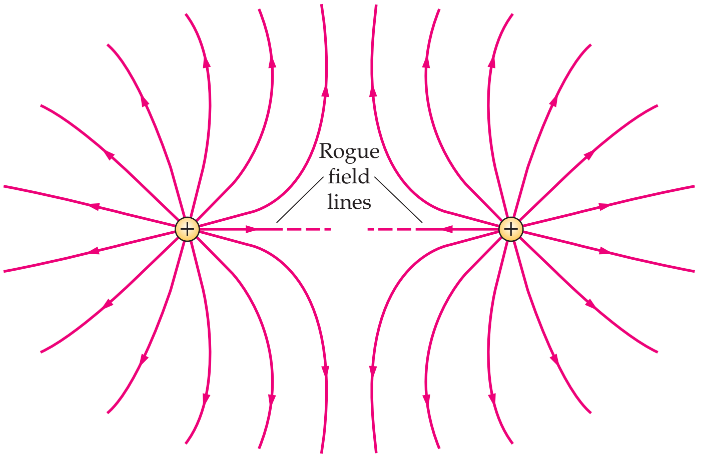


FIGURE 21-22 There are infinitely many field lines emanating from the two charges, two of which are rogue field lines. These rogue field lines terminate at the point midway between the two charges.

We can apply this reasoning to sketch the electric field lines for any system of point charges. Very near each charge, the field lines are equally spaced and emanate from or terminate on the charge radially, depending on the sign of the charge. Very far from all the charges, the detailed configuration of the system of charges is not important, so the field lines are like those of a single point charge having the net charge of the system. The rules for drawing electric field lines are summarized in the following Problem-Solving Strategy.

PROBLEM-SOLVING STRATEGY

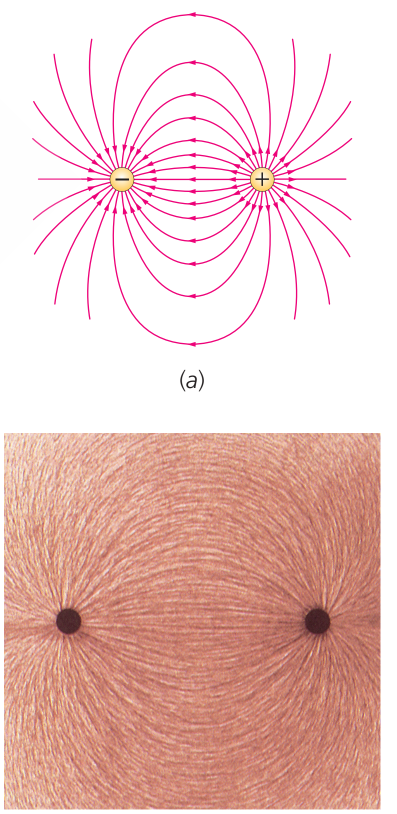
Drawing Field Lines

PICTURE Electric field lines emanate from positive charges and terminate on negative charges.*

SOLVE

1. The lines emanating from (or terminating on) an isolated point charge are drawn uniformly spaced as they emanate (or terminate).
2. The number of lines emanating from a positive charge (or terminating on a negative charge) is proportional to the magnitude of the charge.
3. The density of the lines at any point (the number of lines per unit area through a surface element normal to the lines) is proportional to the magnitude of the field there.
4. At large distances from a system of charges that has a nonzero net charge, the field lines are equally spaced and radial, as if they emanated from (or terminated on) a single point charge equal to the total charge of the system.

CHECK Make sure that the field lines never intersect each other. (If two field lines intersected, that would indicate two directions for $\vec{E}$ at the point of intersection.)



(*a*)

(*b*)

FIGURE 21-23 (*a*) Electric field lines for a dipole. (*b*) The same field lines shown by bits of thread in oil. *(Harold M. Waage.)*

Figure 21-23 shows the electric field lines due to a dipole. Very near the positive charge, the lines are directed radially outward. Very near the negative charge, the

* Rogue field lines are field lines that do not follow this rule. An example of a rogue field line is a line that leaves one of the positive charges in Figure 21-22 and is directed toward the other charge. This field line terminates at the point midway between the two charges-as does a corresponding field line emanating from the second positive charge in the figure. For these two charges there are infinitely many field lines, two of which are rogue field lines.

lines are directed radially inward. Because the charges have equal magnitudes, the number of lines that begin at the positive charge equals the number that end at the negative charge. In this case, the field is strong in the region between the charges, as indicated by the high density of field lines in this region.

Figure 21-24*a* shows the electric field lines for a negative charge $-q$ at a small distance from a positive charge $+2q$. Twice as many lines emanate from the positive charge as terminate on the negative charge. Thus, half the lines emanating from the positive charge $+2q$ terminate on the negative charge $-q$; the other half of the lines emanating from the positive charge continue on indefinitely. Very far from the charges (Figure 21-24*b*), the lines are approximately symmetrically spaced and point radially away from a single point, just as they would for a single positive point charge $+q$.

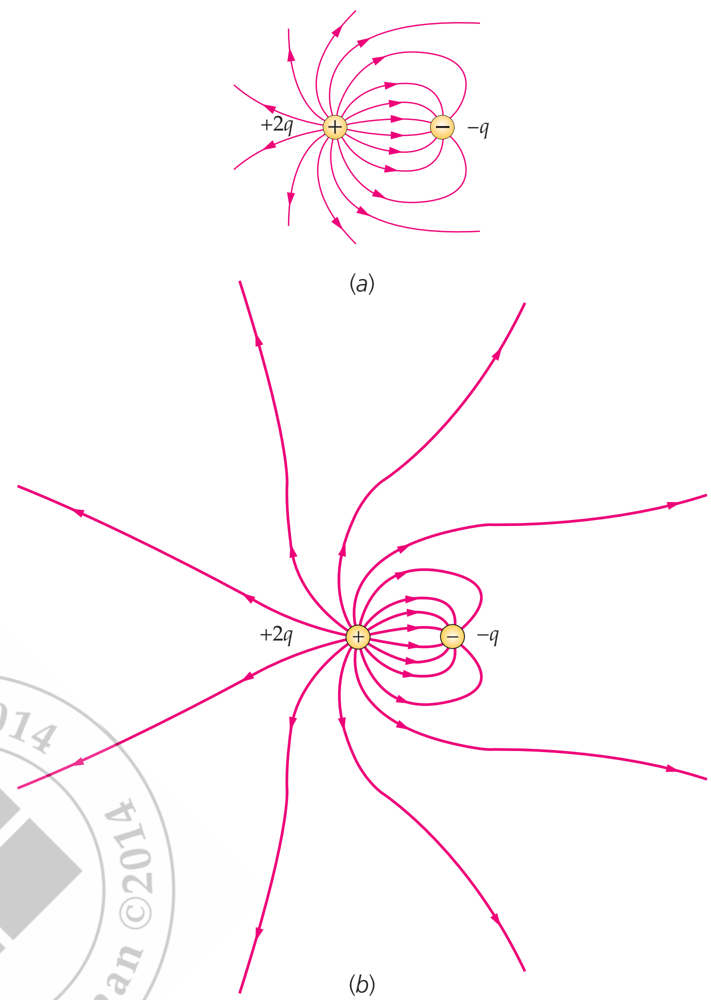


FIGURE 21-24 (*a*) Electric field lines for a point charge $+2q$ and a second point charge $-q$. (*b*) At great distances from the charges, the field lines approach those for a single point charge $+q$ located at the center of charge.

Example 21-10 Field Lines for Two Conducting Spheres — *Conceptual*

The electric field lines for two conducting spheres are shown in Figure 21-25. What is the sign of the charge on each sphere, and what are the relative magnitudes of the charges on the spheres?

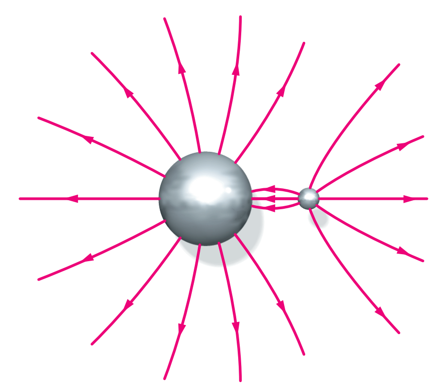


FIGURE 21-25

PICTURE The charge on an object is positive if more field lines emanate from it than terminate on it, and negative if more terminate on it than emanate from it. The ratio of the magnitudes of the charges equals the ratio of the net number of lines emanating from or terminating on the spheres.

SOLVE

1. By counting field lines, determine the net number of field lines emanating from the larger sphere: Because 11 electric field lines emanate from the larger sphere and 3 lines terminate on it, the net number of lines emanating from it is 8.
2. By counting field lines, determine the net number of field lines emanating from the smaller sphere: Because 8 electric field lines emanate from the smaller sphere and no lines terminate on it, the net number of lines emanating from it is 8.
3. Determine the sign of the charge on each sphere: Because both spheres have more field lines emanating from than terminating on them, **both spheres are positively charged.**
4. Determine the relative magnitudes of the charges on the two spheres: Because both spheres have the same net number of lines emanating from them, the **charges on them are equal in magnitude.**

The convention relating the electric field strength to the density of the electric field lines works only because the electric field varies inversely as the square of the distance from a point charge. Because the gravitational field of a point mass also varies inversely as the square of the distance, field-line drawings are also useful for picturing gravitational fields. Near a point mass, the gravitational field lines terminate on the mass just as electric field lines terminate on a negative charge. However, unlike electric field lines near a positive charge, there are no points in space from which gravitational field lines emanate. That is because the gravitational force between two masses is never repulsive.

21-6 ACTION OF THE ELECTRIC FIELD ON CHARGES

A uniform electric field can exert a force on a single charged particle and can exert both a torque and a net force on an electric dipole.

