CHAPTER 2

MOTION IN ONE DIMENSION IS MOTION ALONG A STRAIGHT LINE LIKE THAT OF A CAR ON A STRAIGHT ROAD. THIS DRIVER ENCOUNTERS STOPLIGHTS AND DIFFERENT SPEED LIMITS ON HER COMMUTE ALONG A STRAIGHT HIGHWAY TO SCHOOL. *(Medio Images/Getty Images.)*

Motion in One Dimension

? How can she estimate her arrival time? (See Example 2-3.)

Imagine a car speeding down a highway. There are a number of ways in which you could describe the car's motion to someone else. For example, you could describe the change in the car's position as it travels from one point to another, how fast the car is moving and the direction in which it travels, and whether the car is speeding up or slowing down as it moves. These basic descriptors of motion—known as displacement, velocity, and acceleration—are an essential part of physics. In fact, the attempt to describe the motion of objects gave birth to physics more than 400 years ago.

The study of motion, and the related concepts of force and mass, is called **mechanics.** We begin our investigation into motion by examining **kinematics,** the branch of mechanics that deals with the characteristics of motion. You will need to understand kinematics to understand the rest of this book. Motion permeates all of physics, and an understanding of kinematics is needed to understand how force and mass effect motion. Starting in Chapter 4, we look at **dynamics,** which relates motion, force, and mass.

We study the simplest case of kinematics in this chapter — motion along a straight line. We will develop the models and tools you will need to describe motion in one dimension, and introduce the precise definitions of words commonly used to describe motion, such as displacement, speed, velocity, and acceleration. We will also look at the special case of straight-line motion when acceleration is constant. Finally, we consider the ways in which integration can be used to describe motion. In this chapter, moving objects are restricted to motion along a straight line. To describe such motion, it is not necessary to use the full vector notation developed in Chapter 1. A + or − sign are all that is needed to specify direction along a straight line.

2-1 DISPLACEMENT, VELOCITY, AND SPEED

In a horse race, the winner is the horse whose nose first crosses the finish line. One could argue that all that really matters during the race is the motion of that single point on the horse, and that the size, shape, and motion of the rest of the horse is unimportant. In physics, this type of simplification turns out to be useful for examining the motion of other objects as well. We can often describe the motion of an object by describing the motion of a single point of the object. For example, as a car moves in a straight line along a road, you could describe the motion of the car by examining the motion of a single point on the side of the car. An object that can be represented in this idealized manner is called a **particle.** In kinematics, any object can be considered a particle as long as we are not interested in its size, shape, or internal motion. For example, we can consider cars, trains, and rockets particles. Earth and other planets can also be thought of as particles as they move around the Sun. Even people and galaxies can be treated as particles.

(Bettmann/Corbis.)

POSITION AND DISPLACEMENT

To describe the motion of a particle, we need to be able to describe the **position** of the particle and how that position changes as the particle moves. For one-dimensional motion, we often choose the x axis as the line along which the motion takes place. For example, Figure 2-1 shows a student on a bicycle at position x_i at time t_i. At a later time, t_f, the student is at position x_f. The change in the student's position, $x_f - x_i$, is called a **displacement.** We use the Greek letter Δ (uppercase delta) to indicate the change in a quantity; thus, the change in x can be written as

$$\Delta x = x_f - x_i \qquad \text{2-1}$$

DEFINITION—DISPLACEMENT

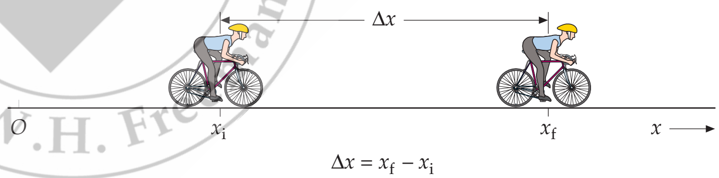


FIGURE 2-1 A student on a bicycle is moving in a straight line. A coordinate axis consists of a line along the path of the bicycle. A point on this line is chosen to be the origin O. Other points on it are assigned a number x, the value of x being proportional to its distance from O. The numbers assigned to points to the right of O are positive as shown, and those assigned to points to the left of O are negative. When the bicycle travels from point x_i to point x_f, its displacement is $\Delta x = x_f - x_i$.

It is important to recognize the difference between displacement and distance traveled. The distance traveled by a particle is the length of the path a particle takes from its initial position to its final position. Distance is a scalar quantity and is always indicated by a positive number. Displacement is the *change in position* of the particle. It is positive if the change in position is in the direction of increasing x (the $+x$ direction), and negative if it is in the $-x$ direction. Displacement can be represented by vectors, as shown in Chapter 1. We will use the full vector notation developed in Chapter 1 when we study motion in two and three dimensions in Chapter 3.

! The notation Δx (read "delta x") stands for a single quantity that is the change in x. Δx is not a product of Δ and x any more than cos θ is a product of cos and θ. By convention, the change in a quantity is always its final value minus its initial value.

Example 2-1 Distance and Displacement of a Dog

You are playing a game of catch with a dog. The dog is initially standing near your feet. Then he jogs 20 feet in a straight line to retrieve a stick, and carries the stick 15 feet back toward you before lying on the ground to chew on the stick. (*a*) What is the total distance the dog travels? (*b*) What is the net displacement of the dog? (*c*) Show that the net displacement for the trip is the sum of the sequential displacements that make up the trip.

PICTURE The total distance, s, is determined by summing the individual distances the dog travels. The displacement is the dog's final position minus the dog's initial position. The dog leaves your side at time 0, gets the stick at time 1, and lies down to chew on it at time 2.

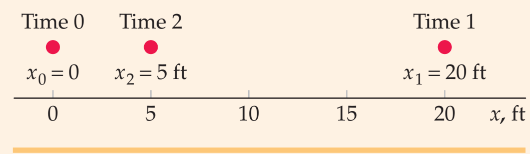


FIGURE 2-2 The red dots represent the dog's position at different times.

SOLVE

(*a*) 1. Make a diagram of the motion (Figure 2-2). Include a coordinate axis:

2. Calculate the total distance traveled:

$$s_{02} = s_{01} + s_{12} = (20\text{ ft}) + (15\text{ ft}) = \boxed{35}\text{ ft}$$

(The subscripts indicate the time intervals, where s_{01} is the distance traveled during the interval from time 0 to time 1, and so forth.)

(*b*) The net displacement is found from its definition, $\Delta x = x_f - x_i$, where $x_i = x_0 = 0$ is the dog's initial position. Five feet from the initial position or $x_f = x_2 = 5$ ft is the dog's final position:

$$\Delta x_{02} = x_2 - x_0 = 5\text{ ft} - 0\text{ ft} = \boxed{5\text{ ft}}$$

where Δx_{02} is the displacement during the interval from time 0 to time 2.

(*c*) The net displacement is also found by adding the displacement for the first leg to the displacement for the second leg.

$$\Delta x_{01} = x_1 - x_0 = 20\text{ ft} - 0\text{ ft} = 20\text{ ft}$$

$$\Delta x_{12} = x_2 - x_1 = 5\text{ ft} - 20\text{ ft} = -15\text{ ft}$$

adding, we obtain

$$\Delta x_{01} + \Delta x_{12} = (x_1 - x_0) + (x_2 - x_1) = x_2 - x_0 = \Delta x_{02}$$

so

$$\Delta x_{02} = \Delta x_{01} + \Delta x_{12} = 20\text{ ft} - 15\text{ ft} = \boxed{5\text{ ft}}$$

CHECK The magnitude of the displacement for any part of the trip is never greater than the total distance traveled for that part. The magnitude of the Part (*b*) result (5 ft) is less than the Part (*a*) result (35 ft), so the Part (*b*) result is plausible.

TAKING IT FURTHER The total distance traveled for a trip is always equal to the sum of the distances traveled for the individual legs of the trip. The total or net displacement for a trip is always equal to the sum of the displacements for the individual legs of the trip.

AVERAGE VELOCITY AND SPEED

We often are interested in the speed something is moving. The **average speed** of a particle is the total distance traveled by the particle divided by the total time from start to finish:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{s}{\Delta t} \qquad 2\text{-}2$$

DEFINITION—AVERAGE SPEED

Because the total distance and total time are both always positive, the average speed is always positive.

Although speed is a useful idea, it does not reveal anything about the direction of motion because neither the total distance nor the total time has an associated direction. A more useful quantity is one that describes both how fast and in what direction an object moves. The term used to describe this quantity is *velocity*. The **average velocity,** $v_{\text{av}\,x}$, of a particle is defined as the ratio of the displacement Δx to the time interval Δt:

$$v_{\text{av}\,x} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad (\text{so } \Delta x = v_{\text{av}\,x}\,\Delta t) \tag{2-3}$$

DEFINITION—AVERAGE VELOCITY

Like displacement, average velocity is a quantity that may be positive or negative. A positive value indicates the displacement is in the $+x$ direction. A negative value indicates the displacement is in the $-x$ direction. The dimensions of velocity are L/T and the SI unit of velocity is meters per second (m/s). Other common units include kilometers per hour (km/h), feet per second (ft/s), and miles per hour (mi/h).

Figure 2-3 is a graph of a particle's position as a function of time. Each point represents the position x of a particle at a particular time t. A straight line connects points P_1 and P_2 and forms the hypotenuse of the triangle having sides $\Delta x = x_2 - x_1$ and $\Delta t = t_2 - t_1$. Notice that the ratio $\Delta x/\Delta t$ is the line's **slope,** which gives us a geometric interpretation of average velocity:

The average velocity for the interval between $t = t_1$ and $t = t_2$ is the slope of the straight line connecting the points (t_1, x_1) and (t_2, x_2) on an x versus t graph.

GEOMETRIC INTERPRETATION OF AVERAGE VELOCITY

See
Math Tutorial ***for more information on***
Linear Equations

Notice that the average velocity depends on the time interval on which it is based. In Figure 2-3, for example, the smaller time interval indicated by t_2' and P_2' gives a larger average velocity, as shown by the greater steepness of the line connecting points P_1 and P_2'.

! The definitions of average speed and average velocity are the most basic of the kinematic parameters. You will need to know these definitions and the definitions that appear later in this chapter to effectively solve kinematics problems.

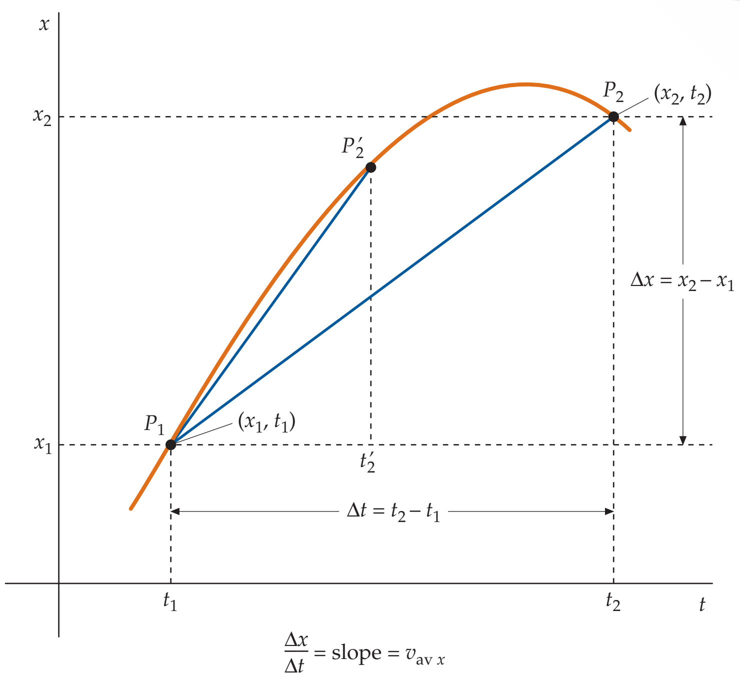


FIGURE 2-3 Graph of x versus t for a particle moving in one dimension. Each point on the curve represents the position x at a particular time t. We have drawn a straight line through points (x_1, t_1) and (x_2, t_2). The displacement $\Delta x = x_2 - x_1$ and the time interval $\Delta t = t_2 - t_1$ between these points are indicated. The straight line between P_1 and P_2 is the hypotenuse of the triangle having sides Δx and Δt, and the ratio $\Delta x/\Delta t$ is its slope. In geometric terms, the slope is a measure of the line's steepness.

Example 2-2 Average Speed and Velocity of the Dog

The dog that you were playing catch with in Example 2-1 jogged 20.0 ft away from you in 1.0 s to retrieve the stick and ambled back 15.0 ft in 1.5 s (Figure 2-4). Calculate (*a*) the dog's average speed, and (*b*) the dog's average velocity for the total trip.

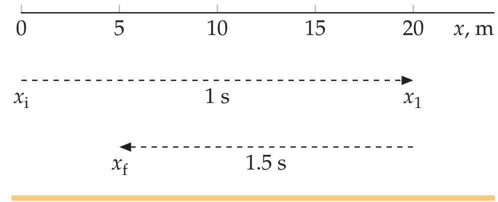


FIGURE 2-4

PICTURE We can solve this problem using the definitions of average speed and average velocity, noting that average *speed* is the total *distance* divided by the total time Δt, whereas the average *velocity* is the net *displacement* divided by Δt:

SOLVE

(*a*) 1. The dog's average speed equals the total distance divided by the total time:

$$\text{Average speed} = \frac{s}{\Delta t}$$

2. Calculate the total distance traveled and the total time:

$$s = s_1 + s_2 = 20.0\text{ ft} + 15.0\text{ ft} = 35.0\text{ ft}$$
$$\Delta t = (t_1 - t_i) + (t_f - t_2) = 1.0\text{ s} + 1.5\text{ s} = 2.5\text{ s}$$

3. Use s and Δt to find the dog's average speed:

$$\text{Average speed} = \frac{35.0\text{ ft}}{2.5\text{ s}} = \boxed{14\text{ ft/s}}$$

(*b*) 1. The dog's average velocity is the ratio of the net displacement Δx to the time interval Δt:

$$v_{\text{av}\,x} = \frac{\Delta x}{\Delta t}$$

2. The dog's net displacement is $x_f - x_i$, where $x_i = 0.0$ ft is the initial position of the dog and $x_f = 5.0$ ft is the dog's final position:

$$\Delta x = x_f - x_i = 5.0\text{ ft} - 0.0\text{ ft} = 5.0\text{ ft}$$

3. Use Δx and Δt to find the dog's average velocity:

$$v_{\text{av}\,x} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ ft}}{2.5\text{ s}} = \boxed{2.0\text{ ft/s}}$$

CHECK An Internet search reveals a greyhound can have an average speed of approximately 66 ft/s (45 mi/h), so our dog should easily be able to jog 14 ft/s (9.5 mi/h). A Part (*a*) result greater than 66 ft/s would not be plausible.

TAKING IT FURTHER Note that the dog's speed is greater than the dog's average velocity because the total distance traveled is greater than the magnitude of the total displacement. Also, note that the total displacement is the sum of the individual displacements. That is, $\Delta x = \Delta x_1 + \Delta x_2 = (20.0\text{ ft}) + (-15.0\text{ ft}) = 5.0\text{ ft}$, which is the Part (*b*), step 2 result.

Example 2-3 Driving to School

It normally takes you 10 min to travel 5.0 mi to school along a straight road. You leave home 15 min before class begins. Delays caused by a broken traffic light slow down traffic to 20 mi/h for the first 2.0 mi of the trip. Will you be late for class?

PICTURE You need to find the total time that it will take you to travel to class. To do so, you must find the time $\Delta t_{2\text{ mi}}$ that you will be driving at 20 mi/h, and the time $\Delta t_{3\text{ mi}}$ for the remainder of the trip, during which you are driving at your usual velocity.

SOLVE

1. The total time equals the time to travel the first 2.0 mi plus the time to travel the remaining 3.0 mi:

$$\Delta t_{\text{tot}} = \Delta t_{2\text{ mi}} + \Delta t_{3\text{ mi}}$$

2. Using $\Delta x = v_{\text{av}\,x}\,\Delta t$, solve for the time taken to travel 2.0 mi at 20 mi/h:

$$\Delta t_{2\text{ mi}} = \frac{\Delta x_1}{v_{\text{av}\,x}} = \frac{2.0\text{ mi}}{20\text{ mi/h}} = 0.10\text{ h} = 6.0\text{ min}$$

3. Using $\Delta x = v_{\text{av}\,x}\,\Delta t$, relate the time taken to travel 3 mi at the usual velocity:

$$\Delta t_{3\text{ mi}} = \frac{\Delta x_2}{v_{\text{av}\,x}} = \frac{3.0\text{ mi}}{v_{\text{usual}\,x}}$$

4. Using $\Delta x = v_{\text{av}\,x}\,\Delta t$, solve for $v_{\text{usual}\,x}$, the velocity needed for you to travel the 5.0 mi in 10 min:

$$v_{\text{usual}\,x} = \frac{\Delta x_{\text{tot}}}{\Delta t_{\text{usual}}} = \frac{5.0\text{ mi}}{10\text{ min}} = 0.50\text{ mi/min}$$

5. Using the results from steps 3 and 4, solve for $\Delta t_{3\text{ mi}}$:
$$\Delta t_{3\text{ mi}} = \frac{\Delta x_2}{v_{\text{usual }x}} = \frac{3.0\text{ mi}}{0.50\text{ mi/min}} = 6.0\text{ min}$$

6. Solve for the total time:
$$\Delta t_{\text{tot}} = \Delta t_{2\text{ mi}} + \Delta t_{3\text{ mi}} = 12\text{ min}$$

7. The trip takes 12 min with the delay, compared to the usual 10 min. Because you wisely allowed yourself 15 min for the trip, $\boxed{\textit{you will not be late for class}}$.

CHECK Note that 20 mi/h = 20 mi/60 min = 1.0 mi/3.0 min. Traveling the entire 5.0 miles at one mile every three minutes, it would take 15 minutes for the trip to school. You allowed yourself 15 minutes for the trip, so you would get there on time even if you traveled at the slow speed of 20 mi/h for the entire 5.0 miles.

Example 2-4 A Train-Hopping Bird

Two trains 60 km apart approach each other on parallel tracks, each moving at 15 km/h. A bird flies back and forth between the trains at 20 km/h until the trains pass each other. How far does the bird fly?

PICTURE In this problem, you are asked to find the total distance flown by the bird. You are given the bird's speed, the trains' speeds, and the initial distance between the trains. At first glance, it might seem like you should find and sum the distances flown by the bird each time it moves from one train to the other. However, a much simpler approach is to use the facts that the time t the bird is flying is the time taken for the trains to meet. The total distance flown is the bird's speed multiplied by the time the bird is flying. Therefore, we can approach this problem by first writing an equation for the quantity to be found, the total distance s flown by the bird.

SOLVE

1. The total distance s_{bird} traveled by the bird equals its speed times the time of flight:
$$s_{\text{bird}} = (\text{average speed})_{\text{bird}} \times t = (\text{speed})_{\text{av bird}} \times t$$

2. The time t that the bird is in the air is the time taken for one of the trains to travel half the initial distance D separating the trains. (Because the trains are traveling at the same speed, each train travels half of the 60 km, which is 30 km, before they meet.):
$$\tfrac{1}{2}D = (\text{speed})_{\text{av train}} \times t$$
so
$$t = \frac{D}{2(\text{speed})_{\text{av train}}}$$

3. Substitute the step-2 result for the time into the step-1 result. The initial separation of the two trains is $D = 60$ km. The total distance traveled by the bird is therefore:
$$s_{\text{bird}} = (\text{speed})_{\text{av bird}}\, t = (\text{speed})_{\text{av bird}} \frac{D}{2(\text{speed})_{\text{av train}}}$$
$$= 20\text{ km/h}\,\frac{60\text{ km}}{2\,(15\text{ km/h})} = \boxed{40\text{ km}}$$

CHECK The speed of each train is three-fourths the speed of the bird, so the distance traveled by one of the trains will be three-fourths the distance the bird travels. Each train travels 30 km. Because 30 km is three-fourths of 40 km, our result of 40 km for the distance the bird travels is very plausible.