MOTION OF POINT CHARGES IN ELECTRIC FIELDS

When a particle that has a charge q is placed in an electric field $\vec{E}$, it experiences a force $q\vec{E}$. If the electric force is the only force acting on the particle, the particle has acceleration

$$\vec{a} = \frac{\Sigma\vec{F}}{m} = \frac{q}{m}\vec{E}$$

where m is the mass of the particle. (If the particle is an electron, its speed in an electric field is often a significant fraction of the speed of light. In such cases, Newton's laws of motion must be modified by Einstein's special theory of relativity.) If the electric field is known, the charge-to-mass ratio of the particle can be determined from the measured acceleration. J. J. Thomson used the deflection of electrons in a uniform electric field in 1897 to demonstrate the existence of electrons and to measure their charge-to-mass ratio. Familiar examples of devices that rely on the motion of electrons in electric fields are oscilloscopes, computer monitors, and television sets that use cathode-ray-tube displays.

A cathode-ray-tube display used for color television. The beams of electrons from the electron gun on the left activate phosphors on the screen at the right, giving rise to bright spots whose colors depend on the relative intensity of each beam. Electric fields between deflection plates in the gun (or magnetic fields from coils surrounding the gun) deflect the beams. The beams sweep across the screen in a horizontal line, are deflected downward, then sweep across again. The entire screen is covered in this way 30 times per second. *(Science & Society Picture Library/Contributor/Getty Images.)*

Example 21-11 Electron Moving Parallel to a Uniform Electric Field

An electron is projected into a uniform electric field $\vec{E} = (1000\text{ N/C})\hat{i}$ with an initial velocity $\vec{v}_0 = (2.00 \times 10^6\text{ m/s})\hat{i}$ in the direction of the field (Figure 21-26). How far does the electron travel before it is brought momentarily to rest?

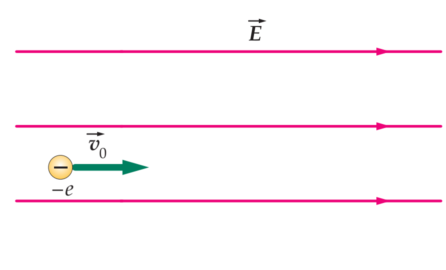


FIGURE 21-26

PICTURE Because the charge of the electron is negative, the force $\vec{F} = -e\vec{E}$ acting on the electron is in the direction opposite that of the field. Because $\vec{E}$ is constant, the force is constant and we can use constant acceleration formulas from Chapter 2. We choose the field to be in the $+x$ direction.

SOLVE

1. The displacement Δx is related to the initial and final velocities: $v_x^2 = v_{0x}^2 + 2a_x\Delta x$

2. The acceleration is obtained from Newton's second law: $a_x = \frac{F_x}{m} = \frac{-eE_x}{m}$

3. When $v_x = 0$, the displacement is:

$$\Delta x = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - v_{0x}^2}{2(-eE_x/m)} = \frac{mv_0^2}{2eE} = \frac{(9.11 \times 10^{-31}\text{ kg})(2.00 \times 10^6\text{ m/s})^2}{2(1.60 \times 10^{-19}\text{ C})(1000\text{ N/C})}$$

$$= 1.14 \times 10^{-2}\text{ m} = \boxed{1.14\text{ cm}}$$

CHECK The displacement Δx is positive, as is expected for something moving in the $+x$ direction.

Example 21-12 Electron Moving Perpendicular to a Uniform Electric Field

An electron enters a uniform electric field $\vec{E} = (-2.0\ \text{kN/C})\hat{j}$ with an initial velocity $\vec{v}_0 = (1.0 \times 10^6\ \text{m/s})\hat{i}$ perpendicular to the field (Figure 21-27). (*a*) Compare the gravitational force acting on the electron to the electric force acting on it. (*b*) By how much has the electron been deflected after it has traveled 1.0 cm in the x direction?

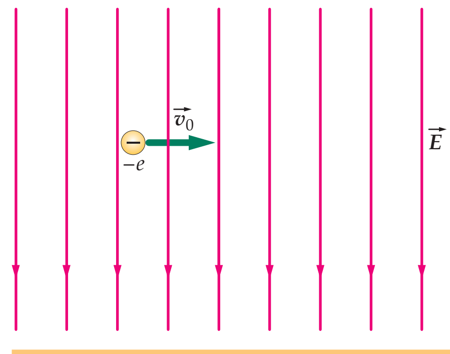


FIGURE 21-27

PICTURE (*a*) Calculate the ratio of the magnitude of the electric force $|q|E = eE$ to that of the gravitational force mg. (*b*) Because mg is, by comparison, negligible, the net force on the electron is equal to the vertically upward electric force. The electron thus moves with constant horizontal velocity v_x and is deflected upward by an amount $\Delta y = \frac{1}{2}at^2$, where t is the time to travel 1.0 cm in the x direction.

SOLVE

(*a*) 1. Calculate the ratio of the magnitude of the electric force, F_e, to the magnitude of the gravitational force, F_g:

$$\frac{F_e}{F_g} = \frac{eE}{mg} = \frac{(1.60 \times 10^{-19}\ \text{C})(2000\ \text{N/C})}{(9.11 \times 10^{-31}\ \text{kg})(9.81\ \text{N/kg})} = \boxed{3.6 \times 10^{13}}$$

(*b*) 1. Express the vertical deflection in terms of the acceleration a and time t:

$$\Delta y = \frac{1}{2}a_y t^2$$

2. Express the time required for the electron to travel a horizontal distance Δx with constant horizontal velocity v_0:

$$t = \frac{\Delta x}{v_0}$$

3. Use this result for t and eE/m for a_y to calculate Δy:

$$\Delta y = \frac{1}{2}\frac{eE}{m}\left(\frac{\Delta x}{v_0}\right)^2 = \frac{1}{2}\frac{(1.6 \times 10^{-19}\ \text{C})(2000\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}}\left(\frac{0.010\ \text{m}}{10^6\ \text{m/s}}\right)^2$$

$$= \boxed{1.8\ \text{cm}}$$

CHECK The step-4 result is positive (upward), as is expected for an object accelerating upward that was initially moving horizontally.

TAKING IT FURTHER (*a*) As is usually the case, the electric force is huge compared with the gravitational force. Thus, it is not necessary to consider gravity when designing a cathode-ray tube, for example, or when calculating the deflection in the problem above. In fact, a television picture tube works equally well upside down and right side up, as if gravity were not even present. (*b*) The path of an electron moving in a uniform electric field is a parabola, the same as the path of a neutral particle moving in a uniform gravitational field.

Example 21-13 The Electric Field in an Ink-Jet Printer *Context-Rich*

You have just finished printing out a long essay for your English professor, and you wonder about how the ink-jet printer knows where to place the ink. You search the Internet and find a picture (Figure 21-28) showing that the ink drops are given a charge and pass between a pair of oppositely charged metal plates that provide a uniform electric field in the region between the plates. Because you have been studying the electric field in physics class, you wonder if you can determine how large a field is used in this type of printer. You search further and find that the 40.0-μm-diameter ink drops have an initial velocity of 40.0 m/s, and that a drop that has a 2.00-nC charge is deflected upward a distance of 3.00 mm as the drop travels through the 1.00-cm-long region between the plates. Find the magnitude of the electric field. (Neglect any effects of gravity on the motion of the drops.)

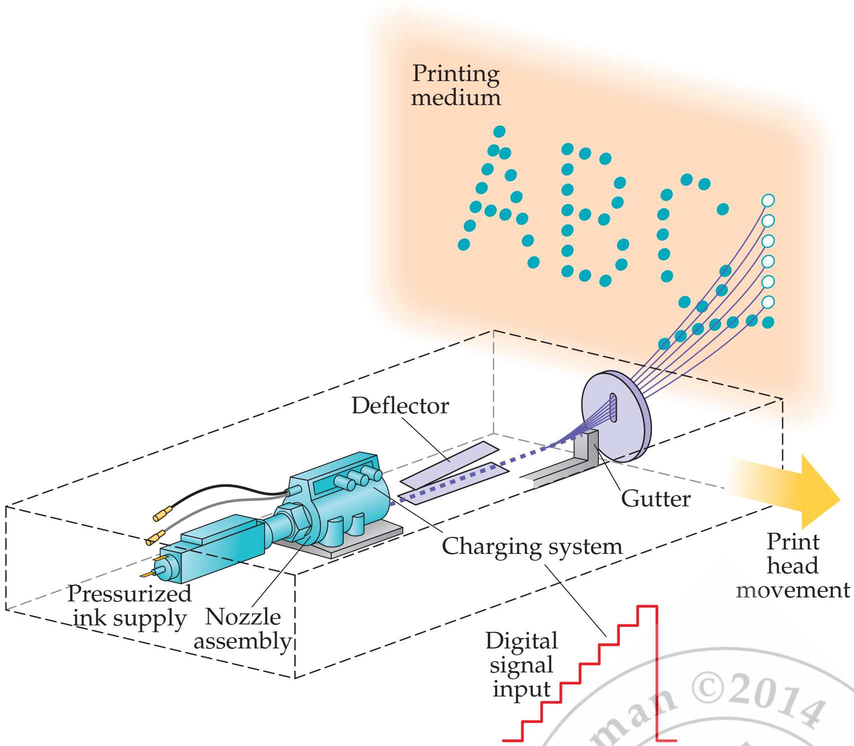


FIGURE 21-28 An ink-jet used for printing. The ink exits the nozzle in discrete droplets. Any droplet destined to form a dot on the image is given a charge. The deflector consists of a pair of oppositely charged plates. The greater the charge a drop receives, the higher the drop is deflected as it passes between the deflector plates. Drops that do not receive a charge are not deflected upward. These drops end up in the gutter, and the ink is returned to the ink reservoir. *(Courtesy of Videojet Systems International.)*

PICTURE The electric field $\vec{E}$ exerts a constant electric force $\vec{F}$ on the drop as it passes between the two plates, where $\vec{F} = q\vec{E}$. We are looking for E. We can get the force $\vec{F}$ by determining the mass and acceleration $\vec{F} = m\vec{a}$. The acceleration can be found from kinematics and mass can be found using the radius. Assume the density ρ of ink is 1000 kg/m³ (the same as the density of water).

SOLVE

1. The electric field strength equals the force to charge ratio: $E = \dfrac{F}{q}$
2. The force, which is in the $+y$ direction (upward), equals the mass multiplied by the acceleration: $F = ma_y$
3. The vertical displacement is obtained using a constant-acceleration kinematic formula with $v_{0y} = 0$: $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}a_y t^2$
4. The time is how long it takes for the drop to travel the $\Delta x = 1.00$ cm at $v_0 = 40.0$ m/s: $\Delta x = v_{0x}t = v_0 t \quad \text{so} \quad t = \Delta x/v_0$
5. Solving for a_y gives: $a_y = \dfrac{2\Delta y}{t^2} = \dfrac{2\Delta y}{(\Delta x/v_0)^2} = \dfrac{2v_0^2\Delta y}{(\Delta x)^2}$
6. The mass equals the density multiplied by the volume: $m = \rho V = \rho\frac{4}{3}\pi r^3$
7. Solve for E:

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{\rho\frac{4}{3}\pi r^3}{q}\frac{2v_0^2\Delta y}{(\Delta x)^2} = \frac{8\pi}{3}\frac{\rho r^3 v_0^2 \Delta y}{q(\Delta x)^2}$$

$$= \frac{8\pi}{3}\frac{(1000\text{ kg/m}^3)(20.0\times10^{-6}\text{ m})^3(40.0\text{ m/s})^2(3.00\times10^{-3}\text{ m})}{(2.00\times10^{-9}\text{ C})(0.0100\text{ m})^2} = \boxed{1.61\text{ kN/C}}$$

CHECK The units in last line of step 7 are kg·m/(C·s²). The units work out because 1 N = 1 kg·m/s².

TAKING IT FURTHER The ink-jet in this example is called a multiple-deflection continuous ink-jet. It is used in some industrial printers. The low-cost ink-jet printers sold for home use do not use charged droplets deflected by an electric field.

DIPOLES IN ELECTRIC FIELDS

In Example 21-9 we found the electric field produced by a dipole, a system of two equal and opposite point charges that are close together. Here we consider the behavior of a dipole in an external electric field. Some molecules have permanent dipole moments due to a nonuniform distribution of charge within the molecule. Such molecules are called **polar molecules.** An example is HCl, which is essentially a positive hydrogen ion of charge $+e$ combined with a negative chloride ion of charge $-e$. The center of charge of the positive ion does not coincide with the center of charge for the negative ion, so the molecule has a permanent dipole moment. Another example is water (Figure 21-29).

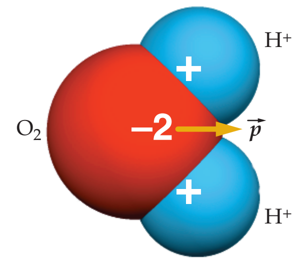


FIGURE 21-29 An H_2O molecule has a permanent dipole moment that points in the direction from the center of negative charge to the center of positive charge.

A uniform external electric field exerts no net force on a dipole, but it does exert a torque that tends to rotate the dipole so as to align it with the direction of the external field. We see in Figure 21-30 that the torque $\vec{\tau}$ calculated about the position of either charge has the magnitude $F_1 L \sin\theta = qEL \sin\theta = pE \sin\theta$.* The direction of the torque vector is into the paper such that it tends to rotate the dipole moment vector $\vec{p}$ so it aligns with the direction of $\vec{E}$. The torque can be expressed most concisely as the cross product:

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad \text{21-11}$$

If the dipole rotates through angle $d\theta$, the electric field does work:

$$dW = -\tau d\theta = -pE \sin\theta \, d\theta$$

(The minus sign arises because the torque opposes any increase in θ.) Setting the negative of this work value equal to the change in potential energy, we have

$$dU = -dW = +pE \sin\theta \, d\theta$$

Integrating, we obtain

$$U = -pE \cos\theta + U_0$$

If we choose the potential energy U to be zero when $\theta = 90°$, then $U_0 = 0$ and the potential energy of the dipole is

$$U = -pE \cos\theta = -\vec{p} \cdot \vec{E} \qquad \text{21-12}$$

POTENTIAL ENERGY OF A DIPOLE IN AN ELECTRIC FIELD

Microwave ovens take advantage of the dipole moment of water molecules to cook food. Like other electromagnetic waves, microwaves have oscillating electric fields that exert torques on dipoles, torques that cause the water molecules to rotate with significant rotational kinetic energy. In this manner, energy is transferred from the microwave radiation to the water molecules at a high rate, accounting for the rapid cooking times that make microwave ovens so convenient.

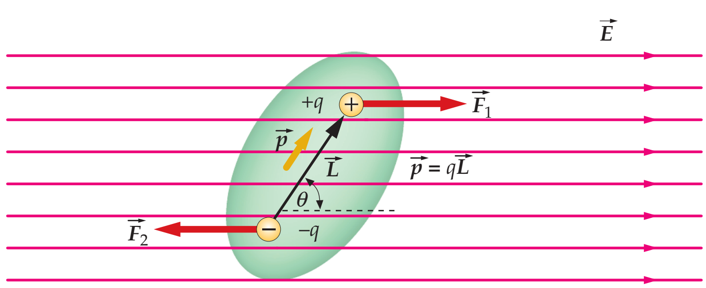


FIGURE 21-30 A dipole in a uniform electric field experiences equal and opposite forces that tend to rotate the dipole so that its dipole moment $\vec{p}$ is aligned with the electric field $\vec{E}$.

* The torque produced by two equal and opposite forces (an arrangement called a couple) is the same about any point in space.

Nonpolar molecules have no permanent dipole moment. However, all neutral molecules have equal amounts of positive and negative charge. In the presence of an external electric field $\vec{E}$, the positive and negative charge centers become separated in space. The positive charges are pushed in the direction of $\vec{E}$ and the negative charges are pushed in the opposite direction. The molecule thus acquires an induced dipole moment parallel to the external electric field and is said to be **polarized.**

In a nonuniform electric field, a dipole experiences a net force because the electric field has different magnitudes at the positive and negative charge centers. Figure 21-31 shows how a positive point charge polarizes a nonpolar molecule and then attracts it. A familiar example is the attraction that holds an electrostatically charged balloon against a wall. The nonuniform field produced by the charge on the balloon polarizes molecules in the wall and attracts them. An equal and opposite force is exerted by the wall molecules on the balloon.

The diameter of an atom or molecule is of the order of 10^{-12} m = 1 pm (one picometer). A convenient unit for the dipole moment of atoms and molecules is the fundamental charge e multiplied by the distance 1 pm. For example, the dipole moment of H_2O in these units has a magnitude of about 40 $e \cdot$pm.

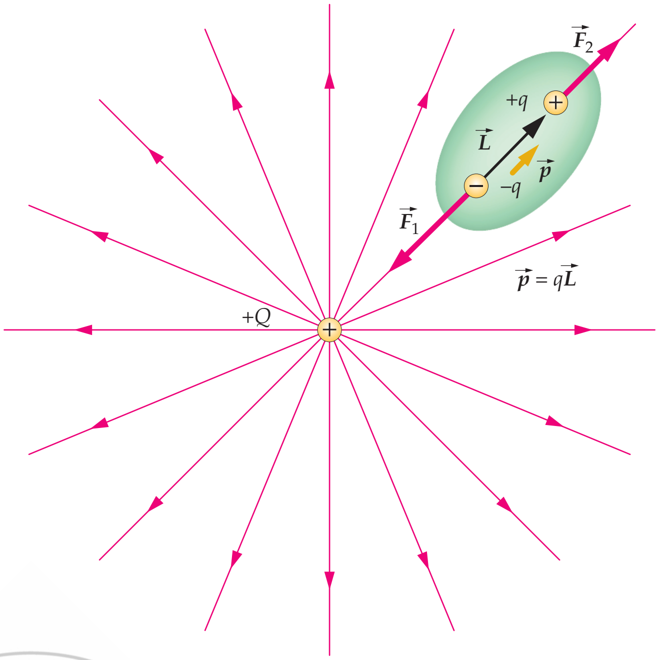


FIGURE 21-31 A nonpolar molecule in the nonuniform electric field of a positive point charge $+Q$. The point charge attracts the negative charges (the electrons) in the molecule and repels the positive charges (the protons). As a result the center of negative charge $-q$ is closer to $+Q$ than is the center of positive charge $+q$ and the induced dipole moment $\vec{p}$ is parallel to the field of the point charge. Because $-q$ is closer to $+Q$ than is $+q$, F_1 is greater than F_2 and the molecule is attracted to the point charge. In addition, if the point charge were negative, the induced dipole moment would be reversed, and the molecule would again be attracted to the point charge.

Example 21-14 Torque and Potential Energy

A polar molecule has a dipole moment of magnitude 20 $e \cdot$pm that makes an angle of 20° with a uniform electric field of magnitude 3.0×10^3 N/C (Figure 21-32). Find (*a*) the magnitude of the torque on the dipole, and (*b*) the potential energy of the system.

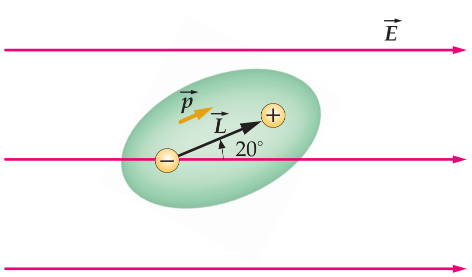


FIGURE 21-32

PICTURE The torque is found from $\vec{\tau} = \vec{p} \times \vec{E}$ and the potential energy is found from $U = -\vec{p} \cdot \vec{E}$.