INSTANTANEOUS VELOCITY AND SPEED

Suppose your average velocity for a long trip was 60 km/h. Because this value is an average, it does not convey any information about how your velocity changed during the trip. For example, there may have been some parts of the journey where you were stopped at a traffic light and other parts where you went faster to make up time. To learn more about the details of your motion, we have to look at the velocity at any given instant during the trip. On first consideration, defining the velocity of a particle at a single instant might seem impossible. At a given instant, a particle is at a single point. If it is at a single point, how can it be moving? If it is not moving, how can it have a velocity? This age-old paradox is resolved when we realize that observing and defining motion requires that we look at the position of the object at more than one instant of time. For example, consider the graph of position versus time in Figure 2-5. As we consider successively shorter time intervals beginning at t_P, the average velocity for the interval approaches the slope of the tangent at t_P. We define the slope of this tangent as the **instantaneous velocity,** $v_x(t)$, at t_P. This tangent is the limit of the ratio $\Delta x/\Delta t$ as Δt, and therefore Δx, approaches zero. So we can say:

> The instantaneous velocity v_x is the limit of the ratio $\Delta x/\Delta t$ as Δt approaches zero.
>
> $$v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$
>
> $= $ slope of the line tangent to the x-versus-t curve   2-4
>
> DEFINITION—INSTANTANEOUS VELOCITY

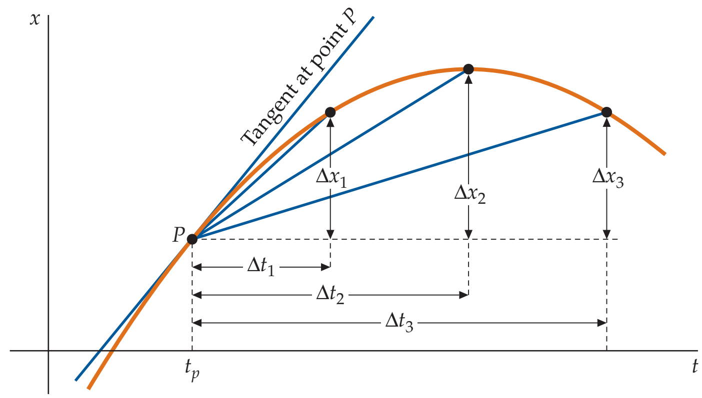


FIGURE 2-5 Graph of x versus t. Note the sequence of successively smaller time intervals, $\Delta t_3, \Delta t_2, \Delta t_1, \ldots$. The average velocity of each interval is the slope of the straight line for that interval. As the time intervals become smaller, these slopes approach the slope of the tangent to the curve at point t_P. The slope of this tangent line is defined as the instantaneous velocity at time t_P.

In calculus, this limit is called the **derivative** of x with respect to t and is written dx/dt. Using this notation, Equation 2-4 becomes:

$$v_x(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad \text{2-5}$$

See **Math Tutorial** *for more information on* **Differential Calculus**

A line's slope may be positive, negative, or zero; consequently, instantaneous velocity (in one-dimensional motion) may be positive (x increasing), negative (x decreasing), or zero (no motion). For an object moving with constant velocity, the object's instantaneous velocity is equal to its average velocity. The position versus time graph of this motion (Figure 2-6) will be a straight line whose slope equals the velocity.

The instantaneous velocity is a vector, and the magnitude of the instantaneous velocity is the **instantaneous speed.** Throughout the rest of the text, we shall use "velocity" in place of "instantaneous velocity" and "speed" in place of "instantaneous speed," except when emphasis or clarity is better served by the use of the adjective "instantaneous."

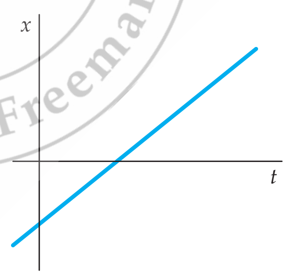


FIGURE 2-6 The position-versus-time graph for a particle moving at constant velocity.

Example 2-5 Position of a Particle as a Function of Time — *Try It Yourself*

The position of a particle as a function of time is given by the curve shown in Figure 2-7. Find the instantaneous velocity at time $t = 1.8$ s. When is the velocity greatest? When is it zero? Is it ever negative?

PICTURE In Figure 2-7, we have sketched the line tangent to the curve at $t = 1.8$ s. The tangent line's slope is the instantaneous velocity of the particle at the given time. You can measure the slope of the tangent line directly off this figure.

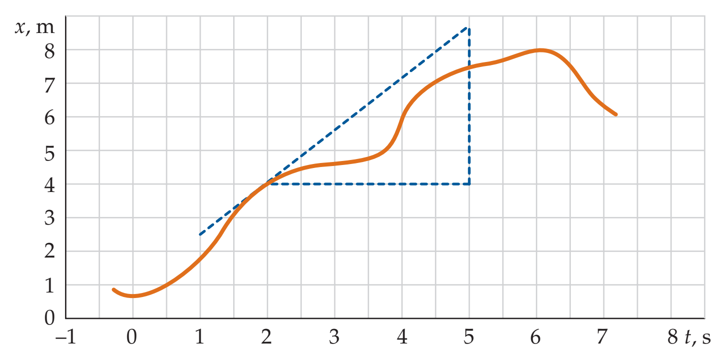


FIGURE 2-7

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps	Answers
1. Find the values x_1 and x_2 for the points on the tangent line at times $t_1 = 2.0$ s and $t_2 = 5.0$ s.	$x_1 \approx 4.0$ m, $x_2 \approx 8.5$ m
2. Compute the slope of the tangent line from these values. This slope equals the instantaneous velocity at $t = 2.0$ s.	$v_x = \text{slope} \approx \dfrac{8.5\text{ m} - 4.0\text{ m}}{5.0\text{ s} - 2.0\text{ s}} = \boxed{1.5\text{ m/s}}$
3. From the figure, the tangent line is steepest, and, therefore, the slope is greatest at about $t = 4.0$ s. The velocity is $\boxed{\text{greatest at } t \approx 4.0\text{ s}}$. The slope and the velocity both are $\boxed{\text{zero at } t = 0.0 \text{ and } t = 6.0\text{ s}}$ and are $\boxed{\text{negative for } t < 0.0 \text{ and } t > 6.0\text{ s}}$.	

CHECK The position of the particle changes from about 1.8 m at 1.0 s to 4.0 m at 2.0 s, so the average velocity for the interval from 1.0 s to 2.0 s is 2.2 m/s. This is the same order of magnitude as the value for the instantaneous velocity at 1.8 s, so the step-2 result is plausible.

PRACTICE PROBLEM 2-1 Estimate the average velocity of this particle between $t = 2.0$ s and $t = 5.0$ s.

Example 2-6 A Stone Dropped from a Cliff

The position of a stone dropped from a cliff is described approximately by $x = 5t^2$, where x is in meters and t is in seconds. The $+x$ direction is downwards and the origin is at the top of the cliff. Find the velocity of the stone during its fall as a function of time t.

PICTURE We can compute the velocity at some time t by computing the derivative dx/dt directly from the definition in Equation 2-4. The corresponding curve giving x versus t is shown in Figure 2-8. Tangent lines are drawn at times t_1, t_2, and t_3. The slopes of these tangent lines increase steadily with increasing time, indicating that the instantaneous velocity increases steadily with time.

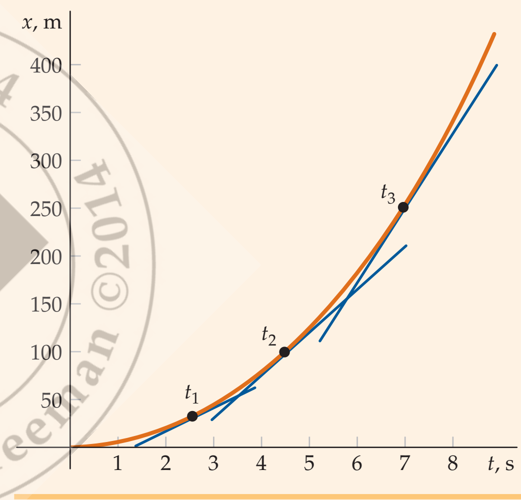


FIGURE 2-8

SOLVE

1. By definition the instantaneous velocity is

$$v_x(t) = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} = \lim_{\Delta t\to 0}\frac{x(t+\Delta t) - x(t)}{\Delta t}$$

2. We compute the displacement Δx from the position function $x(t)$:

$$x(t) = 5t^2$$

3. At a later time $t + \Delta t$, the position is $x(t + \Delta t)$, given by:

$$x(t+\Delta t) = 5(t+\Delta t)^2 = 5[t^2 + 2t\,\Delta t + (\Delta t)^2] = 5t^2 + 10t\,\Delta t + 5(\Delta t)^2$$

4. The displacement for this time interval is thus:

$$\Delta x = x(t+\Delta t) - x(t) = [5t^2 + 10t\,\Delta t + 5(\Delta t)^2] - 5t^2 = 10t\,\Delta t + 5(\Delta t)^2$$

5. Divide Δx by Δt to find the average velocity for this time interval:

$$v_{\text{av}\,x} = \frac{\Delta x}{\Delta t} = \frac{10t\,\Delta t + 5(\Delta t)^2}{\Delta t} = 10t + 5\,\Delta t$$

6. As we consider shorter and shorter time intervals, Δt approaches zero and the second term $5\Delta t$ approaches zero, though the first term, $10t$, remains unchanged:

$$v_x(t) = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} = \lim_{\Delta t\to 0}(10t + 5\,\Delta t) = \boxed{10t}$$

where v_x is in m/s and t is in s.

CHECK The stone starts at rest and goes faster and faster as it moves in the positive direction. Our result for the velocity, $v_x = 10t$, is zero at $t = 0$ and gets larger as t increases. Thus, $v_x = 10t$ is a plausible result.

TAKING IT FURTHER If we had set $\Delta t = 0$ in steps 4 and 5, the displacement would be $\Delta x = 0$, in which case the ratio $\Delta x/\Delta t$ would be undefined. Instead, we leave Δt as a variable until the final step, when the limit $\Delta t \to 0$ is well defined.

To find derivatives quickly, we use rules based on the limiting process above (see Appendix Table A-4). A particularly useful rule is

$$\text{If} \qquad x = Ct^n, \qquad \text{then} \qquad \frac{dx}{dt} = Cnt^{n-1} \qquad \text{2-6}$$

where C and n are any constants. Using this rule in Example 2-6, we have $x = 5t^2$ and $v_x = dx/dt = 10t$, in agreement with our previous results.

2-2 ACCELERATION

When you step on your car's gas pedal or brake, you expect your velocity to change. An object whose velocity changes is said to be accelerating. **Acceleration** is the rate of change of velocity with respect to time. The **average acceleration,** $a_{\text{av}\,x}$, for a particular time interval Δt is defined as the change in velocity, Δv, divided by that time interval:

$$a_{\text{av}\,x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{\text{f}\,x} - v_{\text{i}\,x}}{t_\text{f} - t_\text{i}} \qquad (\text{so } \Delta v_x = a_{\text{av}\,x}\,\Delta t) \qquad \text{2-7}$$

DEFINITION—AVERAGE ACCELERATION

Notice that acceleration has the dimensions of velocity (L/T) divided by time (T), which is the same as length divided by time squared (L/T^2). The SI unit is meters per second squared, m/s^2. Furthermore, like displacement and velocity, acceleration is a vector quantity. For one-dimensional motion, we can use + and − to indicate the direction of the acceleration. Equation 2-7 tells us that for $a_{\text{av}\,x}$ to be positive, Δv_x must be positive, and for $a_{\text{av}\,x}$ to be negative, Δv_x must be negative.

Instantaneous acceleration is the limit of the ratio $\Delta v_x/\Delta t$ as Δt approaches zero. On a plot of velocity versus time, the instantaneous acceleration at time t is the slope of the line tangent to the curve at that time:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$
$$= \text{slope of the line tangent to the } v\text{-versus-}t \text{ curve} \qquad \text{2-8}$$

DEFINITION—INSTANTANEOUS ACCELERATION

Thus, acceleration is the derivative of velocity v_x with respect to time, dv_x/dt. Because velocity is the derivative of the position x with respect to t, acceleration is the second derivative of x with respect to t, d^2x/dt^2. We can see the reason for this notation when we write the acceleration as dv_x/dt and replace v_x with dx/dt:

$$a_x = \frac{dv_x}{dt} = \frac{d(dx/dt)}{dt} = \frac{d^2x}{dt^2} \qquad \text{2-9}$$

Notice that when the time interval becomes extremely small, the average acceleration and the instantaneous acceleration become equal. Therefore, we will use the word acceleration to mean "instantaneous acceleration."

It is important to note that the sign of an object's acceleration does not tell you whether the object is speeding up or slowing down. To determine this, you need to compare the signs of both the velocity and the acceleration of the object. If v_x and a_x are both positive, v_x is positive and becoming more positive so the speed is increasing. If v_x and a_x are both negative, v_x is negative and becoming more negative so the speed is again increasing. When v_x and a_x have opposite signs, the object is slowing down. If v_x is positive and a_x is negative, v_x is positive but is becoming less positive

! Deceleration does not mean the acceleration is negative. Deceleration does mean that v_x and a_x have opposite signs.

CONCEPT CHECK 2-1

You are following the car in front of you at high speed when the driver of the car in front of you brakes hard, bringing his car to a stop to avoid running over a huge pothole. Three tenths of a second after you see the brake lights on the lead car flash, you too brake hard. Assume that the two cars are initially traveling at the same speed, and that once both cars are braking hard, they lose speed at the same rate. Does the distance between the two cars remain constant while the two cars are both braking hard?

so the speed is decreasing. If v_x is negative and a_x is positive, v_x is negative but is becoming less negative so the speed is again decreasing. In summary, if v_x and a_x have the same sign, the speed is increasing; if v_x and a_x have opposite signs, the speed is decreasing. When an object is slowing down, we sometimes say it is decelerating.

If acceleration remains zero, there is no change in velocity over time—velocity is constant. In this case, the plot of x versus t is a straight line. If acceleration is nonzero and constant, as in Example 2-13 , then velocity varies linearly with time and x varies quadratically with time.

Example 2-7 A Fast Cat

A cheetah can accelerate from 0 to 96 km/h (60 mi/h) in 2.0 s, whereas a Corvette requires 4.5 s. Compute the average accelerations for the cheetah and Corvette and compare them with the free-fall acceleration, $g = 9.81\ \mathrm{m/s^2}$.

(Stu Porter/Shutterstock.)

PICTURE Because we are given the initial and final velocities, as well as the change in time for both the cat and the car, we can simply use Equation 2-7 to find the acceleration for each object.

SOLVE

1. Convert 96 km/h into a velocity of m/s:

$$96\ \frac{\cancel{\mathrm{km}}}{\cancel{\mathrm{h}}}\left(\frac{1\ \cancel{\mathrm{h}}}{3600\ \mathrm{s}}\right)\left(\frac{1000\ \mathrm{m}}{1\ \cancel{\mathrm{km}}}\right) = 26.7\ \mathrm{m/s}$$

2. Find the average acceleration from the information given:

$$\text{cat}\quad a_{\mathrm{av}\,x} = \frac{\Delta v_x}{\Delta t} = \frac{26.7\ \mathrm{m/s} - 0}{2.0\ \mathrm{s}} = 13.3\ \mathrm{m/s^2} = \boxed{13\ \mathrm{m/s^2}}$$

$$\text{car}\quad a_{\mathrm{av}\,x} = \frac{\Delta v_x}{\Delta t} = \frac{26.7\ \mathrm{m/s} - 0}{4.5\ \mathrm{s}} = 5.93\ \mathrm{m/s^2} = \boxed{5.9\ \mathrm{m/s^2}}$$

3. To compare the result with the acceleration due to gravity, multiply each by the conversion factor $1g/9.81\ \mathrm{m/s^2}$:

$$\text{cat}\quad 13.3\ \mathrm{m/s^2} \times \frac{1g}{9.81\ \mathrm{m/s^2}} = 1.36g = \boxed{1.4g}$$

$$\text{car}\quad 5.93\ \mathrm{m/s^2} \times \frac{1g}{9.81\ \mathrm{m/s^2}} = 0.604g = \boxed{0.60g}$$

CHECK Because the car takes slightly more than twice as long as the cheetah to accelerate to the same velocity, it makes sense that the car's acceleration is slightly less than half that of the cat's.

TAKING IT FURTHER To reduce round-off errors, calculations are carried out using values with at least three digits even though the answers are reported using only two significant digits. These extra digits used in the calculations are called *guard digits.*

PRACTICE PROBLEM 2-2 A car is traveling at 45 km/h at time $t = 0$. It accelerates at a constant rate of 10 km/(h·s). (*a*) How fast is it traveling at $t = 2.0$ s? (*b*) At what time is the car traveling at 70 km/h?

Example 2-8 Velocity and Acceleration as Functions of Time

The position of a particle is given by $x = Ct^3$, where C is a constant. Find the dimensions of C. In addition, find both the velocity and the acceleration as functions of time.

PICTURE We can find the velocity by applying $dx/dt = Cnt^{n-1}$ (Equation 2-6) to the position of the particle, where n in this case equals 3. Then, we repeat the process to find the time derivative of velocity.

SOLVE

1. The dimensions of x and t are L and T, respectively:

$$C = \frac{x}{t^3} \Rightarrow [C] = \frac{[x]}{[t]^3} = \boxed{\frac{\mathrm{L}}{\mathrm{T}^3}}$$

2. We find the velocity by applying $dx/dt = Cnt^{n-1}$ (Equation 2-6):

$$x = Ct^n = Ct^3$$

$$v_x = \frac{dx}{dt} = Cnt^{n-1} = C3t^{3-1} = \boxed{3Ct^2}$$

3. The time derivative of velocity gives the acceleration: $a = \dfrac{dv_x}{dt} = Cnt^{n-1} = 3C(2)(t^{2-1}) = \boxed{6Ct}$

CHECK We can check the dimensions of our answers. For velocity, $[v_x] = [C][t^2] = (\mathrm{L/T^3})(\mathrm{T^2}) = \mathrm{L/T}$. For acceleration, $[a_x] = [C][t] = (\mathrm{L/T^3})(\mathrm{T}) = \mathrm{L/T^2}$.

PRACTICE PROBLEM 2-3 If a car starts from rest at $x = 0$ with constant acceleration a_x, its velocity v_x depends on a_x and the distance traveled x. Which of the following equations has the correct dimensions and therefore could possibly be an equation relating x, a_x, and v_x?

(*a*) $v_x = 2a_x x$ (*b*) $v_x^2 = 2a_x/x$ (*c*) $v_x = 2a_x x^2$ (*d*) $v_x^2 = 2a_x x$

MOTION DIAGRAMS

In studying physics, you will often wish to estimate the direction of the acceleration vector from a description of the motion. Motion diagrams can help. In a motion diagram the moving object is drawn at a sequence of equally spaced time intervals. For example, suppose you are on a trampoline and, following a high bounce, you are falling back toward the trampoline. As you descend, you keep going faster and faster. A motion diagram of this motion is shown in Figure 2-9*a*. The dots represent your position at equally spaced time intervals, so the space between successive dots increases as your speed increases. The numbers placed next to the dots are there to indicate the progression of time and an arrow representing your velocity is drawn next to each dot. The direction of each arrow represents the direction of your velocity at that instant, and the length of the arrow represents how fast you are going. Your acceleration vector* is in the direction that your velocity vector is changing—which is downward. In general, if the velocity arrows get longer as time progresses, then the acceleration is in the same direction of the velocity. On the other hand, if the velocity arrows get shorter as time progresses (Figure 2-9*b*), the acceleration is in the direction opposite to that of the velocity. Figure 2-9*b* is a motion diagram of your motion as you rise toward the ceiling following a bounce on the trampoline.