SOLVE

1. Calculate the magnitude of the torque:

$$\tau = |\vec{p} \times \vec{E}| = pE\sin\theta = (20\, e\cdot\text{pm})(3 \times 10^3\ \text{N/C})(\sin 20°)$$
$$= (0.02)(1.6 \times 10^{-19}\ \text{C})(10^{-9}\ \text{m})(3 \times 10^3\ \text{N/C})(\sin 20°)$$
$$= \boxed{3.3 \times 10^{-27}\ \text{N}\cdot\text{m}}$$

2. Calculate the potential energy:

$$U = -\vec{p} \cdot \vec{E} = -pE\cos\theta$$
$$= -(0.02)(1.6 \times 10^{-19}\ \text{C})(10^{-9}\ \text{m})(3 \times 10^3\ \text{N/C})\cos 20°$$
$$= \boxed{-9.0 \times 10^{-27}\ \text{J}}$$

CHECK The sign of the potential energy is negative. That is because the reference orientation of the potential energy function $U = -\vec{p} \cdot \vec{E}$ is $U = 0$ for $\theta = 90°$. For $\theta = 20°$ the potential energy is less than zero. The system has more potential energy if $\theta = 20°$ than it does if $\theta = 90°$.

Physics Spotlight

Powder Coating—Industrial Static

Children around the world take advantage of triboelectric properties. The Ohio Art Company introduced the Etch A Sketch™ at about 1960.[*] Styrene beads provide a charge to very fine aluminum powder when shaken. The charged powder is attracted to the translucent screen of the toy. A stylus is then used to draw lines in the powder. The toy is based on the fact that the aluminum and screen attract each other with opposite charges.

Although charged powder can be a toy, it is serious business in many industries. Unprotected metal tends to corrode, so to prevent corrosion, metal parts of automobiles, metal appliances, and other metal objects are coated. In the past, coating involved paints, lacquers, varnishes, and enamels that were put on as liquids and dried. These liquids have disadvantages.[†] The solvents take a long time to dry or release unwanted volatile compounds. Surfaces at different angles can be coated unevenly. Liquid spray causes waste and cannot be easily recycled. Electrostatic powder coating reduces many of these problems.[‡] This coating process was first used in the 1950s and is now popular with manufacturers adhering to environmental regulations by reducing the use of volatile chemicals.

A fine powder is attracted to the back of the screen by electrostatic. Turning the knobs results in the power being rubbed off by a small stylus. *(Courtesy of The Ohio Art Company.)*

Powder coating is applied by giving a charge to the item that will be coated.[#] To do this reliably, it is simplest if the object to be coated is conductive. Then very small (from 1 μm to 100 μm) particles[°] in a powder are given an opposite charge. The coating particles are strongly attracted to the object to be coated. Loose particles can be recycled and used again. Once the particles are on the object, the coating is then cured, either by increased temperature or by ultraviolet light. The curing process locks the molecules of the coating together, and the particles and the object lose their charges.

Coating particles are given a charge by either corona discharge or triboelectric charging.[§] Corona discharge blows the particles through a plasma of electrons, giving them a negative charge. Triboelectric charging blows the particles through a tube that is made from a material on the opposite end of the triboelectric spectrum, often Teflon. The coating particles are given a positive charge from this rapid contact. The item to be coated is given a charge that depends on the coating method used. Depending on the coating and additives, coating charges range from 500 to 1000 μC/kg.[¶] The curing process differs according to the coating materials and the coated item. The curing time can be anywhere from 1 to 30 minutes.[**]

Although powder coating is economical and environmentally friendly, it has difficulties. The abilities of the coating particles to hold a charge[††] can vary with humidity, which must be precisely controlled.[‡‡] If the electric field for corona discharge is too strong, the powder sprays too quickly toward the item to be coated, leaving a bare spot in the middle of a built-up ring, which gives an uneven "orange peel" finish.[##] Electrostatic powders can be child's play, but electrostatic powder coating is a complex, useful, and evolving process.

* Grandjean, A., "Tracing Device." *U.S. Patent No. 3,055,113*, Sept. 25, 1962.
† Matheson, R. D. "20th- to 21st-Century Technological Challenges in Soft Coatings." *Science*, Aug. 9, 2002, Vol. 297, No. 5583, pp. 976–979.
‡ Hammerton, D., and Buysens, K., "UV-Curable Powder Coatings: Benefits and Performance." *Paint and Coatings Industry*, Aug. 2000, p. 58.
\# Zeren, S., and Renoux, D., "Powder Coatings Additives." *Paint and Coatings Industry*, Oct. 2002, p. 116.
° Hemphill, R., "Deposition of $BaTiO_3$ Nanoparticles by Electrostatic Spray Powder Charging." *Paint and Coatings Industry*, Apr. 2006, pp. 74–78.
§ Czyzak, S. J., and Williams, D. T., "Static Electrification of Solid Particles by Spraying." *Science*, Jul. 20, 1951, Vol 14, pp. 66–68.
¶ Zeren, S., and Renoux, D., op. cit.
** Hammerton, D., and Buysens, K., op. cit.
†† O'Konski, C. T., "The Exponential Decay Law in Spray De-electrification." *Science*, Oct. 5, 1951, Vol. 114, p. 368.
‡‡ Sharma, R., et al., "Effect of Ambient Relative Humidity and Surface in Modification on the Charge Decay Properties of Polymer Powders in Powder Coating." *IEEE Transactions on Industry Applications*, Jan./Feb. 2003, Vol. 39, No. 1, pp. 87–95.
\#\# Wostratzky, D., Lord, S., and Sitzmann, E. V., "Power!" *Paint and Coatings Industry*, Oct. 2000, p. 54.

Summary

1. Quantization and conservation are fundamental properties of electric charge.
2. Coulomb's law is the fundamental law of interaction between charges at rest.
3. The electric field describes the condition in space set up by a charge distribution.

TOPIC	RELEVANT EQUATIONS AND REMARKS	
1. Charge	There are two kinds of charge, positive and negative. Charges of like sign repel, those of opposite sign attract.	
Quantization	Charge is quantized—it always occurs in integer multiples of the fundamental charge unit e. The charge of the electron is $-e$ and that of the proton is $+e$.	
Magnitude	$e = 1.60 \times 10^{-19}$ C	
Conservation	Charge is conserved. When charged particles are created or annihilated, the total amount of charge carried by the created or annihilated particles is zero.	
2. Conductors and Insulators	In metals, about one electron per atom is delocalized (free to move about the entire material). In insulators, all the electrons are bound to nearby atoms.	
Ground	A very large conductor (such as Earth) that can supply or absorb a virtually unlimited amount of charge is called a ground.	
3. Charging by Induction	To charge a conductor by induction: connect a ground to the conductor, hold an external charge near the conductor (to attract or repel the conduction electrons), then disconnect the conductor from ground, and finally move the external charge away from the conductor.	
4. Coulomb's Law	The force exerted by point charge q_1 on point charge q_2 a distance r_{12} away is given by $$\vec{F}_{12} = \frac{kq_1q_2}{r_{12}^2}\hat{r}_{12}$$ where unit vector $\hat{r}_{12}$ points from q_1 toward q_2.	21-4
Coulomb constant	$k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$	21-3
5. Electric Field	The electric field due to a system of charges at a point is defined as the net force $\vec{F}$, exerted by those charges on a very small positive test charge q_0, divided by q_0: $$\vec{E} = \frac{\vec{F}}{q_0}$$	21-5
Due to a point charge	$$\vec{E}_{iP} = \frac{kq_i}{r_{iP}^2}\hat{r}_{iP}$$	21-7
Due to a system of point charges	The electric field at P due to several charges is the vector sum of the fields at P due to the individual charges: $$\vec{E}_P = \sum_i \vec{E}_{iP}$$	21-8
6. Electric Field Lines	The electric field can be represented by electric field lines that emanate from positive charges and terminate on negative charges. The strength of the electric field is indicated by the density of the electric field lines.	
7. Dipole	A dipole is a system of two equal but opposite charges separated by a small distance.	
Dipole moment	$$\vec{p} = q\vec{L}$$ where $\vec{L}$ is the position of the positive charge relative to the negative charge.	21-9
Field due to dipole	The electric field strength far from a dipole is proportional to the magnitude of the dipole moment and decreases with the cube of the distance.	
Torque on a dipole	In a uniform electric field, the net force on a dipole is zero, but there is a torque that tends to align the dipole in the direction of the field. $$\vec{\tau} = \vec{p} \times \vec{E}$$	21-11

TOPIC	RELEVANT EQUATIONS AND REMARKS	
Potential energy of a dipole	$U = -\vec{p}\cdot\vec{E} + U_0$ where U_0 is usually taken to be zero.	21-12
8. Polar and Nonpolar Molecules	Polar molecules, such as H_2O and HCl, have permanent dipole moments because their centers of positive and negative charge do not coincide. They behave like simple dipoles in an electric field. Nonpolar molecules do not have permanent dipole moments, but they acquire induced dipole moments in the presence of an electric field.	

Answers to Concept Checks

21-1 (*a*) $+\frac{1}{2}Q$. Because the spheres are identical, they must share the total charge equally. (*b*) $+2Q$, which is necessary to satisfy the conservation of charge

21-2 $Q_1 = +Q/2$, $Q_2 = -Q/4$, and $Q_3 = -Q/4$

Answers to Practice Problems

21-1 $N = Q/e = (50 \times 10^{-9}\text{ C})/(1.6 \times 10^{-19}\text{ C}) = 3.1 \times 10^{11}$. Charge quantization cannot be detected in a charge of this size; even adding or subtracting a million electrons produces a negligibly small effect.