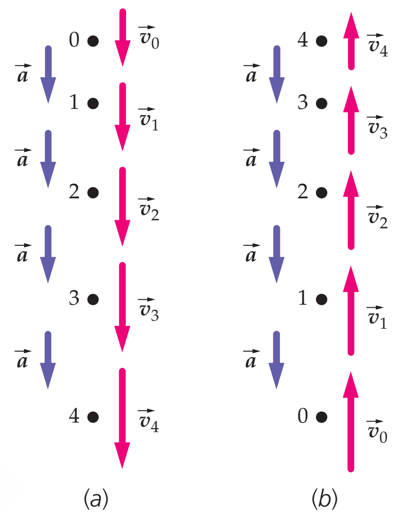


FIGURE 2-9 Motion diagrams. The time intervals between successive dots are identical. (*a*) The velocity vector is increasing, so the acceleration is in the direction of the velocity vector. (*b*) The velocity vector is decreasing, so the acceleration is in the direction opposite to that of the velocity vector.

2-3 MOTION WITH CONSTANT ACCELERATION

The motion of a particle that has nearly constant acceleration is found in nature. For example, near Earth's surface all unsupported objects fall vertically with nearly constant acceleration (provided air resistance is negligible). Other examples of near constant acceleration might include a plane accelerating along a runway for takeoff, and the motion of a car braking for a red light or speeding up at a green light. For a moving particle, the final velocity v_x equals the initial velocity plus the change in velocity, and the change in velocity equals the average acceleration multiplied by the time. That is,

$$v_x = v_{0x} + \Delta v = v_{0x} + a_{\mathrm{av}\,x}\,\Delta t \qquad \text{2-10}$$

If a particle has constant acceleration a_x, it follows that the instantaneous acceleration and the average acceleration are equal. That is,

$$a_x = a_{\mathrm{av}\,x} \qquad (a_x \text{ is constant}) \qquad \text{2-11}$$

Because situations involving nearly constant acceleration are common, we can use the equations for acceleration and velocity to derive a special set of **kinematic equations** for problems involving one-dimensional motion at constant acceleration.

* The velocity vector and the acceleration vector were introduced in Chapter 1 and are further developed in Chapter 3.

DERIVING THE CONSTANT-ACCELERATION KINEMATIC EQUATIONS

Suppose a particle moving with constant acceleration a_x has a velocity v_{0x} at time $t_0 = 0$, and v_x at some later time t. Combining Equations 2-10 and 2-11, we have

$$v_x = v_{0x} + a_x t \qquad (a_x \text{ is constant}) \tag{2-12}$$

CONSTANT ACCELERATION: $v_x(t)$

A v_x-versus-t plot (Figure 2-10) of this equation is a straight line. The line's slope is the acceleration a_x.

To obtain an equation for the position x as a function of time, we first look at the special case of motion with a constant velocity $v_x = v_{0x}$ (Figure 2-11). The change in position Δx during an interval of time Δt is

$$\Delta x = v_{0x}\,\Delta t \qquad (a_x = 0)$$

The area of the shaded rectangle under the v_x-versus-t curve (Figure 2-11*a*) is its height v_{0x} times its width Δt, so the area under the curve is the displacement Δx. If v_{0x} is negative (Figure 2-11*b*), both the displacement Δx and the area under the curve are negative. We normally think of area as a quantity that cannot be negative. However, in this context that is not the case. If v_{0x} is negative, the "height" of the curve is negative and the "area under the curve" is the negative quantity $v_{0x}\,\Delta t$.

The geometric interpretation of the displacement as the area under the v_x-versus-t curve is true not only for constant velocity, but it is true in general, as illustrated in Figure 2-12. To show that this statement is true, we first divide the time interval into numerous small intervals, Δt_1, Δt_2, and so on. Then, we draw a set of rectangles as shown. The area of the rectangle corresponding to the ith time interval Δt_i (shaded in the figure) is $v_i\,\Delta t_i$, which is approximately equal

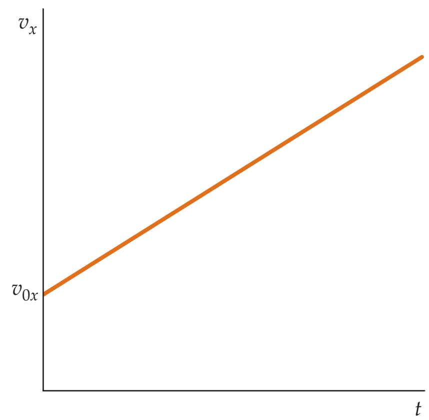


FIGURE 2-10 Graph of velocity versus time for constant acceleration.

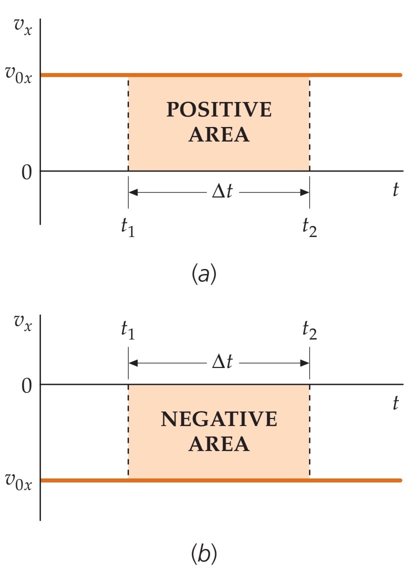


FIGURE 2-11 Motion with constant velocity.

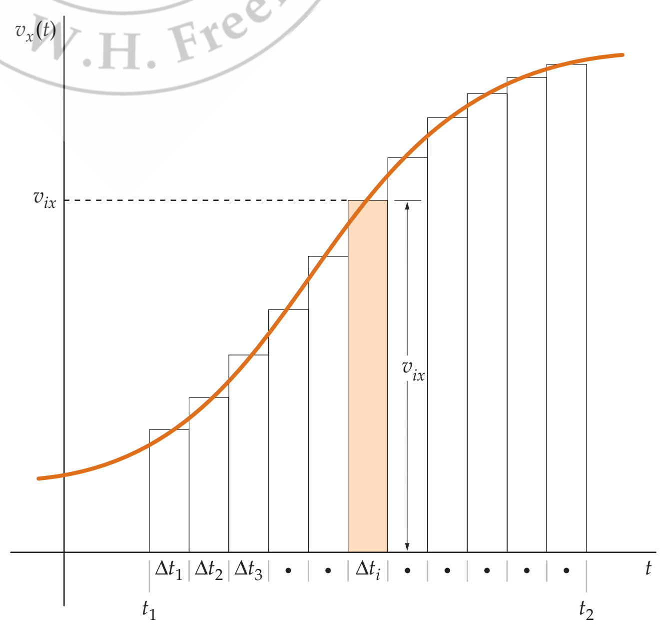


FIGURE 2-12 Graph of a general $v_x(t)$-versus-t curve. The total displacement from t_1 to t_2 is the area under the curve for this interval, which can be approximated by summing the areas of the rectangles.

to the displacement Δx_i during the interval Δt_i. The sum of the rectangular areas is therefore approximately the sum of the displacements during the time intervals and is approximately equal to the total displacement from time t_1 to t_2. We can make the approximation as accurate as we wish by putting enough rectangles under the curve, each rectangle having a sufficiently small value for Δt. For the limit of smaller and smaller time intervals (and more and more rectangles), the resulting sum approaches the area under the curve, which in turn equals the displacement. The displacement Δx is thus the area under the v_x-versus-t curve.

For motion with constant acceleration (Figure 2-13*a*), Δx is equal to the area of the shaded region. This region is divided into a rectangle and a triangle of areas $v_{1x}\,\Delta t$ and $\frac{1}{2}a_x(\Delta t)^2$, respectively, where $\Delta t = t_2 - t_1$. It follows that

$$\Delta x = v_{1x}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad \text{2-13}$$

If we set $t_1 = 0$ and $t_2 = t$, then Equation 2-13 becomes

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad \text{2-14}$$

CONSTANT ACCELERATION: $x(t)$

where x_0 and v_{0x} are the position and velocity at time $t = 0$, and $x = x(t)$ is the position at time t. The first term on the right, $v_{0x}t$, is the displacement that would occur if a_x were zero, and the second term, $\frac{1}{2}a_x t^2$, is the additional displacement due to the constant acceleration.

We next use Equations 2-12 and 2-14 to obtain two additional kinematic equations for constant acceleration. Solving Equation 2-12 for t, and substituting for t, in Equation 2-14 gives

$$\Delta x = v_{0x}\frac{v_x - v_{0x}}{a_x} + \frac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

Multiplying both sides by $2a_x$ we obtain

$$2a_x\,\Delta x = 2v_{0x}(v_x - v_{0x}) + (v_x - v_{0x})^2$$

Simplifying and rearranging terms gives

$$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x \qquad \text{2-15}$$

CONSTANT ACCELERATION: $v_x(x)$

The definition of average velocity (Equation 2-3) is:

$$\Delta x = v_{\text{av}\,x}\,\Delta t$$

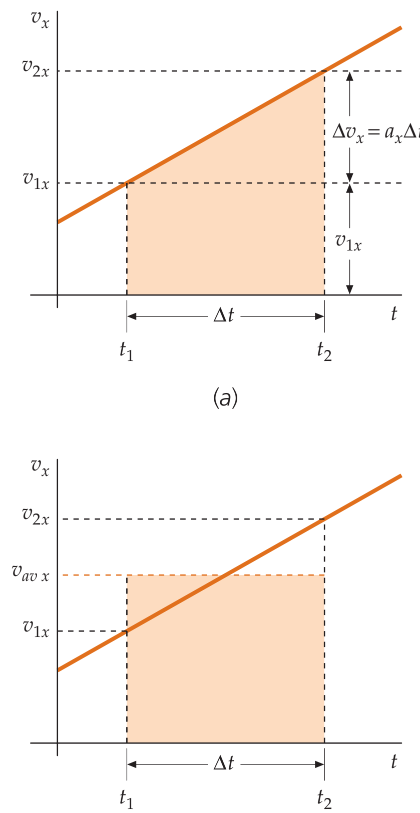
where $v_{\text{av}\,x}\,\Delta t$ is the area under the horizontal line at height $v_{\text{av}\,x}$ in Figure 2-13*b* and Δx is the area under the v_x versus t curve in Figure 2-13*a*. We can see that if $v_{\text{av}\,x} = \frac{1}{2}(v_{1x} + v_{2x})$, the area under the line at height v_{av} in Figure 2-13*a* and the area under the v_x versus t curve in Figure 2-13*b* will be equal. Thus,

$$v_{\text{av}\,x} = \tfrac{1}{2}(v_{1x} + v_{2x}) \qquad \text{2-16}$$

CONSTANT ACCELERATION: $v_{\text{av}\,x}$ AND v_x

For motion with constant acceleration, the average velocity is the mean of the initial and final velocities.

For an example of an instance where Equation 2-16 is not applicable, consider the motion of a runner during a 10.0-km run that takes 40.0 min to complete. The



(*b*)

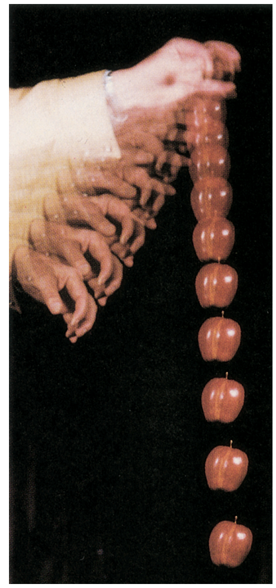
FIGURE 2-13 Motion with constant acceleration.

"It goes from zero to 60 in about 3 seconds." (*© Sydney Harris.*)

! Equation 2-16 is applicable only for time intervals during which the acceleration remains constant.

average velocity for the run is 0.250 km/min, which we compute using the definition of average velocity ($v_{\text{av }x} = \Delta x/\Delta t$). The runner starts from rest ($v_{1x} = 0$), and during the first one or two seconds his velocity increases rapidly, reaching a constant value v_{2x} that is sustained for the remainder of the run. The value of v_{2x} is just slightly greater than 0.250 km/min, so Equation 2-16 gives a value of about 0.125 km/min for the average velocity, a value almost 50% below the value given by the definition of average velocity. Equation 2-16 is not applicable because the acceleration does not remain constant for the entire run.

Equations 2-12, 2-14, 2-15, and 2-16 can be used to solve kinematics problems involving one-dimensional motion with constant acceleration. The choice of which equation or equations to use for a particular problem depends on what information you are given in the problem and what you are asked to find. Equation 2-15 is useful, for example, if we want to find the final velocity of a ball dropped from rest at some height x and we are not interested in the time the fall takes.



A falling apple captured by strobe photography at 60 flashes per second. The acceleration of the apple is indicated by the widening of the spaces between the images. *(Estate of Harold E. Edgerton/Palm Press.)*

USING THE CONSTANT-ACCELERATION KINEMATIC EQUATIONS

Review the Problem-Solving Strategy for solving problems using kinematic equations. Then, examine the examples involving one-dimensional motion with constant acceleration that follow.

PROBLEM-SOLVING STRATEGY

1-D Motion with Constant Acceleration

PICTURE Determine if a problem is asking you to find the time, distance, velocity, or acceleration for an object.

SOLVE Use the following steps to solve problems that involve one-dimensional motion and constant acceleration.

1. Draw a figure showing the particle in its initial and final positions. Include a coordinate axis and label the initial and final coordinates of the position. Indicate the + and − directions for the axis. Label the initial and final velocities, and label the acceleration.
2. Select one of the constant-acceleration kinematic equations (Equations 2-12, 2-14, 2-15, and 2-16). Substitute the given values into the selected equation and, if possible, solve for the desired value.
3. If necessary, select another of the constant-acceleration kinematic equations, substitute the given values into it, and solve for the desired value.

CHECK You should make sure that your answers are dimensionally consistent and the units of the answers are correct. In addition, check to make sure the magnitudes and signs of your answers agree with your expectations.

Problems with one object We will begin by looking at several examples that involve the motion of a single object.

Example 2-9 A Car's Stopping Distance

On a highway at night you see a stalled vehicle and brake your car to a stop. As you brake, the velocity of your car decreases at a constant rate of (5.0 m/s)/s. What is the car's stopping distance if your initial velocity is (*a*) 15 m/s (about 34 mi/h) or (*b*) 30 m/s?

PICTURE Use the Problem-Solving Strategy that precedes this example. The car is drawn as a dot to indicate a particle. We choose the direction of motion as $+x$ direction, and we choose the initial position $x_0 = 0$. The initial velocity is $v_{0x} = +15$ m/s and the final velocity $v_x = 0$. Because the velocity is decreasing, the acceleration is negative. It is $a_x = -5.0$ m/s². We seek the distance traveled, which is the magnitude of the displacement Δx. We are neither given nor asked for the time, so $v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$ (Equation 2-15) will provide a one-step solution.

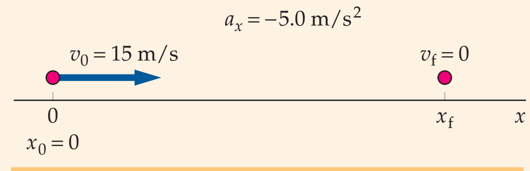


FIGURE 2-14

SOLVE

(*a*) 1. Draw the car (as a small dot) in its initial and final positions (Figure 2-14). Include a coordinate axis and label the drawing with the kinematic parameters.

2. Using Equation 2-15, calculate the displacement Δx:

$$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$$
$$0 = (15\text{ m/s})^2 + 2(-5.0\text{ m/s}^2)\Delta x$$
$$\Delta x = 22.5\text{ m} = \boxed{23\text{ m}}$$

(*b*) Substitute an initial speed of 30 m/s into the expression for Δx obtained in Part (*a*) (see Figure 2-14):

$$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$$
$$0 = (30\text{ m/s})^2 + 2(-5.0\text{ m/s}^2)\Delta x$$
$$\Delta x = \boxed{90\text{ m}}$$

CHECK The car's velocity decreases by 5.0 m/s each second. If its initial velocity is 15 m/s, it would take 3.0 s for it to come to rest. During the 3.0 s, it has an average velocity of half 15 m/s, so it would travel $\frac{1}{2}$(15 m/s)(3.0 s) = 23 m. This confirms our Part (*a*) result. Our Part (*b*) result can be confirmed in the same manner.

Example 2-10 Stopping Distance

Try It Yourself

In the situation described in Example 2-9, (*a*) how much time does it take for the car to stop if its initial velocity is 30 m/s, and (*b*) how far does the car travel in the last second?

PICTURE Use the Problem-Solving Strategy that precedes Example 2-9. (*a*) In this part of the problem, you are asked to find the time it takes the car to stop. You are given the initial velocity $v_{0x} = 30$ m/s. From Example 2-9, you know the car has an acceleration $a_x = -5.0$ m/s². A relationship between time, velocity, and acceleration is given by Equation 2-12. (*b*) Because the car's velocity decreases by 5.0 m/s each second, the velocity 1.0 s before the car stops must be 5.0 m/s. Find the average velocity during the last second and use that to find the distance traveled.

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps	Answers
(*a*) 1. Draw the car (as a small dot) in its initial and final positions (Figure 2-15). Include a coordinate axis and label the drawing with the kinematic parameters.	
2. Use Equation 2-12 to find the total stopping time Δt.	$\Delta t = \boxed{6.0\text{ s}}$
(*b*) 1. Draw the car (as a small dot) in its initial and final positions (Figure 2-16). Include a coordinate axis.	
2. Find the average velocity during the last second from $v_{\text{av}\,x} = \frac{1}{2}(v_{\text{i}\,x} + v_{\text{f}\,x})$.	$v_{\text{av}\,x} = 2.5$ m/s
3. Compute the distance traveled from $\Delta x = v_{\text{av}\,x}\,\Delta t$.	$\Delta x = v_{\text{av}\,x}\,\Delta t = \boxed{2.5\text{ m}}$

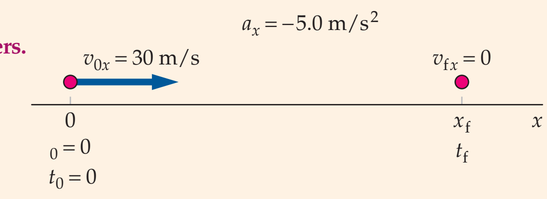


FIGURE 2-15

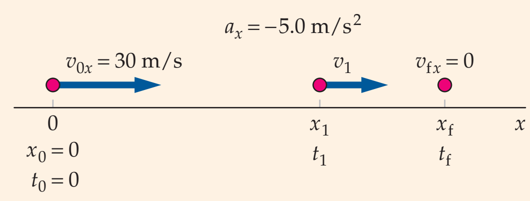
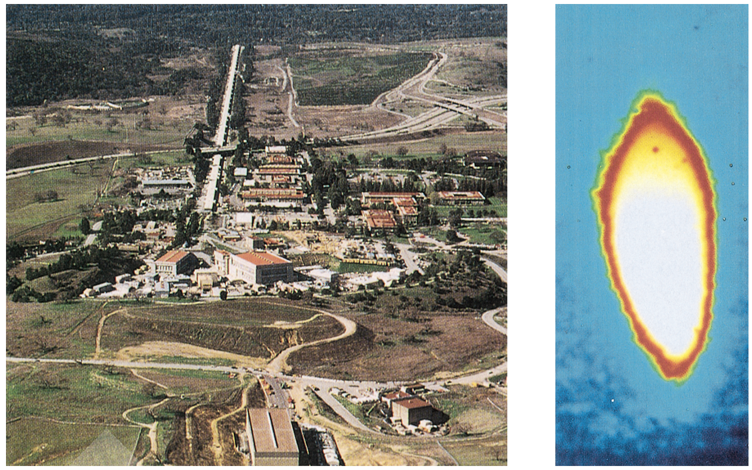


FIGURE 2-16

CHECK We would not expect the car to travel very far during the last second because it is moving relatively slowly. The Part (*b*) result of 2.5 m is a plausible result.

Example 2-11 A Traveling Electron *Try It Yourself*

An electron in a cathode-ray tube accelerates from rest with a constant acceleration of $5.33 \times 10^{12}\ \mathrm{m/s^2}$ for $0.150\ \mu\mathrm{s}$ ($1\ \mu\mathrm{s} = 10^{-6}$ s). The electron then drifts with constant velocity for $0.200\ \mu\mathrm{s}$. Finally, it slows to a stop with an acceleration of $-2.67 \times 10^{13}\ \mathrm{m/s^2}$. How far does the electron travel?



The two-mile-long linear accelerator at Stanford University used to accelerate electrons and positrons in a straight line to nearly the speed of light. Cross section of the accelerator's electron beam as shown on a video monitor. *(Stanford Linear Accelerator, U.S. Department of Energy.)*

PICTURE The equations for constant acceleration do not apply to the full duration of the electron's motion because the acceleration changes twice during that time. However, we can divide the electron's motion into three intervals, each with a different constant acceleration, and use the final position and velocity for the first interval as the initial conditions for the second interval, and the final position and velocity for the second interval as the initial conditions for the third. Apply the Problem-Solving Strategy that precedes Example 2-9 to each of the three constant-acceleration intervals. We will choose the origin to be at the electron's starting position, and the $+x$ direction to be the direction of motion.

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps

1. Draw the electron in its initial and final positions for each constant-acceleration interval (Figure 2-17). Include a coordinate axis and label the drawing with the kinematic parameters.

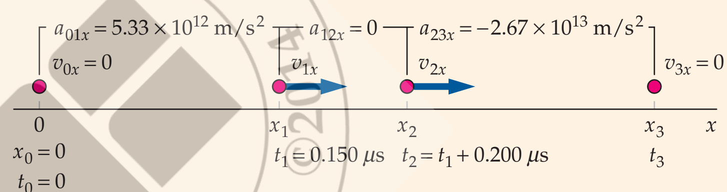


FIGURE 2-17

Answers

2. Set $v_{0x} = 0$ (because the electron starts from rest), use Equations 2-12 and 2-14 to find position x_1 and velocity v_{1x} at the end of the first 0.150-μs interval.
   $x_1 = 6.00\ \mathrm{cm}$, $v_{1x} = 8.00 \times 10^5\ \mathrm{m/s}$
3. The acceleration is zero during the second interval, so the velocity remains constant.
   $v_{2x} = v_{1x} = 8.00 \times 10^5\ \mathrm{m/s}$
4. The velocity remains constant during the second interval, so the displacement Δx_{12} equals the velocity v_{1x} multiplied by $0.200\ \mu\mathrm{s}$.
   $\Delta x_{12} = 16.0\ \mathrm{cm}$, so $x_2 = 22.0\ \mathrm{cm}$
5. To find the displacement for the third interval, use Equation 2-15 with $v_{3x} = 0$.
   $\Delta x_{23} = 1.20\ \mathrm{cm}$, so $x_3 = \boxed{23.2\ \mathrm{cm}}$

CHECK The average velocities are large, but the time intervals are small. Thus, the distances traveled are modest as we might expect.

Sometimes valuable insight can be gained about the motion of an object by asserting that the constant-acceleration formulas still apply even when the acceleration is not constant. The results are then estimates and not exact calculations. This is the case in the following example.

Example 2-12 The Crash Test *Context-Rich*

In a crash test that you are performing, a car traveling 100 km/h (about 62 mi/h) hits an immovable concrete wall. What is the acceleration of the car during the crash?

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PICTURE In this example, different parts of the vehicle will have different accelerations as the car crumples to a halt. The front bumper stops virtually instantly while the rear bumper stops some time later. We will solve for the acceleration of a part of the car that is in the passenger compartment and out of the crumple zone. A bolt holding the driver's seat belt to the floor is such a part. We do not really expect the acceleration of this bolt to be constant. We need additional information to solve this problem–either the stopping distance or the stopping time. We can estimate the stopping distance using common sense. Upon impact, the center of the car will certainly move forward less than half the length of the car. We will choose 0.75 m as a reasonable estimate of the distance the center of the car will move during the crash. Because the problem neither asks for nor provides the time, we will use the equation $v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$.

SOLVE

1. Draw the bolt (as a small circle) at the center of the car at its initial and final positions (Figure 2-18). Include a coordinate axis and label the drawing with the kinematic parameters.

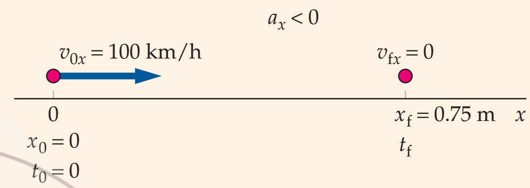

FIGURE 2-18

2. Convert the velocity from km/h to m/s.

$$(100\ \cancel{\text{km}}/\cancel{\text{h}}) \times \left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right) \times \left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right) \times \left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right) = 27.8\ \text{m/s}$$

3. Using $v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$, solve for the acceleration:

$$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$$

so

$$a_x = \frac{v_x^2 - v_{0x}^2}{2\Delta x} = \frac{0^2 - (27.8\ \text{m/s})^2}{2(0.75\ \text{m})}$$

4. Complete the calculation of the acceleration:

$$a_x = -\frac{(27.8\ \text{m/s})^2}{1.5\ \text{m}} = -514\ \text{m/s}^2 \approx \boxed{-500\ \text{m/s}^2}$$

CHECK The magnitude of the acceleration is about 50 times greater than the acceleration caused by the car breaking hard on a dry concrete road. The result is plausible because a large acceleration is expected for a high-speed head-on crash into an immovable object.

PRACTICE PROBLEM 2-4 Estimate the stopping time of the car.

Free-Fall Many practical problems deal with objects in free-fall, that is, objects that fall freely under the influence of gravity only. All objects in free-fall with the same initial velocity move identically. As shown in Figure 2-19, an apple and a feather, simultaneously released from rest in a large vacuum chamber, fall with identical motions. Thus, we know that the apple and the feather fall with the same acceleration. The magnitude of this acceleration, designated by g, has the approximate value $a = g \approx 9.81\ \text{m/s}^2 = 32.2\ \text{ft/s}^2$. If downward is designated as the $+y$ direction, then $a_y = +g$; if upward is designated as the $+y$ direction, then $a_y = -g$.

FIGURE 2-19 In a vacuum the apple and the feather, released simultaneously from rest, fall with identical motion. *(James Sugar/Black Star.)*

! Because g is the *magnitude* of the acceleration, g is *always* positive.

Example 2-13 The Flying Cap

Upon graduation, a joyful physics student throws her cap straight upward with an initial speed of 14.7 m/s. Given that its acceleration has a magnitude of 9.81 m/s² and is directed downward (we neglect air resistance), (*a*) how long does it take for the cap to reach its highest point? (*b*) What is the distance to the highest point? (*c*) Assuming the cap is caught at the same height from which it was released, what is the total time the cap is in flight?

PICTURE When the cap is at its highest point, its instantaneous velocity is zero. (When a problem specifies that an object is "at its highest point," translate this condition into the mathematical condition $v_y = 0$.)

SOLVE

(*a*) 1. Make a sketch of the cap in its initial position and again at its highest point (Figure 2-20). Include a coordinate axis and label the origin and the two specified positions of the cap.

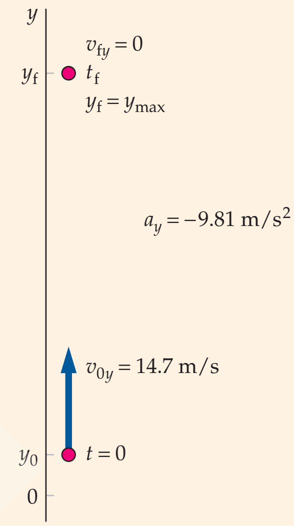

FIGURE 2-20

2. The time is related to the velocity and acceleration:

$$v_y = v_{0y} + a_y t$$

3. Set $v_y = 0$ and solve for t:

$$t = \frac{0 - v_{0y}}{a_y} = \frac{-14.7\ \mathrm{m/s}}{-9.81\ \mathrm{m/s^2}} = \boxed{1.50\ \mathrm{s}}$$

(*b*) We can find the displacement from the time t and the average velocity:

$$\Delta y = v_{\mathrm{av}\,y} t = \tfrac{1}{2}(v_{0y} + v_y)\Delta t$$
$$= \tfrac{1}{2}(14.7\ \mathrm{m/s} + 0)(1.50\ \mathrm{s}) = \boxed{11.0\ \mathrm{m}}$$

(*c*) 1. Set $y = y_0$ in Equation 2-14 and solve for t:

$$\Delta y = v_{0_y} t + \tfrac{1}{2}a_y t^2$$
$$0 = (v_{0y} + \tfrac{1}{2}a_y t)t$$

2. There are two solutions for t when $y = y_0$. The first corresponds to the time at which the cap is released, the second to the time at which the cap is caught:

$$t = 0 \quad \text{(first solution)}$$
$$t = -\frac{2v_{0y}}{a_y} = -\frac{2(14.7\ \mathrm{m/s})}{-9.81\ \mathrm{m/s^2}} = \boxed{3.00\ \mathrm{s}}$$
(second solution)

CHECK On the way up, the cap loses speed at the rate of 9.81 m/s each second. Because its initial speed is 14.7 m/s, we expect it rise for more than 1.00 s, but less than 2.00 s. Thus, a rise time of 1.50 s is quite plausible.

TAKING IT FURTHER On the plot of velocity versus time (Figure 2-21*b*) note that the slope is the same at all times, including the instant that $v_y = 0$. The slope is equal to the instantaneous acceleration, which is a constant $-9.81\ \mathrm{m/s^2}$. On the plot of height versus time (Figure 2-21*a*), note that the rise time equals the fall time. In reality, the cap will not have a constant acceleration because air resistance has a significant effect on a light object like a cap. If air resistance is not negligible, the fall time will exceed the rise time.

PRACTICE PROBLEM 2-5 Find $y_{max} - y_0$ using Equation 2-15. Find the velocity of the cap when it returns to its starting point.

PRACTICE PROBLEM 2-6 What is the velocity of the cap at the following points in time? (*a*) 0.100 s before it reaches its highest point; (*b*) 0.100 s after it reaches its highest point. (*c*) Compute $\Delta v_y/\Delta t$ for this 0.200-s-long time interval.

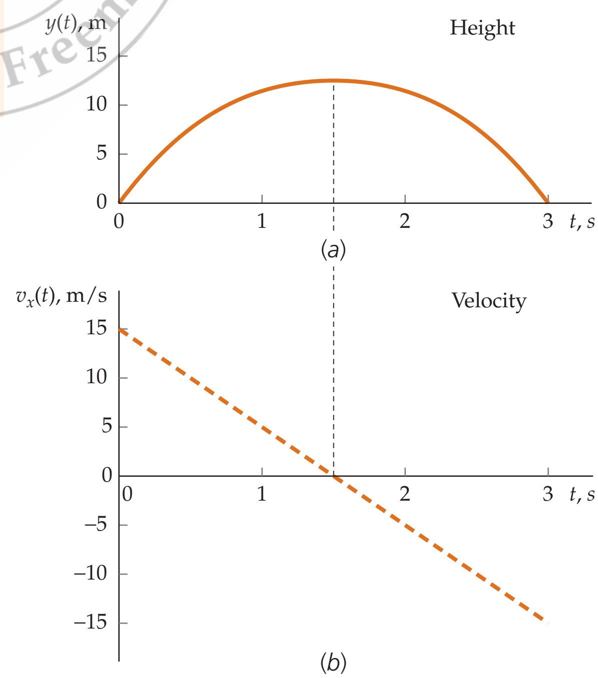

FIGURE 2-21 The height and velocity graphs are drawn one above the other so that both the height and the velocity can be observed at each instant of time.

Problems with two objects We now give some examples of problems involving two objects moving with constant acceleration.

Example 2-14 Catching a Speeding Car

A car is speeding at a constant 25 m/s (= 90 km/h ≈ 56 mi/h) in a school zone. A police car starts from rest just as the speeder passes by it and accelerates at a constant rate of 5.0 m/s². (*a*) When does the police car catch the speeding car? (*b*) How fast is the police car traveling when it catches up with the speeder?

PICTURE To determine when the two cars will be at the same position, we write the positions of the speeder x_S and of the police car x_P as functions of time and solve for the time t_c when $x_S = x_P$. Once we determine when the police car will catch up to the speeder, we can determine the velocity of the police car when it catches up to the speeder using the equation $v_x = a_x t$.

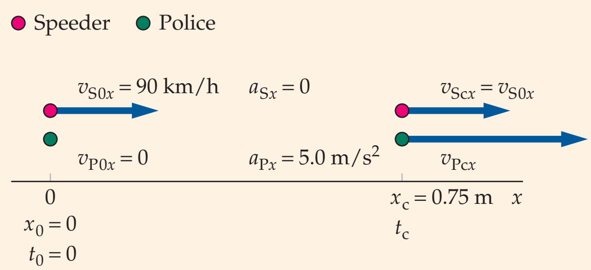

FIGURE 2-22 The speeder and the police car have the same position at $t = 0$ and again at $t = t_c$.

SOLVE

(*a*) 1. Draw the two cars at their initial positions (at $t = 0$) and again at their final positions (at $t = t_c$) (Figure 2-22). Include a coordinate axis and label the drawing with the kinematic parameters.

2. Write the position functions for the speeder and the police car:

$$x_S = v_{Sx}t \quad \text{and} \quad x_P = \tfrac{1}{2}a_{Px}t^2$$

3. Set $x_S = x_P$ and solve for the time t_c, for $t_c > 0$:

$$v_{Sx}t_c = \tfrac{1}{2}a_{Px}t_c^2 \Rightarrow v_{Sx} = \tfrac{1}{2}a_{Px}t_c \qquad t_c \neq 0$$

$$t_c = \frac{2v_{Sx}}{a_{Px}} = \frac{2(25\ \text{m/s})}{5.0\ \text{m/s}^2} = \boxed{10\ \text{s}}$$

(*b*) The velocity of the police car is given by $v_x = v_{0x} + a_x t$, with $v_{0x} = 0$:

$$v_{Px} = a_{Px}t_c = (5.0\ \text{m/s}^2)(10\ \text{s}) = \boxed{50\ \text{m/s}}$$

CHECK Notice that the final velocity of the police car in (*b*) is exactly twice that of the speeder. Because the two cars covered the same distance in the same time, they must have had the same average velocity. The speeder's average velocity, of course, is 25 m/s. For the police car to start from rest, maintain a constant acceleration, and have an average velocity of 25 m/s, it must reach a final velocity of 50 m/s.

PRACTICE PROBLEM 2-7 How far have the cars traveled when the police car catches the speeder?

Example 2-15 The Police Car

Try It Yourself

How fast is the police car in Example 2-14 traveling when it is 25 m behind the speeding car?

PICTURE The speed is given by $v_P = a_x t_1$, where t_1 is the time at which $x_S - x_P = 25$ m.

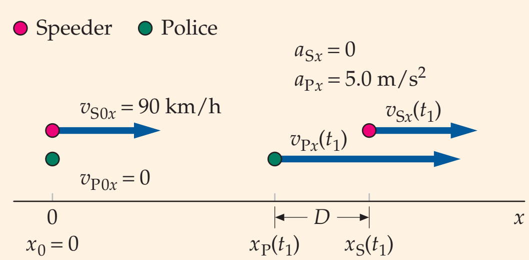

FIGURE 2-23

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps

1. Sketch an x-versus-t graph showing the positions of the two cars (Figure 2-23). On this graph identify the distance $D = x_S - x_P$ between the two cars at a given instant.

Answers

2. Using the equations for x_P and x_S from Example 2-14, solve for t_1 when $x_S - x_P = 25$ m. We expect two solutions, one shortly after the start time and one shortly before the speeder is caught. — $t_1 = (5 \pm \sqrt{15})$ s
3. Use $v_{P1} = a_{Px}t_1$ to compute the speed of the police car when $x_S - x_P = 25$ m. — $v_{P1} = \boxed{5.64 \text{ m/s}}$ and $\boxed{44.4 \text{ m/s}}$

CHECK We see from Figure 2-24 that the distance between the cars starts at zero, increases to a maximum value, and then decreases. We would expect two speeds for a given separation distance.

TAKING IT FURTHER The separation at any time is $D = x_S - x_P = v_{Sx}t - \frac{1}{2}a_{Px}t^2$. At maximum separation, which occurs at $t = 5.0$ s, $dD/dt = 0$. At equal time intervals before and after $t = 5.0$ s, the separations are equal.

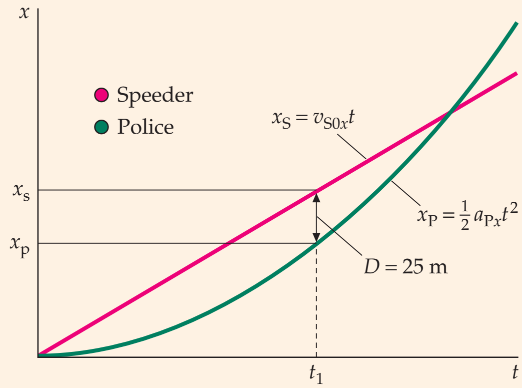

FIGURE 2-24

Example 2-16 A Moving Elevator

While standing in an elevator, you see a screw fall from the ceiling. The ceiling is 3.0 m above the floor. How long does it take the screw to hit the floor if the elevator is moving upward and gaining speed at a constant rate of 4.0 m/s² at the instant the screw leaves the ceiling?

PICTURE When the screw hits the floor, the positions of the screw and the floor are equal. Equate the these positions and solve for the time.

SOLVE

1. Draw a diagram showing the initial and final positions of the screw and the elevator floor (Figure 2-25). Include a coordinate axis and label the drawing with the kinematic parameters. The screw and the floor have the same initial velocity, but different accelerations. Choose the origin to be the initial position of the floor, and designate "upward" as the positive y direction. The screw hits the floor at time t_f:

2. Write equations specifying the *position* y_F of the elevator floor and the *position* y_S of the screw as functions of time. The screw and the elevator have the same initial velocity v_{0y}:

$$y_F - y_{F0} = v_{F0y}t + \tfrac{1}{2}a_{Fy}t^2$$
$$y_F - 0 = v_{0y}t + \tfrac{1}{2}a_{Fy}t^2$$
$$y_S - y_{S0} = v_{S0y}t + \tfrac{1}{2}a_{Sy}t^2$$
$$y_S - h = v_{0y}t + \tfrac{1}{2}(-g)t^2$$

3. Equate the expressions for y_S and y_F at $t = t_f$ and simplify:

$$y_S = y_F$$
$$h + v_{0y}t_f - \tfrac{1}{2}gt_f^2 = v_{0y}t_f + \tfrac{1}{2}a_{Fy}t_f^2$$
$$h - \tfrac{1}{2}gt_f^2 = \tfrac{1}{2}a_{Fy}t_f^2$$

4. Solve for the time and substitute the given values:

$$h = \tfrac{1}{2}(a_F + g)t_f^2 \quad \text{so}$$
$$t_f = \sqrt{\frac{2h}{a_F + g}} = \sqrt{\frac{2(3.0\text{ m})}{4.0\text{ m/s}^2 + 9.81\text{ m/s}^2}} = 0.659\text{ s} = \boxed{0.66\text{ s}}$$

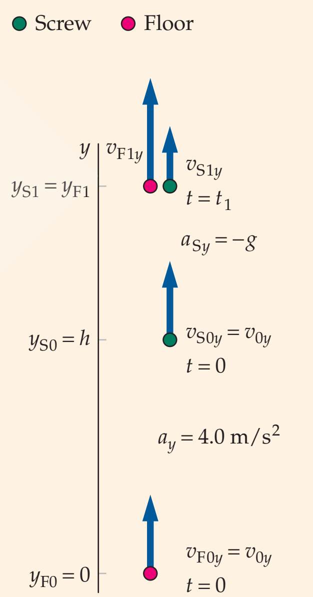

FIGURE 2-25 The y axis is fixed to the building.