21-2 About 3.5×10^{-8} percent

21-3 2.25×10^{-3} N

21-4 $+(6.3\ \mu\text{N})\hat{i}$

21-5 $\hat{r}_{10} = (\hat{i} + \hat{j})/\sqrt{2}$

21-6 No, but suppose it were. Because the x component of $\vec{r}_{10}$ is less than the magnitude of $\vec{r}_{10}$, the denominator of kq_1q_0/x_{10}^2 is less than the denominator of kq_1q_0/r_{10}^2. This would imply that the x component of $\vec{F}_{10}$ is greater than the magnitude of $\vec{F}_{10}$, an impossibility because the component of a vector is never greater than the magnitude of the vector. Therefore, the x component of the force $\vec{F}_{10} = (kq_1q_0/r_{10}^2)\hat{r}_{10}$ is not necessarily equal to $F_{10x} = kq_1q_0/x_{10}^2$.

21-7 $\vec{E} = \vec{F}/q_0 = (4.0 \times 10^4\text{ N/C})\hat{i}$

21-8 $\vec{F} = -(6.4 \times 10^{-15}\text{ N})\hat{i}$

21-9 $x = 1.80$ m

Problems

In a few problems, you are given more data than you actually need; in a few other problems, you are required to supply data from your general knowledge, outside sources, or informed estimate.

Interpret as significant all digits in numerical values that have trailing zeros and no decimal points.

- • Single-concept, single-step, relatively easy
- •• Intermediate-level, may require synthesis of concepts
- ••• Challenging
- SSM Solution is in the *Student Solutions Manual*
- Consecutive problems that are shaded are paired problems.

CONCEPTUAL PROBLEMS

1 • Objects are composed of atoms which are composed of charged particles (protons and electrons); however, we rarely observe the effects of the electrostatic force. Explain why we do not observe these effects.

2 • A carbon atom can become a carbon *ion* if it has one or more of its electrons removed during a process called *ionization.* What is the net charge on a carbon atom that has had two of its electrons removed? (*a*) $+e$, (*b*) $-e$, (*c*) $+2e$, (*d*) $-2e$

3 •• You do a simple demonstration for your high school physics teacher in which you claim to disprove Coulomb's law. You first run a rubber comb through your dry hair, then use it to attract tiny neutral pieces of paper on the desk. You then say, "Coulomb's law states that for there to be electrostatic forces of attraction between two objects, both objects need to be charged. However, the paper was not charged. So according to Coulomb's law, there should be no electrostatic forces of attraction between them, yet there clearly was." You rest your case. (*a*) What is wrong with your assumptions? (*b*) Does attraction between the paper and the comb require that the net charge on the comb be negative? Explain your answer.

4 •• You have a positively charged insulating rod and two metal spheres on insulating stands. Give step-by-step directions of how the rod, without actually touching either sphere, can be used to give one of the spheres (*a*) a negative charge and (*b*) a positive charge.

5 •• (*a*) Two point particles that have charges of $+4q$ and $-3q$ are separated by distance d. Use field lines to draw a visualization of the electric field in the neighborhood of this system. (*b*) Draw the field lines at distances much greater than d from the charges.

6 •• A metal sphere is positively charged. Is it possible for the sphere to electrically attract another positively charged ball? Explain your answer.

7 •• A simple demonstration of electrostatic attraction can be done by dangling a small ball of crumpled aluminum foil on a string and bringing a charged rod near the ball. The ball initially will be attracted to the rod, but once they touch, the ball will be strongly repelled from it. Explain these observations.

8 •• Two positive point charges that are equal in magnitude are fixed in place, one at $x = 0.00$ m and the other at $x = 1.00$ m, on the x axis. A third positive point charge is placed at an equilibrium position. (*a*) Where is this equilibrium position? (*b*) Is the equilibrium position stable if the third particle is constrained to move parallel with the x axis? (*c*) What if it is constrained to move parallel with the y axis? Explain your answer.

9 •• Two neutral conducting spheres are in contact and are supported on a large wooden table by insulated stands. A positively charged rod is brought up close to the surface of one of the spheres on the side opposite its point of contact with the other sphere. (*a*) Describe the induced charges on the two conducting spheres and sketch the charge distributions on them. (*b*) The two spheres are separated and then the charged rod is removed. The spheres are then separated far apart. Sketch the charge distributions on the separated spheres.

10 •• Three point charges, $+q$, $+Q$, and $-Q$, are placed at the corners of an equilateral triangle as shown in Figure 21-33. No other charged objects are nearby. (*a*) What is the direction of the net force on charge $+q$ due to the other two charges? (*b*) What is the total electric force on the system of three charges? Explain.

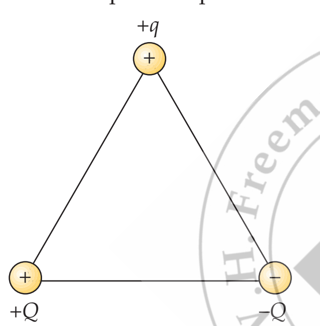


FIGURE 21-33 Problem 10

11 •• A positively charged particle is free to move in a region with a nonzero electric field $\vec{E}$. Which statement(s) must be true?

(*a*) The particle is accelerating in the direction perpendicular to $\vec{E}$.
(*b*) The particle is accelerating in the direction of $\vec{E}$.
(*c*) The particle is moving in the direction of $\vec{E}$.
(*d*) The particle could be momentarily at rest.
(*e*) The force on the particle is opposite the direction of $\vec{E}$.
(*f*) The particle is moving opposite the direction of $\vec{E}$.

12 •• Four charges are fixed in place at the corners of a square as shown in Figure 21-34. No other charges are nearby. Which of the following statements is true?

(*a*) $\vec{E}$ is zero at the midpoints of all four sides of the square.
(*b*) $\vec{E}$ is zero at the center of the square.
(*c*) $\vec{E}$ is zero midway between the top two charges and midway between the bottom two charges.

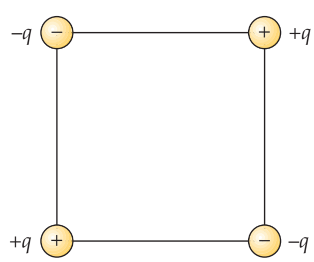


FIGURE 21-34 Problem 12

13 •• Two point particles that have charges of $+q$ and $-3q$ are separated by distance d. (*a*) Use field lines to sketch the electric field in the neighborhood of this system. (*b*) Draw the field lines at distances much greater than d from the charges. SSM

14 •• Three equal positive point charges (each charge $+q$) are fixed at the vertices of an equilateral triangle that has sides of length a. The origin is at the midpoint of one side the triangle, the center of the triangle on the x axis at $x = x_1$, and the vertex opposite the origin is on the x axis at $x = x_2$. (*a*) Express x_1 and x_2 in terms of a. (*b*) Write an expression for the electric field on the x axis a distance x from the origin on the interval 0 $x < x_2$. (*c*) Show that the expression you obtained in (*b*) gives the expected results for $x = 0$ and for $x = x_1$.

15 •• A molecule has a dipole moment given by $\vec{p}$. The molecule is momentarily at rest with $\vec{p}$ making an angle θ with a uniform electric field $\vec{E}$. Describe the subsequent motion of the dipole moment.

16 •• True or false:

(*a*) The electric field of a point charge always points away from the charge.
(*b*) The electric force on a charged particle in an electric field is always in the same direction as the field.
(*c*) Electric field lines never intersect.
(*d*) All molecules have dipole moments in the presence of an external electric field.

17 •• Two molecules have dipole moments of equal magnitude. The dipole moments are oriented in various configurations as shown in Figure 21-35. Determine the electric-field direction at each of the numbered locations. Explain your answers. SSM

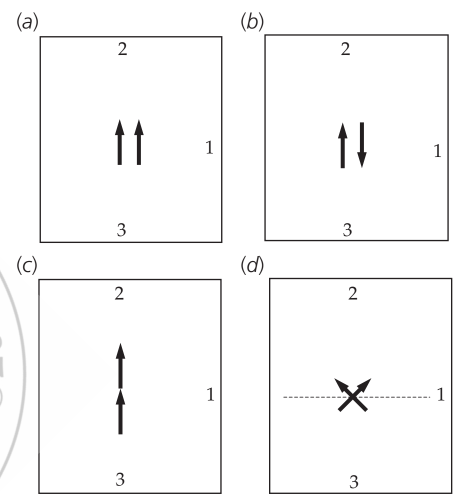


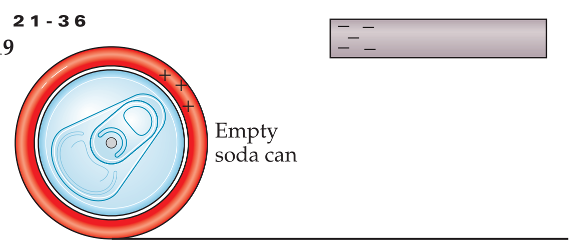
FIGURE 21-35 Problem 17

ESTIMATION AND APPROXIMATION

18 •• Estimate the force required to bind the two protons in the He nucleus together. *Hint: Model the protons as point charges. You will need to have an estimate of the distance between them.*

19 •• A popular classroom demonstration consists of rubbing a plastic rod with fur to give the rod charge, and then placing the rod near an empty soda can that is on its side (Figure 21-36). Explain why the can will roll toward the rod.

FIGURE 21-36 Problem 19



20 •• Sparks in air occur when ions in the air are accelerated to such a high speed by an electric field that when the ions impact on neutral gas molecules, the neutral molecules become ions. If the electric field strength is large enough, the ionized collision products are themselves accelerated and produce more ions on impact, and so forth. This avalanche of ions is what we call a spark. (*a*) Assume that an ion moves, on average, exactly one mean free path through the air before hitting a molecule. If the ion needs to acquire approximately 1.0 eV of kinetic energy in order to ionize a molecule, estimate the

minimum strength of the electric field required at standard room pressure and temperature. Assume that the cross-sectional area of an air molecule is about 0.10 nm^2. (*b*) How does the strength of the electric field in Part (*a*) depend on temperature? (*c*) How does the strength of the electric field in Part (*a*) depend on pressure?