CHECK If the elevator was stationary, the distance the screw falls is given by $h = \frac{1}{2}gt_f^2$. With $h = 3.0$ m, the resulting fall time would be $t_f = 0.78$ s. Because of the elevator's upward acceleration, we would expect it to take less than 0.78 s for the screw to hit the floor. Our 0.66-s result meets this expectation.

Example 2-17 The Moving Elevator

Try It Yourself

Consider the elevator and screw in Example 2-16. Assume the velocity of the elevator is 16 m/s upward when the screw separates from the ceiling. (*a*) How far does the elevator rise while the screw is in freefall? What is the displacement of the screw during freefall? (*b*) What are the velocity of the screw and the velocity of the elevator at impact?

PICTURE The time of flight of the screw is obtained in the solution of Example 2-16. Use this time to solve Parts (*a*) and (*b*).

SOLVE

Cover the column to the right and try these on your own before looking at the answers.

Steps	Answers
(*a*) 1. Using Equation 2-13, find the distance the floor rises between $t = 0$ and $t = t_f$, where t_f is calculated in step-4 of Example 2-16.	$\Delta y_F = v_{Fi}t_f + \frac{1}{2}a_F t_t^2 = \boxed{11.4\text{ m}}$
2. Between $t = 0$ and $t = t_f$, the displacement of the screw is less then that of the floor by 3.0 m.	$\Delta y_S = \boxed{+8.4\text{ m}}$
(*b*) Using $v_y = v_{0y} + a_y t$ (Equation 2-12), find the velocities of the screw and of the floor at impact.	$v_{Sy} = v_{Siy} - gt_f = \boxed{9.5\text{ m/s}}$ $v_{Fy} = v_{Fiy} + a_{Fy}t_f = \boxed{19\text{ m/s}}$

CHECK The Part (*b*) results (velocity of the screw and the velocity of the floor at impact) are both positive indicating both velocities are directed upward. For impact to occur, the floor must be moving upward faster then the screw so it can catch up with the screw. This result is consistent with our Part (*b*) results.

TAKING IT FURTHER The screw strikes the floor 8.4 m above its position when it leaves the ceiling. At impact, the velocity of the screw relative to the building is positive (upward). Relative to the building, the screw is still rising when it and the floor come in contact.

2-4 INTEGRATION

In this section, we use integral calculus to derive the equations of motion. A concise treatment of calculus can be found in the Math Tutorial.

To find the velocity from a given acceleration, we note that the velocity is the function $v_x(t)$ whose time derivative is the acceleration $a_x(t)$:

$$\frac{dv_x(t)}{dt} = a_x(t)$$

If the acceleration is constant, the velocity is that function of time which, when differentiated, equals this constant. One such function is

$$v_x = a_x t \qquad a_x \text{ is constant}$$

More generally, we can add any constant to $a_x t$ without changing the time derivative. Calling this constant c, we have

$$v_x = a_x t + c$$

When $t = 0$, $v_x = c$. Thus, c is the velocity v_{0x} at time $t = 0$.

Similarly, the position function $x(t)$ is that function whose derivative is the velocity:

$$\frac{dx}{dt} = v_x = v_{0x} + a_x t$$

We can treat each term separately. The function whose derivative is the constant v_{0x} is $v_{0x}t$ plus any constant. The function whose derivative is $a_x t$ is $\frac{1}{2}at^2$ plus any constant. Writing x_0 for the combined arbitrary constants, we have

$$x = x_0 + v_{0x}t + \tfrac{1}{2}at_x^2$$

When $t = 0$, $x = x_0$. Thus, x_0 is the position at time $t = 0$.

Whenever we find a function from its derivative, we must include an arbitrary constant in the general form of the function. Because we go through the integration process twice to find $x(t)$ from the acceleration, two constants arise. These constants are usually determined from the velocity and position at some given time, which is usually chosen to be $t = 0$. They are therefore called the **initial conditions.** A common problem, called the **initial-value problem,** takes the form "given $a_x(t)$ and the initial values of x and v_x, find $x(t)$." This problem is particularly important in physics because the acceleration of a particle is determined by the forces acting on it. Thus, if we know the forces acting on a particle and the position and velocity of the particle at some particular time, we can find its position and velocity at all other times.

A function $F(t)$ whose derivative (with respect to t) equals the function $f(t)$ is called the **antiderivative** of $f(t)$. (Because $v_x = dx/dt$ and $a_x = dv_x/dt$, x is the antiderivative of v_x and v_x is the antiderivative of a_x.) Finding the antiderivative of a function is related to the problem of finding the area under a curve.

In deriving Equation 2-14 it was shown that the change in position Δx is equal to the area under the velocity-versus-time curve. To show this (see Figure 2-12), we first divided the time interval into numerous small intervals, Δt_1, Δt_2, and so on. Then, we drew a set of rectangles as shown. The area of the rectangle corresponding to the ith time interval Δt_i (shaded in the figure) is $v_{ix}\,\Delta t_i$, which is approximately equal to the displacement Δx_i during the interval Δt_i. The sum of the rectangular areas is therefore approximately the sum of the displacements during the time intervals and is approximately equal to the total displacement from time t_1 to t_2. Mathematically, we write this as

$$\Delta x \approx \sum_i v_{ix}\,\Delta t_i$$

For the limit of smaller and smaller time intervals (and more and more rectangles), the resulting sum approaches the area under the curve, which in turn equals the displacement. The limit of the sum as Δt approaches zero (and the number of rectangles approaches infinity) is called an **integral** and is written

$$\Delta x = x(t_2) - x(t_1) = \lim_{\Delta t \to 0}\left(\sum_i v_{ix}\,\Delta t_i\right) = \int_{t_1}^{t_2} v_x\,dt \qquad \text{2-17}$$

See **Math Tutorial** *for more information on* **Integrals**

It is helpful to think of the integral sign $\int$ as an elongated S indicating a sum. The limits t_1 and t_2 indicate the initial and final values of the integration variable t.

The process of computing an integral is called **integration.** In Equation 2-17, v_x is the derivative of x, and x is the antiderivative of v_x. This is an example of the fundamental theorem of calculus, whose formulation in the seventeenth century greatly accelerated the mathematical development of physics. If

$$f(t) = \frac{dF(t)}{dt}, \quad \text{then} \quad F(t_2) - F(t_1) = \int_{t_1}^{t_2} f(t)\,dt \qquad \text{2-18}$$

FUNDAMENTAL THEOREM OF CALCULUS

The antiderivative of a function is also called the indefinite integral of the function and is written without limits on the integral sign, as in

$$x = \int v_x \, dt$$

Finding the function x from its derivative v_x (that is, finding the antiderivative) is also called integration. For example, if $v_x = v_{0x}$, a constant, then

$$x = \int v_{0x} \, dt = v_{0x}t + x_0$$

where x_0 is the arbitrary constant of integration. We can find a general rule for the integration of a power of t from Equation 2-6, which gives the general rule for the derivative of a power. The result is

$$\int t^n \, dt = \frac{t^{n+1}}{n+1} + C, \quad n \neq -1 \qquad \text{2-19}$$

where C is an arbitrary constant. This equation can be checked by differentiating the right side using the rule of Equation 2-6. (For the special case $n = -1$, $\int t^{-1} dt = \ln t + C$, where $\ln t$ is the natural logarithm of t.)

Because $a_x = dv_x/dt$, the change in velocity for some time interval can similarly be interpreted as the area under the a_x-versus-t curve for that interval. This change is written

$$\Delta v_x = \lim_{\Delta t \to 0} \left(\sum_i a_{ix} \Delta t_i \right) = \int_{t_1}^{t_2} a_x \, dt \qquad \text{2-20}$$

We can now derive the constant-acceleration equations by computing the indefinite integrals of the acceleration and velocity. If a_x is constant, we have

$$v_x = \int a_x dt = a_x \int dt = v_{0x} + a_x t \qquad \text{2-21}$$

where we have expressed the product of a_x and the constant of integration as v_{0x}. Integrating again, and writing x_0 for the constant of integration, gives

$$x = \int (v_{0x} + a_x t) dt = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad \text{2-22}$$

It is instructive to derive Equations 2-21 and 2-22 using definite integrals instead of indefinite ones. For constant acceleration, Equation 2-20, with $t_1 = 0$, gives

$$v_x(t_2) - v_x(0) = a_x \int_0^{t_2} dt = a_x(t_2 - 0)$$

where the time t_2 is arbitrary. Because it is arbitrary, we can set $t_2 = t$ to obtain

$$v_x = v_{0x} + a_x t$$

where $v_x = v_x(t)$ and $v_{0x} = v_x(0)$. To derive Equation 2-22, we substitute $v_{0x} + a_x t$ for v_x in Equation 2-17 with $t_1 = 0$. This gives

$$x(t_2) - x(0) = \int_0^{t_2} (v_{0x} + a_x t) dt$$

This integral is equal to the area under the v_x-versus-t curve (Figure 2-26). Evaluating the integral and solving for x gives

$$x(t_2) - x(0) = \int_0^{t_2} (v_{0x} + a_x t) dt = v_{0x}t + \tfrac{1}{2}a_x t^2 \Big|_0^{t_2} = v_{0x}t_2 + \tfrac{1}{2}a_x t_2^2$$

where t_2 is arbitrary. Setting $t_2 = t$, we obtain

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2$$

where $x = x(t)$ and $x_0 = x(0)$.

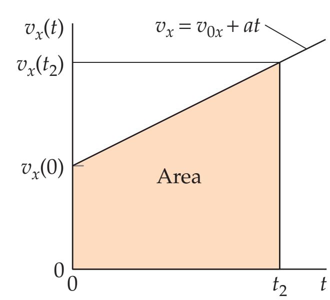

FIGURE 2-26 The area under the v_x-versus-t curve equals the displacement $\Delta x = x(t_2) - x(0)$.

The definition of average velocity is $\Delta x = v_{\text{av}\,x}\,\Delta t$ (Equation 2-3). In addition, $\Delta x = \int_{t_1}^{t_2} v_x\,dt$ (Equation 2-17). Equating the right sides of these equations and solving for $v_{\text{av}\,x}$ gives

$$v_{\text{av}\,x} = \frac{1}{\Delta t}\int_{t_1}^{t_2} v_x\,dt \qquad 2\text{-}23$$

ALTERNATIVE DEFINITION OF AVERAGE VELOCITY

where $\Delta t = t_2 - t_1$. Equation 2-23 is mathematically equivalent to the definition of average velocity, so either equation can serve as a definition of average velocity.

Example 2-18 A Coasting Boat

A Shelter Island ferryboat moves with constant velocity $v_{0x} = 8.0$ m/s for 60 s. It then shuts off its engines and coasts. Its coasting velocity is given by $v_x = v_{0x}t_1^2/t^2$, where $t_1 = 60$ s. What is the displacement of the boat for the interval $0 < t < \infty$?

(Gene Mosca.)

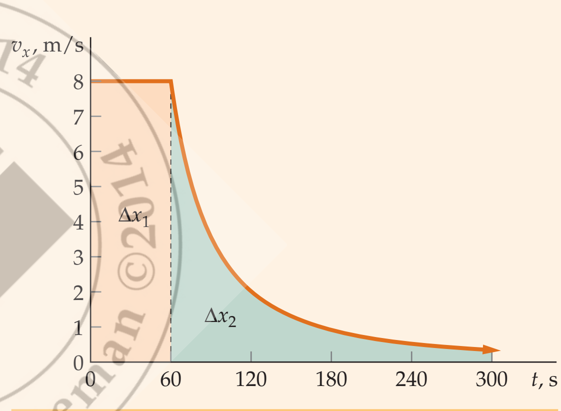

FIGURE 2-27

PICTURE The velocity function for the boat is shown in Figure 2-27. The total displacement is calculated as the sum of the displacement Δx_1 during the interval $0 < t < t_1 = 60$ s and the displacement Δx_2 during the interval $t_1 < t < \infty$.

SOLVE

1. The velocity of the boat is constant during the first 60 s; thus the displacement is simply the velocity times the elapsed time:

$$\Delta x_1 = v_{0x}\,\Delta t = v_{0x}t_1 = (8.0\text{ m/s})(60\text{ s}) = 480\text{ m}$$

2. The remaining displacement is given by the integral of the velocity from $t = t_1$ to $t = \infty$. We use Equation 2-17 to calculate the integral:

$$\Delta x_2 = \int_{t_1}^{\infty} v_x\,dt = \int_{t_1}^{\infty}\frac{v_{0x}t_1^2}{t^2}\,dt = v_{0x}t_1^2\int_{t_1}^{\infty} t^{-2}\,dt$$

$$= v_{0x}t_1^2\,\frac{t^{-1}}{-1}\bigg|_{t_1}^{\infty} = -v_{0x}t_1^2\left(\frac{1}{\infty} - \frac{1}{t_1}\right)$$

$$= -(0 - v_0t_1) = (8\text{ m/s})(60\text{ s}) = 480\text{ m}$$

3. The total displacement is the sum of the displacements found above:

$$\Delta x = \Delta x_1 + \Delta x_2 = 480\text{ m} + 480\text{ m} = \boxed{960\text{ m}}$$

CHECK The expressions obtained for the displacements in both steps 1 and 2 are velocity multiplied by time, so they are both dimensionally correct.

TAKING IT FURTHER Note that the area under the v_x-versus-t curve (Figure 2-27) is finite. Thus, even though the boat never stops moving, it travels only a finite distance. A better representation of the velocity of a coasting boat might be the exponentially decreasing function $v_x = v_{0x}e^{-b(t-t_1)}$, where b is a positive constant. In that case, the boat would also coast a finite distance in the interval $t_1 \le t \le \infty$.

Physics Spotlight

Linear Accelerators

Linear accelerators are instruments that accelerate electrically charged particles to high speeds along a long, straight track to collide with a target. Large accelerators can impart very high kinetic energies (on the order of billions of electron volts) to charged particles that serve as probes for studying the fundamental particles of matter and the forces that hold them together. (The energy required to remove an electron from an atom is on the order of one electron volt.) In the two-mile-long linear accelerator at Stanford University, electromagnetic waves boost the speed of electrons or positrons as they move through an evacuated copper pipe. When the high-speed particles collide with a target, several different kinds of subatomic particles are produced along with X rays and gamma rays. These particles then pass into devices called particle detectors.

The beige cylinder in the background is the linear accelerator at the heart of the Naval Academy Tandem Accelerator Laboratory. A beam of high-speed protons travels from the accelerator to the target area in the foreground. *(Gene Mosca.)*

Through experiments with such accelerators, physicists have determined that protons and neutrons, once thought to be the ultimate particles of the nucleus, are themselves composed of more fundamental particles called quarks. Another group of particles known as leptons, which include electrons, neutrinos, and a few other particles, have also been identified. Most large accelerator research centers such as the Fermi National Accelerator Laboratory in Batavia, Illinois, use a series of linear and circular accelerators to achieve higher particle speeds. As the speed of a particle approaches the speed of light, the energy required to accelerate it to that speed approaches infinity.

Although the big accelerators may have a high profile, thousands of linear accelerators are used worldwide for a host of practical applications. One of the most common applications is the cathode ray tube (CRT) of a television set or computer monitor. In a CRT, electrons from the cathode (a heated filament) are accelerated in a vacuum toward a positively charged anode. Electromagnets control the direction of the electron beam onto the inside of a screen coated with a phosphor, a material that emits light when struck by electrons. The kinetic energy of electrons in a CRT ranges to a maximum of about 30,000 electron volts. The speed of an electron that has this kinetic energy is about one third of the speed of light.

In the field of medicine, linear accelerators about a thousand times more powerful than a CRT are used for radiation treatment of cancer. "The linear accelerator uses microwave technology (similar to that used for radar) to accelerate electrons in a part of the accelerator called the 'wave guide,' then allows these electrons to collide with a heavy metal target. As a result of the collisions, high-energy x-rays are scattered from the target. A portion of these x-rays is collected and then shaped to form a beam that matches the patient's tumor."*

Other applications of accelerators include the production of radioisotopes for tracers in medicine and biology, sterilization of surgical tools, and analysis of materials to determine their composition. For example, in a technique called particle-induced X-ray emission (PIXE), an ion beam, often consisting of protons, causes target atoms to emit X rays that identify the type of atoms present. This technique has been applied to the study of archeological materials and variety of other types of samples.

* The American College of Radiology and the Radiological Society of North America, http//www.radiologyinfo.org/content/therapy/linear_accelerator.htm.

SUMMARY

Displacement, velocity, and acceleration are important *defined* kinematic quantities.

TOPIC	RELEVANT EQUATIONS AND REMARKS	
1. Displacement	$\Delta x = x_2 - x_1$	2-1
Graphical interpretation	Displacement is the area under the v_x-versus-t curve.	
2. Velocity		
Average velocity	$v_{\text{av}\,x} = \dfrac{\Delta x}{\Delta t}$ or $v_{\text{av}\,x} = \dfrac{1}{\Delta t}\displaystyle\int_{t_1}^{t_2} v_x\,dt$	2-3, 2-23
Instantaneous velocity	$v_x(t) = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$	2-5
Graphical interpretation	The instantaneous velocity is the slope of the x-versus-t curve.	
3. Speed		
Average speed	$\text{average speed} = \dfrac{\text{total distance}}{\text{total time}} = \dfrac{s}{t}$	2-2
Instantaneous speed	Instantaneous speed is the magnitude of the instantaneous velocity $\text{speed} = \lvert v_x \rvert$	
4. Acceleration		
Average acceleration	$a_{\text{av}\,x} = \dfrac{\Delta v_x}{\Delta t}$	2-7
Instantaneous acceleration	$a_x = \dfrac{dv_x}{dt} = \dfrac{d^2x}{dt^2}$	2-9
Graphical interpretation	The instantaneous acceleration is the slope of the v_x-versus-t curve.	
Acceleration due to gravity	The acceleration of an object near the surface of Earth in free-fall under the influence of gravity alone is directed downward and has magnitude $g = 9.81\ \text{m/s}^2 = 32.2\ \text{ft/s}^2$	
5. Kinematic equations for constant acceleration		
Velocity	$v_x = v_{0x} + a_x t$	2-12
Average velocity	$v_{\text{av}\,x} = \frac{1}{2}(v_{0x} + v_x)$	2-16
Displacement in terms of $v_{\text{av}\,x}$	$\Delta x = x - x_0 = v_{\text{av}\,x} t = \frac{1}{2}(v_{0x} + v_x)t$	
Displacement as a function of time	$\Delta x = x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$	2-14
v_x^2 as a function of Δx	$v_x^2 = v_{0x}^2 + 2a_x\,\Delta x$	2-15
6. Displacement and velocity as integrals	Displacement is represented graphically as the area under the v_x-versus-t curve. This area is the integral of v_x over time from some initial time t_1 to some final time t_2 and is written $\Delta x = \lim\limits_{\Delta t \to 0} \sum_i v_{i\,x}\,\Delta t_i = \displaystyle\int_{t_1}^{t_2} v_x\,dt$ (2-17) Similarly, change in velocity is represented graphically as the area under the a_x-versus-t curve: $\Delta v_x = \lim\limits_{\Delta t \to 0} \sum_i a_{i\,x}\,\Delta t_i = \displaystyle\int_{t_1}^{t_2} a_x\,dt$ (2-20)	2-17, 2-20

Answers to Concept Checks

2-1 No. The distance between cars will not remain constant. Instead, the distance will continuously decrease. When you first begin breaking, the speed of your car is greater than the speed of the car in front. That is because the car in front began breaking 0.3 s earlier. Because the cars lose speed at the same rate, the speed of your car will remain greater than the speed of the car in front throughout the braking period.

Answers to Practice Problems

2-1 1.2 m/s

2-2 (*a*) 65 km/h (*b*) 2.5 s

2-3 Only (*d*) has the same dimensions on both sides of the equation. Although we cannot obtain the exact equation from dimensional analysis, we can often obtain the functional dependence.

2-4 54 ms

2-5 (a) and (*b*) $y_{\max} - y_0 = 11.0$ m (*c*) -14.7 m/s; notice that the final speed is the same as the initial speed

2-6 (*a*) $+0.981$ m/s (*b*) -0.981 m/s
(*c*) $[(-0.981 \text{ m/s}) - (+0.981 \text{ m/s})]/(0.200 \text{ s}) = -9.81 \text{ m/s}^2$

2-7 250 m

PROBLEMS

In a few problems, you are given more data than you actually need; in a few other problems, you are required to supply data from your general knowledge, outside sources, or informed estimates.

Interpret as significant all digits in numerical values that have trailing zeroes and no decimal points.

For all problems, use $g = 9.81 \text{ m/s}^2$ for the free-fall acceleration due to gravity and neglect friction and air resistance unless instructed to do otherwise.

- • Single-concept, single-step, relatively easy
- •• Intermediate-level, may require synthesis of concepts
- ••• Challenging
- SSM Solution is in the Student Solutions Manual
- Consecutive problems that are shaded are paired problems.

CONCEPTUAL PROBLEMS

1 • What is the average velocity over the "round trip" of an object that is launched straight up from the ground and falls straight back down to the ground?

2 • An object thrown straight up falls back and is caught at the same place it is launched from. Its time of flight is T; its maximum height is H. Neglect air resistance. The correct expression for its average speed for the entire flight is (*a*) H/T, (*b*) 0, (*c*) $H/(2T)$, (*d*) $2H/T$.

3 • Using the information in the previous question, what is its average speed just for the first half of the trip? What is its average velocity for the second half of the trip? (Answer in terms of H and T.)

4 • Give an everyday example of one-dimensional motion where (*a*) the velocity is westward and the acceleration is eastward, and (*b*) the velocity is northward and the acceleration is northward.

5 • Stand in the center of a large room. Call the direction to your right "positive," and the direction to your left "negative." Walk across the room along a straight line, using a constant acceleration to quickly reach a steady speed along a straight line in the negative direction. After reaching this steady speed, keep your velocity negative but make your acceleration positive. (*a*) Describe how your speed varied as you walked. (*b*) Sketch a graph of x versus t for your motion. Assume you started at $x = 0$. (*c*) Directly under the graph of Part (*b*), sketch a graph of v_x versus t. SSM

6 • True/false: The displacement *always* equals the product of the average velocity and the time interval. Explain your choice.

7 • Is the statement "for an object's velocity to remain constant, its acceleration *must* remain zero" true or false? Explain your choice.

8 •• **MULTISTEP** Draw careful graphs of the position and velocity and acceleration over the time period $0 \le t \le 30$ s for a cart that, in succession, has the following motion. The cart is moving at the constant speed of 5.0 m/s in the $+x$ direction. It passes by the origin at $t = 0.0$ s. It continues on at 5.0 m/s for 5.0 s, after which it gains speed at the constant rate of 0.50 m/s each second for 10.0 s. After gaining speed for 10.0 s, the cart loses speed at the constant rate of 0.50 m/s for the next 15.0 s.

9 • True/false: Average velocity *always* equals one-half the sum of the initial and final velocities. Explain your choice.

10 • Identical twin brothers standing on a horizontal bridge each throw a rock straight down into the water below. They throw rocks at exactly the same time, but one hits the water before the other. How can this be? Explain what they did differently. Ignore any effects due to air resistance.

11 •• Dr. Josiah S. Carberry stands at the top of the Sears Tower in Chicago. Wanting to emulate Galileo, and ignoring the safety of the pedestrians below, he drops a bowling ball from the top of the tower. One second later, he drops a second bowling ball. While the balls are in the air, does their separation (*a*) increase over time, (*b*) decrease, (*c*) stay the same? Ignore any effects due to air resistance. SSM

12 •• Which of the position-versus-time curves in Figure 2-28 best shows the motion of an object (*a*) with positive acceleration, (*b*) with constant positive velocity, (*c*) that is always at rest, and

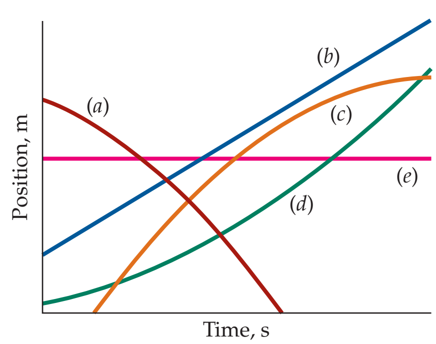

FIGURE 2-28
Problem 12

(*d*) with negative acceleration? (There may be more than one correct answer for each part of the problem.)

13 •• Which of the velocity-versus-time curves in Figure 2-29 best describes the motion of an object (*a*) with constant positive acceleration, (*b*) with positive acceleration that is decreasing with time, (*c*) with positive acceleration that is increasing with time, and (*d*) with no acceleration? (There may be more than one correct answer for each part of the problem.) SSM

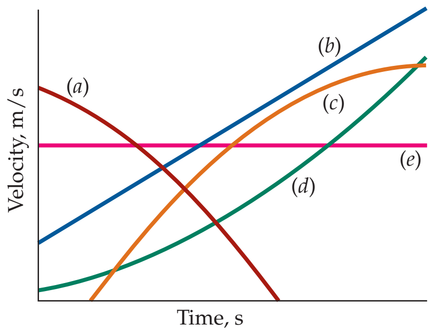

FIGURE 2-29 Problem 13

14 •• The diagram in Figure 2-30 tracks the location of an object moving in a straight line along the x axis. Assume that the object is at the origin at $t = 0$. Of the five times shown, which time (or times) represents when the object is (*a*) farthest from the origin, (*b*) at rest for an instant, (*c*) in the midst of being at rest for a while, and (*d*) moving away from the origin?

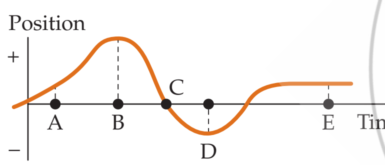

FIGURE 2-30 Problems 14 and 15

15 •• An object moves along a straight line. Its position-versus-time graph is shown in Figure 2-30. At which time or times is its (*a*) speed at a minimum, (*b*) acceleration positive, and (*c*) velocity negative? SSM

16 •• For each of the four graphs of x versus t in Figure 2-31 answer the following questions. (*a*) Is the velocity at time t_2 greater than, less than, or equal to the velocity at time t_1? (*b*) Is the speed at time t_2 greater than, less than, or equal to the speed at time t_1?

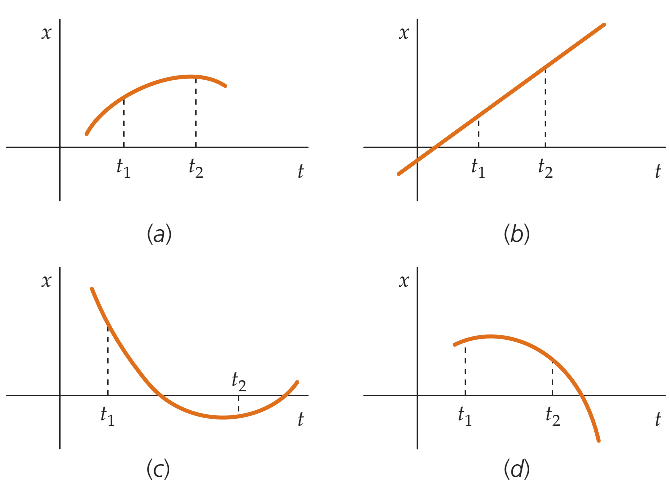

FIGURE 2-31 Problem 16

17 •• True/false:

(*a*) If the acceleration of an object is always zero, then it cannot be moving.
(*b*) If the acceleration of an object is always zero, then its x-versus-t curve must be a straight line.
(*c*) If the acceleration of an object is nonzero at an instant, it may be momentarily at rest at that instant.

Explain your reasoning for each answer. If an answer is *true*, give an example.

18 •• A hard-thrown tennis ball is moving horizontally when it bangs into a vertical concrete wall at perpendicular incidence. The ball rebounds straight back off the wall. Neglect any effects due to gravity for the small time interval described here. Assume that toward the wall is the $+x$ direction. What are the directions of its velocity and acceleration (*a*) just before hitting the wall, (*b*) at maximum impact, and (*c*) just after leaving the wall?

19 •• A ball is thrown straight up. Neglect any effects due to air resistance. (*a*) What is the velocity of the ball at the top of its flight? (*b*) What is its acceleration at that point? (*c*) What is different about the velocity and acceleration at the top of the flight if instead the ball impacts a horizontal ceiling very hard and then returns. SSM

20 •• An object that is launched straight up from the ground, reaches a maximum height H, and falls straight back down to the ground, hitting it T seconds after launch. Neglect any effects due to air resistance. (*a*) Express the average speed for the entire trip as a function of H and T. (*b*) Express the average speed for the same interval of time as a function of the initial launch speed v_0.

21 •• A small lead ball is thrown directly upward. True or false: (Neglect any effects due to air resistance.) (*a*) The magnitude of its acceleration decreases on the way up. (*b*) The direction of its acceleration on its way down is opposite to the direction of its acceleration on its way up. (*c*) The direction of its velocity on its way down is opposite to the direction of its velocity on its way up.

22 •• At $t = 0$, object A is dropped from the roof of a building. At the same instant, object B is dropped from a window 10 m below the roof. Air resistance is negligible. During the descent of B to the ground, the distance between the two objects (*a*) is proportional to t, (*b*) is proportional to t^2, (*c*) decreases, (*d*) remains 10 m throughout.

23 •• CONTEXT-RICH You are driving a Porsche that accelerates uniformly from 80.5 km/h (50 mi/h) at $t = 0.00$ s to 113 km/h (70 mi/h) at $t = 9.00$ s. (*a*) Which graph in Figure 2-32 best describes the velocity of your car? (*b*) Sketch a position-versus-time graph showing the location of your car during these nine seconds, assuming we let its position x be zero at $t = 0$.

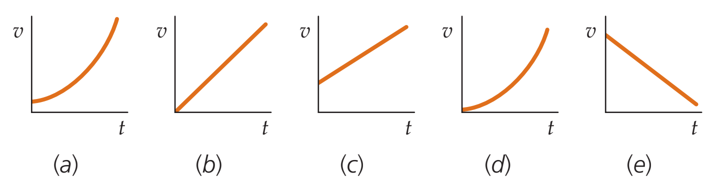

FIGURE 2-32 Problem 23

24 •• A small heavy object is dropped from rest and falls a distance D in a time T. After it has fallen for a time $2T$, what will be its (*a*) fall distance from its initial location, (*b*) its speed, and (*c*) its acceleration? (Neglect air resistance.)

25 •• In a race, at an instant when two horses are running right next to each other and in the same direction (the $+x$ direction), Horse A's instantaneous velocity and acceleration are +10 m/s and +2.0 m/s^2, respectively, and horse B's instantaneous velocity and acceleration are +12 m/s and −1.0 m/s^2, respectively. Which horse is passing the other at this instant? Explain.

26 •• True or false: (*a*) The equation $x - x_o = v_{ox}t + \frac{1}{2}a_x t^2$ is always valid for particle motion in one dimension. (*b*) If the velocity at a given instant is zero, the acceleration at that instant must also be zero. (*c*) The equation $\Delta x = v_{av}\,\Delta t$ holds for all particle motion in one dimension.

27 •• If an object is moving in a straight line at constant acceleration, its instantaneous velocity halfway through any time interval is (*a*) greater than its average velocity, (*b*) less than its average velocity, (*c*) equal to its average velocity, (*d*) half its average velocity, (*e*) twice its average velocity.

28 •• A turtle, seeing his owner put some fresh lettuce on the opposite side of his terrarium, begins to accelerate (at a constant rate) from rest at time $t = 0$, heading directly toward the food. Let t_1 be the time at which the turtle has covered half the distance to his lunch. Derive an expression for the ratio of t_2 to t_1, where t_2 is the time at which the turtle reaches the lettuce.

29 •• The positions of two cars in parallel lanes of a straight stretch of highway are plotted as functions of time in the Figure 2-33. Take positive values of x as being to the right of the origin. Qualitatively answer the following: (*a*) Are the two cars ever side by side? If so, indicate that time (those times) on the axis. (*b*) Are they always traveling in the same direction, or are they moving in opposite directions for some of the time? If so, when? (*c*) Are they ever traveling at the same velocity? If so, when? (*d*) When are the two cars the farthest apart? (*e*) Sketch (no numbers) the velocity versus time curve for each car. SSM

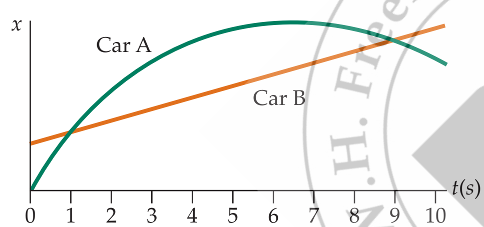

FIGURE 2-33 **Problem 29**

30 •• A car driving at constant velocity passes the origin at time $t = 0$. At that instant, a truck, at rest at the origin, begins to accelerate uniformly from rest. Figure 2-34 shows a qualitative plot of the velocities of truck and car as functions of time. Compare their displacements (from the origin), velocities, and accelerations at the instant that their curves intersect.

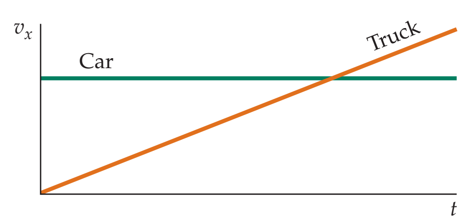

FIGURE 2-34 **Problem 30**

31 •• Reginald is out for a morning jog, and during the course of his run on a straight track, he has a velocity that depends upon time as shown in Figure 2-35. That is, he begins at rest, and ends at rest, peaking at a maximum velocity v_{max} at an arbitrary time t_{max}. A second runner, Josie, runs throughout the time interval $t = 0$ to $t = t_f$ at a constant speed v_J, so that each has the same displacement during the time interval. Note: t_f is NOT twice t_{max}, but represents an arbitrary time. What is the relation between v_J *and* v_{max}?

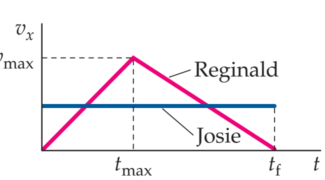

FIGURE 2-35 **Problem 31**

32 •• Which graph (or graphs), if any, of v_x versus t in Figure 2-36 best describes the motion of a particle with (*a*) positive velocity and increasing speed, (*b*) positive velocity and zero acceleration, (*c*) constant nonzero acceleration, and (*d*) a speed decrease?

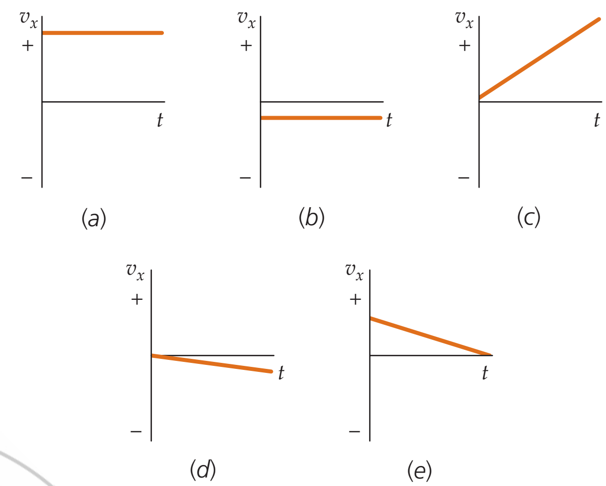

FIGURE 2-36 **Problems 32 and 33**

33 •• Which graph (or graphs), if any, of v_x versus t in Figure 2-36 best describes the motion of a particle with (*a*) negative velocity and increasing speed, (*b*) negative velocity and zero acceleration, (*c*) variable acceleration, and (*d*) increasing speed?

34 •• Sketch a v-versus-t curve for each of the following conditions: (*a*) Acceleration is zero and constant while velocity is not zero. (*b*) Acceleration is constant but not zero. (*c*) Velocity and acceleration are both positive. (*d*) Velocity and acceleration are both negative. (*e*) Velocity is positive and acceleration is negative. (*f*) Velocity is negative and acceleration is positive. (*g*) Velocity is momentarily zero but the acceleration is not zero.

35 •• Figure 2-37 shows nine graphs of position, velocity, and acceleration for objects in motion along a straight line. Indicate the graphs that meet the following conditions: (*a*) Velocity is constant,

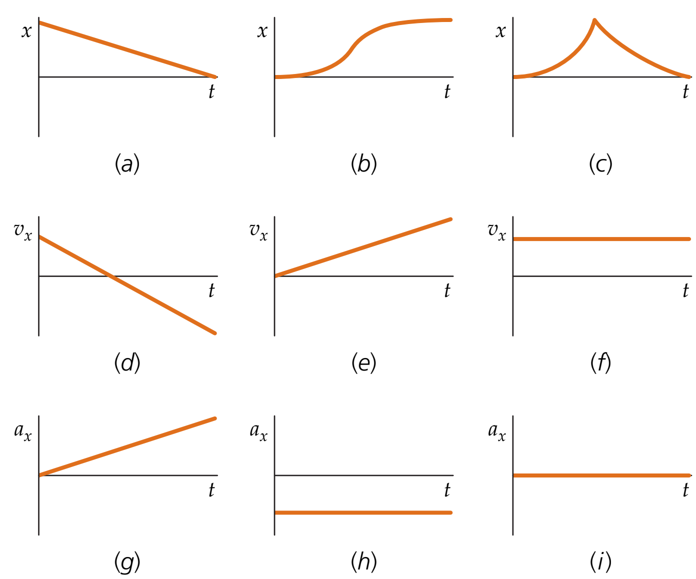

FIGURE 2-37 **Problem 35**

(*b*) velocity reverses its direction, (*c*) acceleration is constant, and (*d*) acceleration is not constant. (*e*) Which graphs of position, velocity, and acceleration are mutually consistent?

ESTIMATION AND APPROXIMATION

36 • **CONTEXT-RICH** While engrossed in thought about the scintillating lecture just delivered by your physics professor you mistakenly walk directly into the wall (rather than through the open lecture hall door). Estimate the magnitude of your average acceleration as you rapidly come to a halt.

37 • **BIOLOGICAL APPLICATION** Occasionally, people can survive falling large distances if the surface they land on is soft enough. During a traverse of the Eiger's infamous Nordvand, mountaineer Carlos Ragone's rock anchor gave way and he plummeted 500 feet to land in snow. Amazingly, he suffered only a few bruises and a wrenched shoulder. Assuming that his impact left a hole in the snow 4.0 ft deep, estimate his average acceleration as he slowed to a stop (that is, while he was impacting the snow). SSM

38 •• When we solve free-fall problems near Earth, it's important to remember that air resistance may play a significant role. If its effects are significant, we may get answers that are wrong by orders of magnitude if we ignore it. How can we tell when it is valid to ignore the effects of air resistance? One way is to realize that air resistance increases with increasing speed. Thus, as an object falls and its speed *increases*, its downward acceleration *decreases*. Under these circumstances, the object's speed will approach, as a limit, a value called its *terminal speed*. This terminal speed depends upon such things as the mass and cross-sectional area of the body. Upon reaching its terminal speed, its acceleration is zero. For a "typical" skydiver falling through the air, a typical terminal speed is about 50 m/s (roughly 120 mph). At half its terminal speed, the skydiver's acceleration will be about $\frac{3}{4}g$. Let us take half the terminal speed as a reasonable "upper bound" beyond which we should not use our constant acceleration free-fall relationships. Assuming the skydiver started from rest, (*a*) estimate how far, and for how long, the skydiver falls before we can no longer neglect air resistance. (*b*) Repeat the analysis for a Ping-Pong ball, which has a terminal speed of about 5.0 m/s. (*c*) What can you conclude by comparing your answers for Parts (*a*) and (*b*)?