CHARGE

21 • A plastic rod is rubbed against a wool shirt, thereby acquiring a charge of $-0.80\ \mu C$. How many electrons are transferred from the wool shirt to the plastic rod?

22 • A charge equal to the charge of Avogadro's number of protons ($N_A = 6.02 \times 10^{23}$) is called a *faraday*. Calculate the number of coulombs in a faraday.

23 • What is the total charge of all of the protons in 1.00 kg of carbon? SSM

24 •• Suppose a cube of aluminum which is 1.00 cm on a side accumulates a net charge of +2.50 pC. (*a*) What percentage of the electrons originally in the cube was removed? (*b*) By what percentage has the mass of the cube decreased because of this removal?

25 •• During a process described by the *photoelectric effect*, ultraviolet light can be used to charge a piece of metal. (*a*) If such light is incident on a slab of conducting material and electrons are ejected with enough energy that they escape the surface of the metal, how long before the metal has a net charge of +1.50 nC if 1.00×10^6 electrons are ejected per second? (*b*) If 1.3 eV is needed to eject an electron from the surface, what is the power rating of the light beam? (Assume this process is 100% efficient.)

COULOMB'S LAW

26 • A point charge $q_1 = 4.0\ \mu C$ is at the origin and a point charge $q_2 = 6.0\ \mu C$ is on the x axis at $x = 3.0$ m. (*a*) Find the electric force on charge q_2. (*b*) Find the electric force on q_1. (*c*) How would your answers for Parts (*a*) and (*b*) differ if q_2 were $-6.0\ \mu C$?

27 • Three point charges are on the x axis: $q_1 = -6.0\ \mu C$ is at $x = -3.0$ m, $q_2 = 4.0\ \mu C$ is at the origin, and $q_3 = -6.0\ \mu C$ is at $x = 3.0$ m. Find the electric force on q_1. SSM

28 •• A 2.0-μC point charge and a 4.0-μC point charge are a distance L apart. Where should a third point charge be placed so that the electric force on that third charge is zero?

29 •• A -2.0-μC point charge and a 4.0-μC point charge are a distance L apart. Where should a third point charge be placed so that the electric force on that third charge is zero?

30 •• Three point charges, each of magnitude 3.00 nC, are at separate corners of a square of edge length 5.00 cm. The two point charges at opposite corners are positive, and the third point charge is negative. Find the force exerted by these point charges on a fourth point charge $q_4 = +3.00$ nC at the remaining corner.

31 •• A point charge of 5.00 μC is on the y axis at $y = 3.00$ cm, and a second point charge of $-5.00\ \mu C$ is on the y axis at $y = -3.00$ cm. Find the electric force on a point charge of 2.00 μC on the x axis at $x = 8.00$ cm.

32 •• A point particle that has a charge of $-2.5\ \mu C$ is located at the origin. A second point particle that has a charge of 6.0 μC is at $x = 1.0$ m, $y = 0.50$ m. A third point particle, an electron, is at a point that has coordinates (x, y). Find the values of x and y such that the electron is in equilibrium.

33 •• A point particle that has a charge of $-1.0\ \mu C$ is located at the origin; a second point particle that has a charge of 2.0 μC is located at $x = 0$, $y = 0.10$ m; and a third point particle that has a charge of 4.0 μC is located at $x = 0.20$ m, $y = 0$. Find the total electric force on each of the three point charges.

34 •• A point particle that has a charge of 5.00 μC is located at $x = 0$, $y = 0$ and a point particle that has a charge q is located at $x = 4.00$ cm, $y = 0$. The electric force on a point particle at $x = 8.00$ cm, $y = 0$ that has a charge of 2.00 μC is $-(19.7\ \text{N})\hat{\boldsymbol{i}}$. Determine the charge q.

35 ••• Five identical point charges, each having charge Q, are equally spaced on a semicircle of radius R as shown in Figure 21-37. Find the force (in terms of k, Q, and R) on a charge q located equidistant from the five other charges. SSM

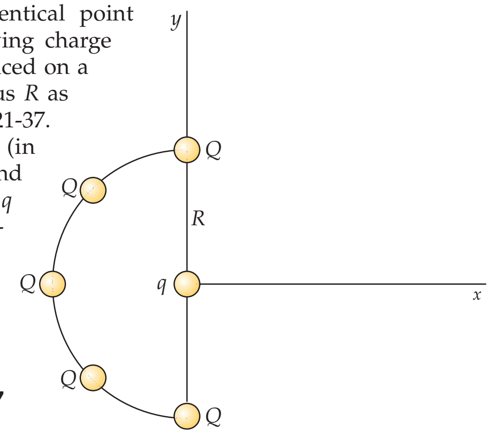


FIGURE 21-37
Problem 35

36 ••• The structure of the NH_3 molecule is approximately that of an equilateral tetrahedron, with three H^+ ions forming the base and an N^{3-} ion at the apex of the tetrahedron. The length of each side is 1.64×10^{-10} m. Calculate the electric force that acts on each ion.

THE ELECTRIC FIELD

37 • A point charge of 4.0 μC is at the origin. What are the magnitude and direction of the electric field on the x axis at (*a*) $x = 6.0$ m and (*b*) $x = -10$ m? (*c*) Sketch the function E_x versus x for both positive and negative values of x. (Remember that E_x is negative when $\vec{E}$ points in the $-x$ direction.) SSM

38 • Two point charges, each +4.0 μC, are on the x axis; one point charge is at the origin and the other is at $x = 8.0$ m. Find the electric field on the x axis at (*a*) $x = -2.0$ m, (*b*) $x = 2.0$ m, (*c*) $x = 6.0$ m, and (*d*) $x = 10$ m. (*e*) At what point on the x axis is the electric field zero? (*f*) Sketch E_x versus x for $-3.0\ \text{m} < x < 11$ m.

39 • When a 2.0-nC point charge is placed at the origin, it experiences an electric force of 8.0×10^{-4} N in the $+y$ direction. (*a*) What is the electric field at the origin? (*b*) What would be the electric force on a -4.0-nC point charge placed at the origin? (*c*) If this force is due to the electric field of a point charge on the y axis at $y = 3.0$ cm, what is the value of that charge?

40 • The electric field near the surface of Earth points downward and has a magnitude of 150 N/C. (*a*) Compare magnitude of the upward electric force on an electron with the magnitude of gravitational force on the electron. (*b*) What charge should be placed on a Ping-Pong ball of mass 2.70 g so that the electric force balances the weight of the ball near Earth's surface?

41 •• Two point charges q_1 and q_2 both have a charge equal to +6.0 nC and are on the y axis at $y_1 = +3.0$ cm and $y_2 = -3.0$ cm, respectively. (*a*) What are the magnitude and direction of the electric field on the x axis at $x = 4.0$ cm? (*b*) What is the force exerted on a third charge $q_0 = 2.0$ nC when it is placed on the x axis at $x = 4.0$ cm? SSM

42 •• A point charge of +5.0 μC is located on the x axis at $x = -3.0$ cm, and a second point charge of $-8.0\ \mu C$ is located on the x axis at $x = +4.0$ cm. Where should a third charge of +6.0 μC be placed so that the electric field at the origin is zero?

43 •• A -5.0-μC point charge is located at $x = 4.0$ m, $y = -2.0$ m, and a 12-μC point charge is located at $x = 1.0$ m, $y = 2.0$ m. (*a*) Find the magnitude and direction of the electric field at $x = -1.0$ m, $y = 0$. (*b*) Calculate the magnitude and direction of the electric force on an electron that is placed at $x = -1.0$ m, $y = 0$.

44 •• Two equal positive charges q are on the y axis; one point charge is at $y = +a$ and the other is at $y = -a$. (*a*) Show that on the x axis the x component of the electric field is given by $E_x = 2kqx/(x^2 + a^2)^{3/2}$. (*b*) Show that near the origin, where x is much smaller than a, $E_x \approx 2kqx/a^3$. (*c*) Show that for values of x much larger than a, $E_x \approx 2kq/x^2$. Explain why a person might expect this result even without deriving it by taking the appropriate limit.

45 •• A 5.0-μC point charge is located at $x = 1.0$ m, $y = 3.0$ m, and a -4.0-μC point charge is located at $x = 2.0$ m, $y = -2.0$ m. (*a*) Find the magnitude and direction of the electric field at $x = -3.0$ m, $y = 1.0$ m. (*b*) Find the magnitude and direction of the force on a proton placed at $x = -3.0$ m, $y = 1.0$ m.

46 •• Two positive point charges, each having charge Q, are on the y axis—one at $y = +a$ and the other at $y = -a$. (*a*) Show that the electric field strength on the x axis is greatest at $x = a/\sqrt{2}$ and $x = -a/\sqrt{2}$ by computing $\partial E_x/\partial x$ and setting the derivative equal to zero. (*b*) Sketch the function E_x versus x using your results for Part (*a*) of this problem and the facts that E_x is approximately $2kqx/a^3$ when x is much smaller than a and E_x is approximately $2kq/x^2$ when x is much larger than a.

47 •• Two point particles, each having a charge q, sit on the base of an equilateral triangle that has sides of length L as shown in Figure 21-38. A third point particle that has a charge equal to $2q$ sits at the apex of the triangle. Where must a fourth point particle that has a charge equal to q be placed in order that the electric field at the center of the triangle be zero? (The center is in the plane of the triangle and equidistant from the three vertices.) SSM

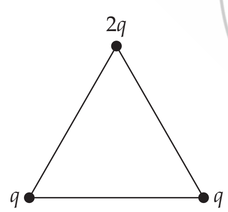


FIGURE 21-38 Problems 47 and 48

48 •• Two point particles, each having a charge equal to q, sit on the base of an equilateral triangle that has sides of length L as shown in Figure 21-38. A third point particle that has a charge equal to $2q$ sits at the apex of the triangle. A fourth point particle that has charge q' is placed at the midpoint of the baseline making the electric field at the center of the triangle equal to zero. What is the value of q'? (The center is in the plane of the triangle and equidistant from all three vertices.)