39 •• **BIOLOGICAL APPLICATION** On June 14, 2005, Asafa Powell of Jamaica set a world's record for the 100-m dash with a time $t = 9.77$ s. Assuming he reached his maximum speed in 3.00 s, and then maintained that speed until the finish, estimate his acceleration during the first 3.00 s.

40 •• The photograph in Figure 2-38 is a short-time exposure (1/30 s) of a juggler with two tennis balls in the air. (*a*) The tennis ball near the top of its trajectory is less blurred than the lower one. Why is that? (*b*) Estimate the speed of the ball that he is just releasing from his right hand. (*c*) Determine how high the ball should have gone above the launch point and compare it to an estimate from the picture. *Hint: You have a built-in distance scale if you assume some reasonable value for the height of the juggler.*

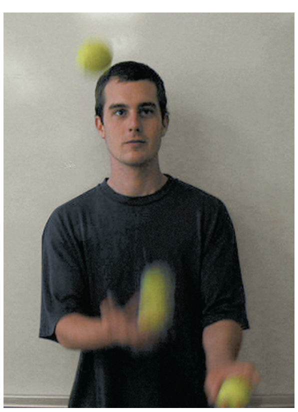

FIGURE 2-38
Problem 40
(Courtesy of Chuck Adler.)

41 •• A rough rule of thumb for determining the distance between you and a lightning strike is to start counting the seconds that elapse ("one-Mississippi, two-Mississippi, . . .") until you hear the thunder (sound emitted by the lightning as it rapidly heats the air around it). Assuming the speed of sound is about 750 mi/h, (*a*) estimate how far away is a lightning strike if you counted about 5 s until you heard the thunder. (*b*) Estimate the uncertainty in the distance to the strike in Part (*a*). Be sure to explain your assumptions and reasoning. *Hint: The speed of sound depends on the air temperature, and your counting is far from exact!*

SPEED, DISPLACEMENT, AND VELOCITY

42 • **ENGINEERING APPLICATION** (*a*) An electron in a television tube travels the 16-cm distance from the grid to the screen at an average speed of 4.0×10^{7} m/s. How long does the trip take? (*b*) An electron in a current-carrying wire travels at an average speed of 4.0×10^{-5} m/s. How long does it take to travel 16 cm?

43 • A runner runs 2.5 km, in a straight line, in 9.0 min and then takes 30 min to walk back to the starting point. (*a*) What is the runner's average velocity for the first 9.0 min? (*b*) What is the average velocity for the time spent walking? (*c*) What is the average velocity for the whole trip? (*d*) What is the average speed for the whole trip? SSM

44 • A car travels in a straight line with an average velocity of 80 km/h for 2.5 h and then with an average velocity of 40 km/h for 1.5 h. (*a*) What is the total displacement for the 4.0-h trip? (*b*) What is the average velocity for the total trip?

45 • One busy air route across the Atlantic Ocean is about 5500 km. The now-retired Concord, a supersonic jet capable of flying at twice the speed of sound, was used for traveling such routes. (*a*) Roughly how long did it take for a one-way flight? (Use 343 m/s for the speed of sound.) (*b*) Compare this time to the time taken by a subsonic jet flying at 0.90 times the speed of sound.

46 • The speed of light, designated by the universally recognized symbol c, has a value, to two significant figures, of 3.0×10^{8} m/s. (*a*) How long does it take for light to travel from the Sun to Earth, a distance of 1.5×10^{11} m? (*b*) How long does it take light to travel from the moon to Earth, a distance of 3.8×10^{8} m?

47 • Proxima Centauri, the closest star to us besides our own Sun, is 4.1×10^{13} km from Earth. From Zorg, a planet orbiting this star, a Gregor places an order at Tony's Pizza in Hoboken, New Jersey, communicating by light signals. Tony's fastest delivery craft travels at $1.00 \times 10^{-4}c$ (see Problem 46). (*a*) How long does it take Gregor's order to reach Tony's Pizza? (*b*) How long does Gregor wait between sending the signal and receiving the pizza? If Tony's has a "1000-years-or-it's-free" delivery policy, does Gregor have to pay for the pizza? SSM

48 • A car making a 100-km journey travels 40 km/h for the first 50 km. How fast must it go during the second 50 km to average 50 km/h?

49 •• **CONTEXT-RICH** Late in ice hockey games, the team that is losing sometimes "pulls" their goalkeeper off the ice to add an additional offensive player and increase their chances of scoring. In such cases, the goalie on the opposing team might have an opportunity to score into the unguarded net 55.0 m away. Suppose you are the goaltender for your university team and are in just such a situation. You launch a shot (in hopes of getting your first career goal) on the frictionless ice. You eventually hear a disappointing "clang" as the puck strikes a goalpost (instead of going in!) exactly 2.50 s later. In this case, how fast did the puck travel? You should assume 343 m/s for the speed of sound.

50 •• Cosmonaut Andrei, your co-worker at the International Space Station, tosses a banana at you at a speed of 15 m/s. At exactly the same instant, you fling a scoop of ice cream at Andrei along exactly the same path. The collision between banana and ice cream produces a banana split 7.2 m from your location 1.2 s after the banana and ice cream were launched. (*a*) How fast did you toss the ice cream? (*b*) How far were you from Andrei when you tossed the ice cream? (Neglect any effects due to gravity.)

51 •• Figure 2-39 shows the position of a particle as a function of time. Find the average velocities for the time intervals *a*, *b*, *c*, and *d* indicated in the figure.

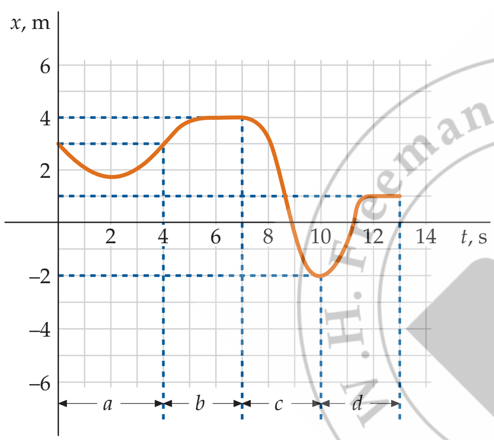

FIGURE 2-39 Problem 51

52 •• **ENGINEERING APPLICATION** It has been found that, on average, galaxies are moving away from Earth at a speed that is proportional to their distance from Earth. This discovery is known as Hubble's law, named for its discoverer, astrophysicist Sir Edwin Hubble. He found that the recessional speed v of a galaxy a distance r from Earth is given by $v = Hr$, where $H = 1.58 \times 10^{-18}\ \mathrm{s}^{-1}$ is called the Hubble constant. What are the expected recessional speeds of galaxies (*a*) 5.00×10^{22} m from Earth, and (*b*) 2.00×10^{25} m from Earth? (*c*) If the galaxies at each of these distances had traveled at their expected recessional speeds, how long ago would they have been at our location?

53 •• The cheetah can run as fast as 113 km/h, the falcon can fly as fast as 161 km/h, and the sailfish can swim as fast as 105 km/h. The three of them run a relay with each covering a distance L at maximum speed. What is the average speed of this relay team for the entire relay? Compare this average speed with the numerical average of the three individual speeds. Explain carefully why the average speed of the relay team is *not* equal to the numerical average of the three individual speeds. SSM

54 •• Two cars are traveling along a straight road. Car A maintains a constant speed of 80 km/h and car B maintains a constant speed of 110 km/h. At $t = 0$, car B is 45 km behind car A. (*a*) How much farther will car A travel before car B overtakes it? (*b*) How much ahead of A will B be 30 s after it overtakes A?

55 •• **MULTISTEP** A car traveling at a constant speed of 20 m/s passes an intersection at time $t = 0$. A second car traveling at a constant speed of 30 m/s in the same direction passes the same intersection 5.0 s later. (*a*) Sketch the position functions $x_1(t)$ and $x_2(t)$ for the two cars for the interval $0 \le t \le 20\,\mathrm{s}$. (*b*) Determine when the second car will overtake the first. (*c*) How far from the intersection will the two cars be when they pull even? (*d*) Where is the first car when the second car passes the intersection? SSM

56 •• **BIOLOGICAL APPLICATION** Bats use echolocation to determine their distance from objects they cannot easily see in the dark. The time between the emission of a high-frequency sound pulse (a click) and the detection of its echo is used to determine such distances. A bat, flying at a constant speed of 19.5 m/s in a straight line toward a vertical cave wall, makes a single clicking noise and hears the echo 0.15 s later. Assuming that she continued flying at her original speed, how close was she to the wall when she received the echo? Assume a speed of 343 m/s for the speed of sound.

57 ••• **ENGINEERING APPLICATION** A submarine can use *sonar* (sound traveling through water) to determine its distance from other objects. The time between the emission of a sound pulse (a "ping") and the detection of its echo can be used to determine such distances. Alternatively, by measuring the time between *successive* echo receptions of a *regularly timed set* of pings, the submarine's *speed* may be determined by comparing the time between echoes to the time between pings. Assume you are the sonar operator in a submarine traveling at a constant velocity underwater. Your boat is in the eastern Mediterranean Sea, where the speed of sound is known to be 1522 m/s. If you send out pings every 2.00 s, and your apparatus receives echoes reflected from an undersea cliff every 1.98 s, how fast is your submarine approaching the cliff?

ACCELERATION

58 • A sports car accelerates in third gear from 48.3 km/h (about 30 mi/h) to 80.5 km/h (about 50 mi/h) in 3.70 s. (*a*) What is the average acceleration of this car in $\mathrm{m/s^2}$? (*b*) If the car maintained this acceleration, how fast would it be moving one second later?

59 • An object is moving along the x axis. At $t = 5.0$ s, the object is at $x = +3.0$ m and has a velocity of $+5.0$ m/s. At $t = 8.0$ s, it is at $x = +9.0$ m and its velocity is -1.0 m/s. Find its average acceleration during the time interval $5.0\ \mathrm{s} < t < 8.0\ \mathrm{s}$. SSM

60 •• A particle moves along the x axis with velocity $v_x = (8.0\ \mathrm{m/s^2})t - 7.0\ \mathrm{m/s}$. (*a*) Find the average acceleration for two different one-second intervals, one beginning at $t = 3.0$ s and the other beginning at $t = 4.0$ s. (*b*) Sketch v_x versus t over the interval $0 < t < 10$ s. (*c*) How do the instantaneous accelerations at the middle of each of the two time intervals specified in Part (*a*) compare to the average accelerations found in Part (*a*)? Explain.

61 •• **MULTISTEP** The position of a certain particle depends on time according to the equation $x(t) = t^2 - 5.0t + 1.0$, where x is in meters if t is in seconds. (*a*) Find the displacement and average velocity for the interval $3.0\,\mathrm{s} \le t \le 4.0\,\mathrm{s}$. (*b*) Find the general formula for the displacement for the time interval from t to $t + \Delta t$. (*c*) Use the limiting process to obtain the instantaneous velocity for any time t. SSM

62 •• The position of an object as a function of time is given by $x = At^2 - Bt + C$, where $A = 8.0\ \mathrm{m/s^2}$, $B = 6.0$ m/s, and $C = 4.0$ m. Find the instantaneous velocity and acceleration as functions of time.

63 ••• The one-dimensional motion of a particle is plotted in Figure 2-40. (*a*) What is the average acceleration in each of the intervals *AB*, *BC*, and *CE*? (*b*) How far is the particle from its starting point after 10 s? (*c*) Sketch the displacement of the particle as a function of time; label the instants *A*, *B*, *C*, *D*, and *E* on your graph. (*d*) At what time is the particle traveling most slowly?

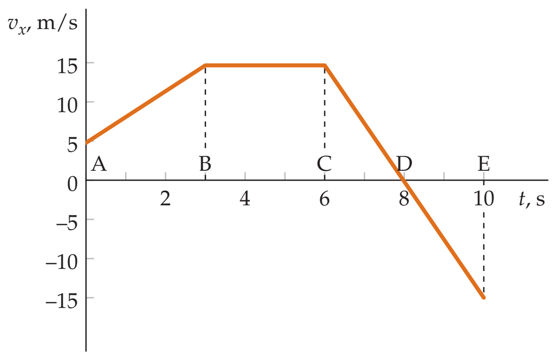

FIGURE 2-40 Problem 63

CONSTANT ACCELERATION AND FREE-FALL

64 • An object projected vertically upward with initial speed v_0 attains a maximum height h above its launch point. Another object projected up with initial speed $2v_0$ from the same height will attain a maximum height of (*a*) $4h$, (*b*) $3h$, (*c*) $2h$, (*d*) h. (Air resistance is negligible.)

65 • A car traveling along the x axis starts from rest at $x = 50$ m and accelerates at a constant rate of 8.0 m/s². (*a*) How fast is it going after 10 s? (*b*) How far has it gone after 10 s? (*c*) What is its average velocity for the interval $0 \le t \le 10$ s?

66 • An object traveling along the x axis with an initial velocity of +5.0 m/s has a constant acceleration of +2.0 m/s². When its speed is 15 m/s, how far has it traveled?

67 • An object traveling along the x axis at constant acceleration has a velocity of +10 m/s when it is at $x = 6.0$ m and of +15 m/s when it is at $x = 10.0$ m. What is its acceleration? SSM

68 • The speed of an object traveling along the x axis increases at the constant rate of +4.0 m/s each second. At $t = 0.0$ s, its velocity is +1.0 m/s and its position is +7.0 m. How fast is it moving when its position is +8.0 m, and how much time has elapsed from the start at $t = 0.0$ s?

69 •• A ball is launched directly upward from ground level with an initial speed of 20 m/s. (Air resistance is negligible.) (*a*) How long is the ball in the air? (*b*) What is the greatest height reached by the ball? (*c*) How many seconds after launch is the ball 15 m above the release point?

70 •• In the Blackhawk landslide in California, a mass of rock and mud fell 460 m down a mountain and then traveled 8.00 km across a level plain. It has been theorized that the rock and mud moved on a cushion of water vapor. Assume that the mass dropped with the free-fall acceleration and then slid horizontally, losing speed at a constant rate. (*a*) How long did the mud take to drop the 460 m? (*b*) How fast was it traveling when it reached the bottom? (*c*) How long did the mud take to slide the 8.00 km horizontally?

71 •• A load of bricks is lifted by a crane at a steady velocity of 5.0 m/s when one brick falls off 6.0 m above the ground. (*a*) Sketch the position of the brick $y(t)$ versus time, from the moment it leaves the pallet until it hits the ground. (*b*) What is the greatest height the brick reaches above the ground? (*c*) How long does it take to reach the ground? (*d*) What is its speed just before it hits the ground? SSM

72 •• A bolt comes loose from underneath an elevator that is moving upward at a constant speed of 6.0 m/s. The bolt reaches the bottom of the elevator shaft in 3.0 s. (*a*) How high above the bottom of the shaft was the elevator when the bolt came loose? (*b*) What is the speed of the bolt when it hits the bottom of the shaft?

73 •• An object is dropped from rest at a height of 120 m. Find the distance it falls during its final second in the air.

74 •• An object is released from rest at a height h. During the final second of its fall, it traverses a distance of 38 m. Determine h.

75 •• A stone is thrown vertically downward from the top of a 200-m cliff. During the last half second of its flight, the stone travels a distance of 45 m. Find the initial speed of the stone. SSM

76 •• An object is released from rest at a height h. It travels $0.4h$ during the first second of its descent. Determine the average velocity of the object during its entire descent.

77 •• A bus accelerates from rest at 1.5 m/s² for 12 s. It then travels at constant velocity for 25 s, after which it slows to a stop with an acceleration of magnitude 1.5 m/s². (*a*) What is the total distance that the bus travels? (*b*) What is its average velocity?

78 •• Al and Bert are jogging side-by-side on a trail in the woods at a speed of 0.75 m/s. Suddenly Al sees the end of the trail 35 m ahead and decides to speed up to reach it. He accelerates at a constant rate of 0.50 m/s² while Bert continues on at a constant speed. (*a*) How long does it take Al to reach the end of the trail? (*b*) Once he reaches the end of the trail, he immediately turns around and heads back along the trail with a constant speed of 0.85 m/s. How long does it take him to meet up with Bert? (*c*) How far are they from the end of the trail when they meet?

79 •• You have designed a rocket to be used to sample the local atmosphere for pollution. It is fired vertically with a constant upward acceleration of 20 m/s². After 25 s, the engine shuts off and the rocket continues rising (in freefall) for a while. (Air resistance is negligible.) The rocket eventually stops rising and then falls back to the ground. You want to get a sample of air that is 20 km above the ground. (*a*) Did you reach your height goal? If not, what would you change so that the rocket raches 20 km? (*b*) Determine the total time the rocket is in the air. (*c*) Find the speed of the rocket just before it hits the ground.

80 •• A flowerpot falls from a windowsill of an apartment that is on the tenth floor of an apartment building. A person in an apartment below, coincidentally in possession of a high-speed high-precision timing system, notices that it takes 0.20 s for the pot to fall past his window, which is 4.0-m from top to bottom. How far above the top of the window is the windowsill from which the pot fell? (Neglect any effects due to air resistance.)

81 •• In a classroom demonstration, a glider moves along an inclined track with constant acceleration. It is projected from the low end of the track with an initial velocity. After 8.00 s have elapsed, it is 100 cm from the low end and is moving along the track at a velocity of −15 cm/s. Find the initial velocity and the acceleration. SSM

82 •• A rock dropped from a cliff covers one-third of its total distance to the ground in the last second of its fall. Air resistance is negligible. How high is the cliff?

83 •• A typical automobile under hard braking loses speed at a rate of about 7.0 m/s^2; the typical reaction time to engage the brakes is 0.50 s. A local school board sets the speed limit in a school zone such that all cars should be able to stop in 4.0 m. (*a*) What maximum speed does this imply for an automobile in this zone? (*b*) What fraction of the 4.0 m is due to the reaction time? SSM

84 •• Two trains face each other on adjacent tracks. They are initially at rest, and their front ends are 40 m apart. The train on the left accelerates rightward at 1.0 m/s^2. The train on the right accelerates leftward at 1.3 m/s^2. (*a*) How far does the train on the left travel before the front ends of the trains pass? (*b*) If the trains are each 150 m in length, how long after the start are they completely past one another, assuming their accelerations are constant?

85 •• Two stones are dropped from the edge of a 60-m cliff, the second stone 1.6 *s* after the first. How far below the top of the cliff is the second stone when the separation between the two stones is 36 m?

86 •• A motorcycle officer hidden at an intersection observes a car driven by an oblivious driver who ignores a stop sign and continues through the intersection at constant speed. The police officer takes off in pursuit 2.0 s after the car has passed the stop sign. She accelerates at 4.2 m/s^2 until her speed is 110 km/h, and then continues at this speed until she catches the car. At that instant, the car is 1.4 km from the intersection. (a) How long did it take for the officer to catch up to the car? (*b*) How fast was the car traveling?

87 •• At $t = 0$, a stone is dropped from the top of a cliff above a lake. Another stone is thrown downward 1.6 s later from the same point with an initial speed of 32 m/s. Both stones hit the water at the same instant. Find the height of the cliff.

88 •• A passenger train is traveling at 29 m/s when the engineer sees a freight train 360 m ahead of his train traveling in the same direction on the same track. The freight train is moving at a speed of 6.0 m/s. (*a*) If the reaction time of the engineer is 0.40 s, what is the minimum (constant) rate at which the passenger train must lose speed if a collision is to be avoided? (*b*) If the engineer's reaction time is 0.80 s and the train loses speed at the minimum rate described in Part (*a*), at what rate is the passenger train approaching the freight train when the two collide? (*c*) For both reaction times, how far will the passenger train have traveled in the time between the sighting of the freight train and the collision?

89 •• **Biological Application** The click beetle can project itself vertically with an acceleration of about 400*g* (an order of magnitude more than a human could survive!). The beetle jumps by "unfolding" its 0.60-cm long legs. (*a*) How high can the click beetle jump? (*b*) How long is the beetle in the air? (Assume constant acceleration while in contact with the ground and neglect air resistance.)