49 •• Two equal positive point charges $+q$ are on the y axis; one is at $y = +a$ and the other is at $y = -a$. The electric field at the origin is zero. A test charge q_0 placed at the origin will therefore be in equilibrium. (*a*) Discuss the stability of the equilibrium for a positive test charge by considering small displacements from equilibrium along the x axis and small displacements along the y axis. (*b*) Repeat Part (*a*) for a negative test charge. (*c*) Find the magnitude and sign of a charge q_0 that when placed at the origin results in a net force of zero on each of the three charges.

50 ••• Two positive point charges $+q$ are on the y axis at $y = +a$ and $y = -a$. A bead of mass m and charge $-q$ slides without friction along a taut thread that runs along the x axis. Let x be the position of the bead. (*a*) Show that for $x \ll a$, the bead experiences a linear restoring force (a force that is proportional to x and directed toward the equilibrium position at $x = 0$) and therefore undergoes simple harmonic motion. (*b*) Find the period of the motion.

POINT CHARGES IN ELECTRIC FIELDS

51 •• The acceleration of a particle in an electric field depends on q/m (the charge-to-mass ratio of the particle). (*a*) Compute q/m for an electron. (*b*) What are the magnitude and direction of the acceleration of an electron in a uniform electric field that has a magnitude of 100 N/C? (*c*) Compute the time it takes for an electron placed at rest in a uniform electric field that has a magnitude of 100 N/C to reach a speed $0.01c$. (When the speed of an electron approaches the speed of light c, relativistic kinematics must be used to calculate its motion, but at speeds of $0.01c$ or less, nonrelativistic kinematics is sufficiently accurate for most purposes.) (*d*) How far does the electron travel in that time? SSM

52 • The acceleration of a particle in an electric field depends on the charge-to-mass ratio of the particle. (*a*) Compute q/m for a proton, and find its acceleration in a uniform electric field that has a magnitude of 100 N/C. (*b*) Find the time it takes for a proton initially at rest in such a field to reach a speed of $0.01c$ (where c is the speed of light). (When the speed of a proton approaches the speed of light c, relativistic kinematics must be used to calculate its motion, but at speeds of $0.01c$ or less, nonrelativistic kinematics is sufficiently accurate for most purposes.)

53 • An electron has an initial velocity of 2.00×10^6 m/s in the $+x$ direction. It enters a region that has a uniform electric field $\vec{E} = (300\text{ N/C})\hat{j}$. (*a*) Find the acceleration of the electron. (*b*) How long does it take for the electron to travel 10.0 cm in the $+x$ direction in the region that has the field? (*c*) Through what angle, and in what direction, is the electron deflected while traveling the 10.0 cm in the x direction?

54 •• An electron is released from rest in a weak electric field given by $\vec{E} = -1.50 \times 10^{-10}\text{ N/C}\,\hat{j}$. After the electron has traveled a vertical distance of 1.0 μm, what is its speed? (Do not neglect the gravitational force on the electron.)

55 •• A 2.00-g charged particle is released from rest in a region that has a uniform electric field $\vec{E} = (300\text{ N/C})\hat{i}$. After traveling a distance of 0.500 m in this region, the particle has a kinetic energy of 0.120 J. Determine the charge of the particle.

56 •• A charged particle leaves the origin with a speed of 3.00×10^6 m/s at an angle of 35° above the x axis. A uniform electric field, given by $\vec{E} = -E_0\hat{j}$, exists throughout the region. Find E_0 such that the particle will cross the x axis at $x = 1.50$ cm if the particle is (*a*) an electron and (*b*) a proton.

57 •• An electron starts at the position shown in Figure 21-39 with an initial speed $v_0 = 5.00 \times 10^6$ m/s at 45° to the x axis. The electric field is in the $+y$ direction and has a magnitude of 3.50×10^3 N/C. The black lines in the figure are charged metal plates. On which plate and at what location will the electron strike? SSM

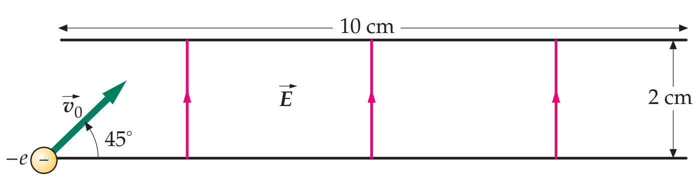


FIGURE 21-39 Problem 57

58 •• **ENGINEERING APPLICATION** An electron that has a kinetic energy equal to 2.00×10^{-16} J is moving to the right along the axis of a cathode-ray tube as shown in Figure 21-40. An electric field $\vec{E} = (2.00 \times 10^4\text{ N/C})\hat{j}$ exists in the region between the deflection plates, and no electric field ($\vec{E} = 0$) exists outside this region. (*a*) How far is the electron from the axis of the tube when it exits the region between the plates? (*b*) At what angle is the electron moving, with respect to the axis, after exiting the region between the plates? (*c*) At what distance from the axis will the electron strike the fluorescent screen?

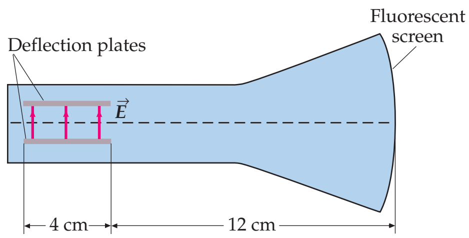


FIGURE 21-40 Problem 58

DIPOLES

59 • Two point charges, $q_1 = 2.0$ pC and $q_2 = -2.0$ pC, are separated by 4.0 μm. (*a*) What is the magnitude of the dipole moment of this pair of charges? (*b*) Sketch the pair and show the direction of the dipole moment.

60 • A dipole of moment 0.50 $e \cdot$nm is placed in a uniform electric field that has a magnitude of 4.0×10^4 N/C. What is the magnitude of the torque on the dipole when (*a*) the dipole is aligned with the electric field, (*b*) the dipole is transverse to (perpendicular to) the electric field, and (*c*) the direction of dipole makes an angle of 30° with the direction of electric field? (*d*) Defining the potential energy to be zero when the dipole is transverse to the electric field, find the potential energy of the dipole for the orientations specified in Parts (*a*) and (*c*).

GENERAL PROBLEMS

61 • Show that it is only possible to place one isolated proton in an ordinary empty coffee cup by considering the following situation. Assume the first proton is fixed at the bottom of the cup. Determine the distance directly above this proton where a second proton would be in equilibrium. Compare this distance to the depth of an ordinary coffee cup to complete the argument. SSM

62 •• Point charges of $-5.00\ \mu$C, $+3.00\ \mu$C, and $+5.00\ \mu$C are located on the x axis at $x = -1.00$ cm, $x = 0$, and $x = +1.00$ cm, respectively. Calculate the electric field on the x axis at $x = 3.00$ cm and at $x = 15.0$ cm. Are there any points on the x axis where the magnitude of the electric field is zero? If so, where are those points?

63 •• Point charges of $-5.00\ \mu$C and $+5.00\ \mu$C are located on the x axis at $x = -1.00$ cm and $x = +1.00$ cm, respectively. (*a*) Calculate the electric field strength at $x = 10.00$ cm. (*b*) Estimate the electric field strength at $x = 10.00$ cm by modeling the two charges as an electric dipole located at the origin and using $E = 2kp/|x|^3$ (Equation 21-10). Compare your result with the result obtained in Part (*a*), and explain the reason for the difference between the two results.

64 •• A fixed point charge of $+2q$ is connected by strings to point charges of $+q$ and $+4q$, as shown in Figure 21-41. Find the tensions T_1 and T_2.

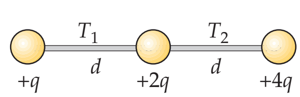


FIGURE 21-41 Problem 64

65 •• A positive charge Q is to be divided into two positive point charges q_1 and q_2. Show that, for a given separation D, the force exerted by one charge on the other is greatest if $q_1 = q_2 = \frac{1}{2}Q$. SSM

66 •• A point charge Q is located on the x axis at $x = 0$, and a point charge $4Q$ is located at $x = 12.0$ cm. The electric force on a point charge of $-2.00\ \mu$C is zero if that charge is placed at $x = 4.00$ cm, and is 126 N in the $+x$ direction if placed at $x = 8.00$ cm. Determine the charge Q.

67 •• Two point particles separated by 0.60 m have a total charge of 200 μC. (*a*) If the two particles repel each other with a force of 80 N, what is the charge on each of the two particles? (*b*) If the two particles attract each other with a force of 80 N, what are the charges on the two particles?

68 •• A point particle that has charge $+q$ and unknown mass m is released from rest in a region that has a uniform electric field $\vec{E}$ that is directed vertically downward. The particle hits the ground at a speed $v = 2\sqrt{gh}$, where h is the initial height of the particle. Find m in terms of E, q, and g.

69 •• A rigid 1.00-m-long rod is pivoted about its center (Figure 21-42). A charge $q_1 = 5.00 \times 10^{-7}$ C is placed on one end of the rod, and a charge $q_2 = -q_1$ is placed a distance $d = 10.0$ cm directly below it. (*a*) What is the force exerted by q_2 on q_1? (*b*) What is the torque (measured about the rotation axis) due to that force? (*c*) To counterbalance the attraction between the two charges, we hang a block 25.0 cm from the pivot as shown. What value should we choose for the mass m of the block? (*d*) We now move the block and hang it a distance of 25.0 cm from the balance point, on the same side of the balance as the charge. Keeping q_1 the same, and d the same, what value should we choose for q_2 to keep this apparatus in balance? SSM

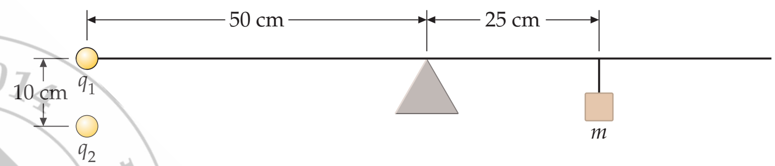


FIGURE 21-42 Problem 69

70 •• Two 3.0-μC point charges are located at $x = 0$, $y = 2.0$ m and at $x = 0$, $y = -2.0$ m. Two other point charges, each with charge Q, are located at $x = 4.0$ m, $y = 2.0$ m and at $x = 4.0$ m, $y = -2.0$ m (Figure 21-43). The electric field at $x = 0, y = 0$ due to the presence of the four charges is $(4.0 \times 10^3\ \text{N/C})\hat{\imath}$. Determine Q.

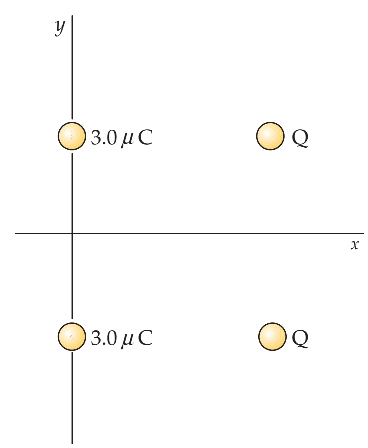


FIGURE 21-43 Problem 70

71 •• Two point charges have a total charge equal to 200 μC and are separated by 0.600 m. (*a*) Find the charge of each particle if the particles repel each other with a force of 120 N. (*b*) Find the force on each particle if the charge on each particle is 100 μC. SSM

72 •• Two point charges have a total charge equal to 200 μC and are separated by 0.600 m. (*a*) Find the charge of each particle if the particles attract each other with a force of 120 N. (*b*) Find the force on each particle if the charge on each particle is 100 μC. SSM

73 •• A point charge of $-3.00\ \mu$C is located at the origin; a point charge of 4.00 μC is located on the x axis at $x = 0.200$ m; a third point charge Q is located on the x axis at $x = 0.320$ m. The electric force on the 4.00–μC charge is 240 N in the $+x$ direction. (*a*) Determine the charge Q. (*b*) With this configuration of three charges, at what location(s) is the electric field zero?

74 •• Two point particles, each of mass m and charge q, are suspended from a common point by threads of length L. Each thread makes an angle θ with the vertical as shown in Figure 21-44. (*a*) Show that $q = 2L\sin\theta\sqrt{(mg/k)\tan\theta}$ where k is the Coulomb constant. (*b*) Find q if $m = 10.0$ g, $L = 50.0$ cm, and $\theta = 10.0°$.

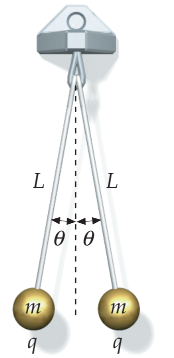


FIGURE 21-44 Problem 74

75 •• Suppose that in Problem 74 $L = 1.5$ m and $m = 0.010$ kg. (*a*) What is the angle that each string makes with the vertical if $q = 0.75\ \mu$C? (*b*) What is the angle that each string makes with the vertical if one particle has a charge of 0.50 μC and the other has a charge of 1.0 μC?

76 •• Four point charges of equal magnitude are arranged at the corners of a square of side L as shown in Figure 21-45. (*a*) Find the magnitude and direction of the force exerted on the charge in the lower left corner by the other three charges. (*b*) Show that the electric field at the midpoint of one of the sides of the square is directed along that side toward the negative charge and has a magnitude E given by

$$E = k\frac{8q}{L^2}\left(1 - \frac{1}{5\sqrt{5}}\right).$$

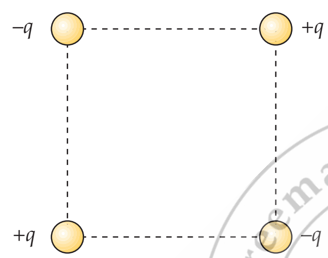


FIGURE 21-45 Problem 76

77 •• Figure 21-46 shows a dumbbell consisting of two identical small particles, each of mass m, attached to the ends of a thin (massless) rod of length a that is pivoted at its center. The particles have charges of $+q$ and $-q$, and the dumbbell is located in a uniform electric field $\vec{E}$. Show that for small values of the angle θ between the direction of the dipole and the direction of the electric field, the system displays a rotational form of simple harmonic motion, and obtain an expression for the period of that motion. SSM

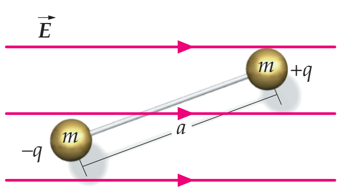


FIGURE 21-46 Problems 77 and 78

78 •• For the dumbbell in Problem 77, let $m = 0.0200$ kg, $a = 0.300$ m, and $\vec{E} = (600\text{ N/C})\hat{i}$. The dumbbell is initially at rest and makes an angle of 60° with the x axis. The dumbbell is then released, and when it is momentarily aligned with the electric field, its kinetic energy is 5.00×10^{-3} J. Determine the magnitude of q.

79 •• An electron (charge $-e$, mass m) and a positron (charge $+e$, mass m) revolve around their common center of mass under the influence of their attractive coulomb force. Find the speed v of each particle in terms of e, m, k, and their separation distance L. SSM

80 ••• A simple pendulum of length 1.0 m and mass 5.0×10^{-3} kg is placed in a uniform, electric field $\vec{E}$ that is directed vertically upward. The bob has a charge of $-8.0\ \mu$C. The period of the pendulum is 1.2 s. What are the magnitude and direction of $\vec{E}$?

81 ••• A point particle of mass m and charge q is constrained to move vertically inside a narrow, frictionless cylinder (Figure 21-47). At the bottom of the cylinder is a point charge Q having the same sign as q. (*a*) Show that the particle whose mass is m will be in equilibrium at a height $y_0 = (kqQ/mg)^{1/2}$. (*b*) Show that if the particle is displaced from its equilibrium position by a small amount and released, it will exhibit simple harmonic motion with angular frequency $\omega = (2g/y_0)^{1/2}$.

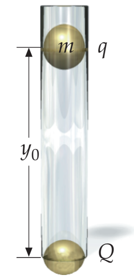


FIGURE 21-47 Problem 81

82 ••• Two neutral molecules on the x axis attract each other. Each molecule has a dipole moment $\vec{p}$, and these dipole moments are on the $+x$ direction and are separated by a distance d. Derive an expression for the force of attraction in terms of p and d.

83 ••• Two equal positive point charges Q are on the x axis at $x = \frac{1}{2}a$ and $x = -\frac{1}{2}a$. (*a*) Obtain an expression for the electric field on the y axis as a function of y. (*b*) A bead of mass m, which has a charge q, moves along the y axis on a thin frictionless taut thread. Find the electric force that acts on the bead as a function of y and determine the sign of q such that this force always points away from the origin. (*c*) The bead is initially at rest at the origin. If it is given a slight nudge in the $+y$ direction, how fast will the bead be traveling the instant the net force on it is a maximum? (Assume any effects due to gravity are negligible.)

84 ••• A gold nucleus is 100 fm (1 fm = 10^{-15} m) from a proton, which initially is at rest. When the proton is released, it speeds away because of the repulsion that it experiences due to the charge on the gold nucleus. What is the proton's speed a large distance (assume to be infinity) from the gold nucleus? (Assume the gold nucleus remains stationary.)

85 ••• During a famous experiment in 1919, Ernest Rutherford shot doubly ionized helium nuclei (also known as alpha particles) at a gold foil. He discovered that virtually all of the mass of an atom resides in an extremely compact nucleus. Suppose that during such an experiment, an alpha particle far from the foil has a kinetic energy of 5.0 MeV. If the alpha particle is aimed directly at the gold nucleus, and the only force acting on it is the electric force of repulsion exerted on it by the gold nucleus, how close will it approach the gold nucleus before turning back? That is, what is the minimum center-to-center separation of the alpha particle and the gold nucleus? SSM

86 ••• During the Millikan experiment used to determine the charge on the electron, a charged polystyrene microsphere is released in still air in a known vertical electric field. The charged microsphere will accelerate in the direction of the net force until it reaches terminal speed. The charge on the microsphere is determined by measuring the terminal speed. During one such experiment, the microsphere has radius $r = 5.50 \times 10^7$ m, and the field has a magnitude $E = 6.00 \times 10^4$ N/C. The magnitude of the drag force on the sphere is given by $F_D = 6\pi\eta r v$, where v is the speed of the sphere and η is the viscosity of air ($\eta = 1.8 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2$). Polystyrene has density $1.05 \times 10^3\ \text{kg/m}^3$. (*a*) If the electric field is pointing down and the polystyrene microsphere is rising with a terminal speed of 1.16×10^{-4} m/s, what is the charge on the sphere? (*b*) How many excess electrons are on the sphere? (*c*) If the direction of the electric field is reversed but its magnitude remains the same, what is the new terminal speed?

87 ••• In Problem 86, there is a description of the Millikan experiment used to determine the charge on the electron. During the experiment, a switch is used to reverse the direction of the electric field without changing its magnitude, so that one can measure the terminal speed of the microsphere both as it is moving upward and as it is moving downward. Let v_u represent the terminal speed when the particle is moving up and v_d the terminal speed when moving down. (*a*) If we let $u = v_u + v_d$, show that $q = 3\pi\eta r u/E$, where q is the microsphere's net charge. For the purpose of determining q, what advantage does measuring both v_u and v_d have over measuring only one terminal speed? (*b*) Because charge is quantized, u can only change by steps of magnitude $N\Delta$, where N is an integer. Using the data from Problem 86, calculate Δ. SSM