90 •• An automobile accelerates from rest at 2.0 m/s^2 for 20 s. The speed is then held constant for 20 s, after which there is an acceleration of -3.0 m/s^2 until the automobile stops. What is the total distance traveled?

91 •• Consider measuring the free-fall motion of a particle (neglect air resistance). Before the advent of computer-driven data-logging software, these experiments typically employed a wax-coated tape placed vertically next to the path of a dropped electrically conductive object. A spark generator would cause an arc to jump between two vertical wires through the falling object and through the tape, thereby marking the tape at fixed time intervals Δt. Show that the change in height during successive time intervals for an object falling from rest follows *Galileo's Rule of Odd Numbers*: $\Delta y_{21} = 3\Delta y_{10}$, $\Delta y_{32} = 5\Delta y_{10}, \ldots$, where Δy_{10} is the change in y during the first interval of duration Δt, Δy_{21} is the change in y during the second interval of duration Δt, etc. SSM

92 •• A particle travels along the x axis with a constant acceleration of $+3.0$ m/s^2. It is at $x = -100$ m at time $t = 4.0$ s. In addition, it has a velocity of $+15$ m/s at a time 6.0 s later. Find its position at this later time.

93 •• If it were possible for a spacecraft to maintain a constant acceleration indefinitely, trips to the planets of the Solar System could be undertaken in days or weeks, while voyages to the nearer stars would only take a few years. (*a*) Using data from the tables at the back of the book, find the time it would take for a one-way trip from Earth to Mars (at Mars' closest approach to Earth). Assume that the spacecraft starts from rest, travels along a straight line, accelerates halfway at 1 *g*, flips around, and decelerates at 1 *g* for the rest of the trip. (*b*) Repeat the calculation for a 4.1×10^{13}-km trip to Proxima Centauri, our nearest stellar neighbor outside of the Sun. (See Problem 47.) SSM

94 •• The Stratosphere Tower in Las Vegas is 1137 ft high. It takes 1 min, 20 s to ascend from the ground floor to the top of the tower using the high-speed elevator. The elevator starts and ends at rest. Assume that it maintains a constant upward acceleration until it reaches its maximum speed, and then maintains a constant acceleration of equal magnitude until it comes to a stop. Find the magnitude of the acceleration of the elevator. Express this acceleration magnitude as a multiple of *g* (the acceleration due to gravity).

95 •• A train pulls away from a station with a constant acceleration of 0.40 m/s^2. A passenger arrives at a point next to the track 6.0 s after the end of the train has passed the very same point. What is the slowest constant speed at which she can run and still catch the train? On a single graph, plot the position versus time curves for both the train and the passenger.

96 ••• Ball A is dropped from the top of a building of height h at the same instant that ball B is thrown vertically upward from the ground. When the balls collide, they are moving in opposite directions, and the speed of A is twice the speed of B. At what height does the collision occur?

97 ••• Solve Problem 96 if the collision occurs when the balls are moving in the same direction and the speed of A is 4 times that of B.

98 ••• Starting at one station, a subway train accelerates from rest at a constant rate of 1.00 m/s^2 for half the distance to the next station, then slows down at the same rate for the second half of the journey. The total distance between stations is 900 m. (*a*) Sketch a graph of the velocity v_x as a function of time over the full journey. (*b*) Sketch a graph of the position as a function of time over the full journey. Place appropriate numerical values on both axes.

99 ••• A speeder traveling at a constant speed of 125 km/h races past a billboard. A patrol car pursues from rest with constant acceleration of (8.0 km/h)/s until it reaches its maximum speed of 190 km/h, which it maintains until it catches up with the speeder. (*a*) How long does it take the patrol car to catch the speeder if it starts moving just as the speeder passes? (*b*) How far does each car travel? (*c*) Sketch $x(t)$ for each car. SSM

100 ••• When the patrol car in Problem 99 (traveling at 190 km/h) is 100 m behind the speeder (traveling at 125 km/h), the speeder sees the police car and slams on his brakes, locking the wheels. (*a*) Assuming that each car can brake at 6.0 m/s^2 and that the driver of the police car brakes instantly as she sees the brake lights of the speeder (reaction time = 0.0 s), show that the cars collide. (*b*) At what time after the speeder applies his brakes do the two cars collide? (*c*) Discuss how reaction time would affect this problem.

101 ••• Leadfoot Lou enters the "Rest-to-Rest" auto competition, in which each contestant's car begins and ends at rest, covering a fixed distance L in as short a time as possible. The intention is to demonstrate driving skills, and to find which car is the best at the *total combination* of speeding up and slowing down. The course is designed so that maximum speeds of the cars are never reached. (*a*) If Lou's car maintains an acceleration (magnitude) of a during speedup, and maintains a deceleration (magnitude) of $2a$ during braking, at what fraction of L should Lou move his foot from the gas pedal to the brake? (*b*) What fraction of the total time for the trip has elapsed at that point? (*c*) What is the fastest speed Lou's car ever reaches? Neglect Lou's reaction time, and answer in terms of a and L.

102 ••• A physics professor, equipped with a rocket backpack, steps out of a helicopter at an altitude of 575 m with zero initial velocity. (Neglect air resistance.) For 8.0 s, she falls freely. At that time, she fires her rockets and slows her rate of descent at 15 m/s^2 until her rate of descent reaches 5.0 m/s. At this point, she adjusts her rocket engine controls to maintain that rate of descent until she reaches the ground. (*a*) On a single graph, sketch her acceleration and velocity as functions of time. (Take upward to be positive.) (*b*) What is her speed at the end of the first 8.0 s? (*c*) What is the duration of her slowing-down period? (*d*) How far does she travel while slowing down? (*e*) How much time is required for the entire trip from the helicopter to the ground? (*f*) What is her average velocity for the entire trip?

INTEGRATION OF THE EQUATIONS OF MOTION

103 • The velocity of a particle is given by $v_x(t) = (6.0\ \text{m/s}^2)t + (3.0\ \text{m/s})$. (*a*) Sketch v versus t and find the area under the curve for the interval $t = 0$ to $t = 5.0$ s. (*b*) Find the position function $x(t)$. Use it to calculate the displacement during the interval $t = 0$ to $t = 5.0$ s. SSM

104 • Figure 2-41 shows the velocity of a particle versus time. (*a*) What is the magnitude, in meters, represented by the area of the shaded box? (*b*) Estimate the displacement of the particle for the two 1-s intervals, one beginning at $t = 1.0$ s and the other at $t = 2.0$ s. (*c*) Estimate the average velocity for the interval $1.0\,\text{s} \le t \le 3.0\,\text{s}$. (*d*) The equation of the curve is $v_x = (0.50\ \text{m/s}^3)t^2$. Find the displacement of the particle for the interval $1.0\,\text{s} \le t \le 3.0\,\text{s}$ by integration and compare this answer with your answer for Part (*b*). Is the average velocity equal to the mean of the initial and final velocities for this case?

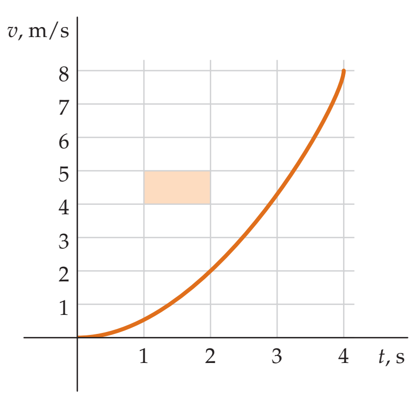

FIGURE 2-41 Problem 104

105 •• The velocity of a particle is given by $v_x = (7.0\ \text{m/s}^3)t^2 - 5.0\ \text{m/s}$. If the particle is at the origin at $t_0 = 0$, find the position function $x(t)$.

106 •• Consider the velocity graph in Figure 2-42. Assuming $x = 0$ at $t = 0$, write correct algebraic expressions for $x(t)$, $v_x(t)$, and $a_x(t)$ with appropriate numerical values inserted for all constants.

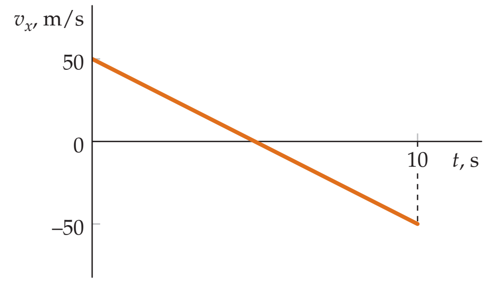

FIGURE 2-42 Problem 106

107 ••• Figure 2-43 shows the acceleration of a particle versus time. (*a*) What is the magnitude, in m/s, of the area of the shaded box? (*b*) The particle starts from rest at $t = 0$. Estimate the velocity at $t = 1.0$ s, 2.0 s, and 3.0 s by counting the boxes under the curve. (*c*) Sketch the curve v_x versus t from your results for Part (*b*); then estimate how far the particle travels in the interval $t = 0$ to $t = 3.0$ s.

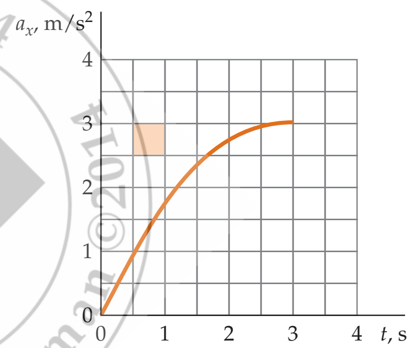

FIGURE 2-43 Problem 107

108 ••• Figure 2-44 is a graph of v_x versus t for a particle moving along a straight line. The position of the particle at time $t = 0$ is $x_0 = 5.0$ m. (*a*) Find x for various times t by counting boxes, and sketch x as a function of t. (*b*) Sketch a graph of the acceleration a_x as a function of the time t. (*c*) Determine the displacement of the particle between $t = 3.0$ s and 7.0 s.

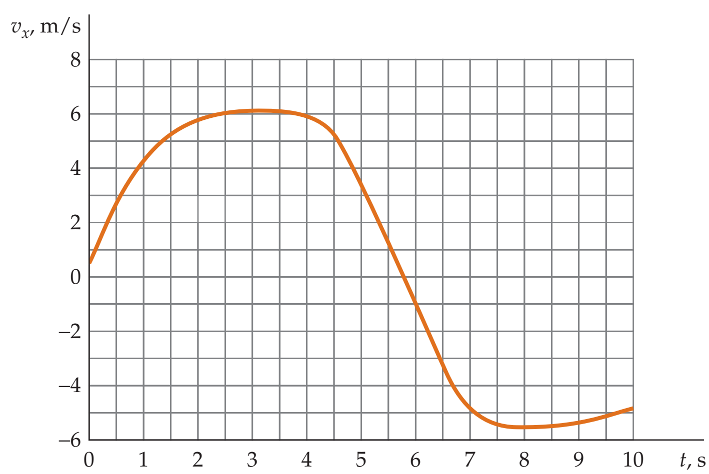

FIGURE 2-44 Problem 108

109 ••• **Conceptual** Figure 2-45 shows a plot of x versus t for an object moving along a straight line. For this motion, sketch graphs (using the same t axis) of (*a*) v_x as a function of t, and (*b*) a_x as a function of t. (*c*) Use your sketches to qualitatively compare the time(s) when the object is at its largest distance from the origin to the time(s) when its speed is greatest. Explain why the times are *not* the same. (*d*) Use your sketches to qualitatively compare the time(s) when the object is moving fastest to the time(s) when its acceleration is the largest. Explain why the times are *not* the same. SSM

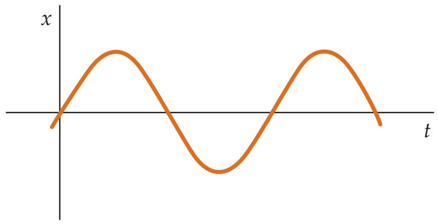

FIGURE 2-45 Problem 109

110 ••• **Multistep** The acceleration of a certain rocket is given by $a_x = bt$, where b is a positive constant. (*a*) Find the position function $x(t)$ if $x = x_0$ and $v_x = v_{0x}$ at $t = 0$. (*b*) Find the position and velocity at $t = 5.0$ s if $x_0 = 0$, $v_{0x} = 0$ and $b = 3.0\ \text{m/s}^3$. (*c*) Compute the average velocity of the rocket between $t = 4.5$ s and 5.5 s at $t = 5.0$ s if $x_0 = 0$, $v_{0x} = 0$ and $b = 3.0\ \text{m/s}^3$. Compare this average velocity with the instantaneous velocity at $t = 5.0$ s.

111 ••• In the time interval from 0.0 s to 10.0 s, the acceleration of a particle traveling in a straight line is given by $a_x = (0.20\ \text{m/s}^3)t$. Let to the right be the $+x$ direction. The particle initially has a velocity to the right of 9.5 m/s and is located 5.0 m to the left of the origin. (*a*) Determine the velocity as a function of time during the interval; (*b*) determine the position as a function of time during the interval; (*c*) determine the average velocity between $t = 0.0$ s and 10.0 s, and compare it to the average of the instantaneous velocities at the start and ending times. Are these two averages equal? Explain. SSM

112 ••• Consider the motion of a particle that experiences a variable acceleration given by $a_x = a_{0x} + bt$, where a_{0x} and b are constants and $x = x_0$ and $v_x = v_{0x}$ at $t = 0$. (*a*) Find the instantaneous velocity as a function of time. (*b*) Find the position as a function of time. (*c*) Find the average velocity for the time interval with an initial time of zero and arbitrary final time t. (*d*) Compare the average of the initial and final velocities to your answer to Part (*c*). Are these two averages equal? Explain.

GENERAL PROBLEMS

113 ••• **Context-Rich** You are a student in a science class that is using the following apparatus to determine the value of g. Two photogates are used. (Note: You may be familiar with photogates in everyday living. You see them in the doorways of some stores. They are designed to ring a bell when someone interrupts the beam while walking through the door.) One photogate is located at the edge of a table that is 1.00 m above the floor, and the second photogate is located directly below the first, at a height 0.500 m above the floor. You are told to drop a marble through these gates, releasing it from rest a negligible distance above the upper gate. The upper gate starts a timer as the ball passes through its beam. The second photogate stops the timer when the ball passes through its beam. (*a*) Prove that the experimental magnitude of free-fall acceleration is given by $g_{exp} = (2\Delta y)/(\Delta t)^2$, where Δy is the vertical distance between the photogates and Δt is the fall time. (*b*) For your setup, what value of Δt would you expect to measure, assuming g_{exp} is the standard value ($9.81\ \text{m/s}^2$)? (*c*) During the experiment, a slight error is made. Instead of locating the first photogate even with the top of the table, your not-so-careful lab partner locates it 0.50 cm lower than the top of the table. However, she does manage to properly locate the second photogate at a height of 0.50 m above the floor. However, she releases the marble from the same height that it was released from when the photogate was 1.00 m above the floor. What value of g_{exp} will you and your partner determine? What percentage difference does this represent from the standard value of g?

114 ••• **Multistep** The position of a body oscillating on a spring is given by $x = A \sin \omega t$, where A and ω (lower case Greek omega) are constants, $A = 5.0$ cm, and $\omega = 0.175\ \text{s}^{-1}$. (*a*) Plot x as a function of t for $0 \le t \le 36$ s. (*b*) Measure the slope of your graph at $t = 0$ to find the velocity at this time. (*c*) Calculate the average velocity for a series of intervals, beginning at $t = 0$ and ending at $t = 6.0$, 3.0, 2.0, 1.0, 0.50, and 0.25 s. (*d*) Compute dx/dt to find the velocity at time $t = 0$. (*e*) Compare your results in Parts (*c*) and (*d*) and explain why your Part (*c*) results approach your Part (*d*) result.

115 ••• **Conceptual** Consider an object that is attached to a horizontally oscillating piston. The object moves with a velocity given by $v = B \sin(\omega t)$, where B and ω (lower case Greek omega) are constants and ω is in s^{-1}. (*a*) Explain why B is equal to the maximum speed v_{max}. (*b*) Determine the acceleration of the object as a function of time. Is the acceleration constant? (*c*) What is the maximum acceleration (magnitude) in terms of ω and v_{max}. (*d*) At $t = 0$, the object's position is known to be x_0. Determine the position as a function of time in terms of t, ω, x_0 and v_{max}. SSM

116 ••• Suppose the acceleration of a particle is a function of x, where $a_x(x) = (2.0\ \text{s}^{-2})x$. (*a*) If the velocity is zero when $x = 1.0$ m, what is the speed when $x = 3.0$ m? (*b*) How long does it take the particle to travel from $x = 1.0$ m to $x = 3.0$ m.

117 ••• A rock falls through water with a continuously decreasing acceleration. Assume that the rock's acceleration as a function of *velocity* has the form $a_y = g - bv_y$ where b is a positive constant. (The $+y$ direction is directly downward.) (*a*) What are the SI units of b? (*b*) Prove mathematically that if the rock is released from rest at time $t = 0$, the acceleration will depend exponentially on *time* according to $a_y(t) = ge^{-bt}$. (c) What is the terminal speed for the rock in terms of g and b? (See Problem 38 for an explanation of the phenomenon of *terminal speed*.) SSM

118 ••• A small rock sinking through water (see Problem 117) experiences an exponentially decreasing acceleration given by $a_y(t) = ge^{-bt}$, where b is a positive constant that depends on the shape and size of the rock and the physical properties of the water. Based upon this, find expressions for the velocity and position of the rock as functions of time. Assume that its initial position and velocity are both zero and that the $+y$ direction is directly downward.

119 ••• **Spreadsheet** The acceleration of a skydiver jumping from an airplane is given by $a_y = g - bv_y^2$, where b is a positive constant that depends on the skydiver's cross-sectional area and the density of the surrounding atmosphere she is diving through. The $+y$ directions is directly downward. (*a*) If her initial speed is zero when stepping from a hovering helicopter, show that her speed as a function of time is given by $v_y(t) = v_t \tanh(t/T)$, where v_t is the terminal speed (see Problem 38) given by $v_t = \sqrt{g/b}$, and $T = v_t/g$ is a time-scale parameter. (*b*) What fraction of the terminal speed is the speed at $t = T$. (*c*) Use a **spreadsheet** program to graph $v_y(t)$ as a function of time, using a terminal speed of 56 m/s (use this value to calculate b and T). Does the resulting curve make sense?

120 ••• **APPROXIMATION** Imagine that you are standing at a wishing well, wishing that you knew how deep the surface of the water was. Cleverly, you make your wish. Then you take a penny from your pocket and drop it into the well. Exactly three seconds after you dropped the penny, you hear the sound it made when it struck the water. If the speed of sound is 343 m/s, how deep is the well? Neglect any effects due to air resistance.

121 ••• **CONTEXT-RICH** You are driving a car at the 25-mi/h speed limit when you observe the light at the intersection 65 m in front of you turn yellow. You know that at that particular intersection the light remains yellow for exactly 5.0 s before turning red. After you think for 1.0 s, you then accelerate the car at a constant rate. You somehow manage to pass your 4.5-m-long car completely through the 15.0-m-wide intersection just as the light turns red, thus narrowly avoiding a ticket for being in an intersection when the light is red. Immediately after passing through the intersection, you take your foot off the accelerator, relieved. However, down the road you are pulled over for speeding. You assume that you were ticketed for the speed of your car as it exited the intersection. Determine this speed and decide whether you should fight this ticket in court. Explain.

122 ••• For a spherical celestial object of radius R, the acceleration due to gravity g at a distance x from the *center* of the object is $g = g_0 R^2/x^2$, where g_0 is the acceleration due to gravity at the object's surface and $x > R$. For the moon, take $g_0 = 1.63\ \text{m/s}^2$ and $R = 3200$ km. If a rock is released from rest at a height of $4R$ above the lunar surface, with what speed does the rock impact the moon? *Hint: Its acceleration is a function of position and increases as the object falls. So do not use constant acceleration free-fall equations, but go back to basics.*