**Tables**

### Table I: Binomial Distribution Cumulative Probabilities

Let $X$ be a binomial random variable with parameters $n$ and $p$: $X \sim \mathrm{B}(n, p)$. This table contains cumulative probabilities: $\mathrm{P}(X \le x) = \sum_{k=0}^{x} \mathrm{P}(X = k) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1) + \mathrm{P}(X = 2) + \cdots + \mathrm{P}(X = x)$.

$n = 5$

| $x$ \ $p$ | 0.01 | 0.05 | 0.10 | 0.20 | 0.25 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.75 | 0.80 | 0.90 | 0.95 | 0.99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.9510 | 0.7738 | 0.5905 | 0.3277 | 0.2373 | 0.1681 | 0.0778 | 0.0313 | 0.0102 | 0.0024 | 0.0010 | 0.0003 | 0.0000 | | |
| 1 | 0.9990 | 0.9774 | 0.9185 | 0.7373 | 0.6328 | 0.5282 | 0.3370 | 0.1875 | 0.0870 | 0.0308 | 0.0156 | 0.0067 | 0.0005 | 0.0000 | |
| 2 | 1.0000 | 0.9988 | 0.9914 | 0.9421 | 0.8965 | 0.8369 | 0.6826 | 0.5000 | 0.3174 | 0.1631 | 0.1035 | 0.0579 | 0.0086 | 0.0012 | 0.0000 |
| 3 | | 1.0000 | 0.9995 | 0.9933 | 0.9844 | 0.9692 | 0.9130 | 0.8125 | 0.6630 | 0.4718 | 0.3672 | 0.2627 | 0.0815 | 0.0226 | 0.0010 |
| 4 | | | 1.0000 | 0.9997 | 0.9990 | 0.9976 | 0.9898 | 0.9688 | 0.9222 | 0.8319 | 0.7627 | 0.6723 | 0.4095 | 0.2262 | 0.0490 |

$n = 10$

| $x$ \ $p$ | 0.01 | 0.05 | 0.10 | 0.20 | 0.25 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.75 | 0.80 | 0.90 | 0.95 | 0.99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.9044 | 0.5987 | 0.3487 | 0.1074 | 0.0563 | 0.0282 | 0.0060 | 0.0010 | 0.0001 | 0.0000 | | | | | |
| 1 | 0.9957 | 0.9139 | 0.7361 | 0.3758 | 0.2440 | 0.1493 | 0.0464 | 0.0107 | 0.0017 | 0.0001 | 0.0000 | 0.0000 | | | |
| 2 | 0.9999 | 0.9885 | 0.9298 | 0.6778 | 0.5256 | 0.3828 | 0.1673 | 0.0547 | 0.0123 | 0.0016 | 0.0004 | 0.0001 | 0.0000 | | |
| 3 | 1.0000 | 0.9990 | 0.9872 | 0.8791 | 0.7759 | 0.6496 | 0.3823 | 0.1719 | 0.0548 | 0.0106 | 0.0035 | 0.0009 | 0.0000 | | |
| 4 | | 0.9999 | 0.9984 | 0.9672 | 0.9219 | 0.8497 | 0.6331 | 0.3770 | 0.1662 | 0.0473 | 0.0197 | 0.0064 | 0.0001 | 0.0000 | |
| 5 | | 1.0000 | 0.9999 | 0.9936 | 0.9803 | 0.9527 | 0.8338 | 0.6230 | 0.3669 | 0.1503 | 0.0781 | 0.0328 | 0.0016 | 0.0001 | |
| 6 | | | 1.0000 | 0.9991 | 0.9965 | 0.9894 | 0.9452 | 0.8281 | 0.6177 | 0.3504 | 0.2241 | 0.1209 | 0.0128 | 0.0010 | 0.0000 |
| 7 | | | | 0.9999 | 0.9996 | 0.9984 | 0.9877 | 0.9453 | 0.8327 | 0.6172 | 0.4744 | 0.3222 | 0.0702 | 0.0115 | 0.0001 |
| 8 | | | | 1.0000 | 1.0000 | 0.9999 | 0.9983 | 0.9893 | 0.9536 | 0.8507 | 0.7560 | 0.6242 | 0.2639 | 0.0861 | 0.0043 |
| 9 | | | | | | 1.0000 | 0.9999 | 0.9990 | 0.9940 | 0.9718 | 0.9437 | 0.8926 | 0.6513 | 0.4013 | 0.0956 |

$n = 15$

| $x$ \ $p$ | 0.01 | 0.05 | 0.10 | 0.20 | 0.25 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.75 | 0.80 | 0.90 | 0.95 | 0.99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.8601 | 0.4633 | 0.2059 | 0.0352 | 0.0134 | 0.0047 | 0.0005 | 0.0000 | | | | | | | |
| 1 | 0.9904 | 0.8290 | 0.5490 | 0.1671 | 0.0802 | 0.0353 | 0.0052 | 0.0005 | 0.0000 | | | | | | |
| 2 | 0.9996 | 0.9638 | 0.8159 | 0.3980 | 0.2361 | 0.1268 | 0.0271 | 0.0037 | 0.0003 | 0.0000 | | | | | |
| 3 | 1.0000 | 0.9945 | 0.9444 | 0.6482 | 0.4613 | 0.2969 | 0.0905 | 0.0176 | 0.0019 | 0.0001 | 0.0000 | | | | |
| 4 | | 0.9994 | 0.9873 | 0.8358 | 0.6865 | 0.5155 | 0.2173 | 0.0592 | 0.0093 | 0.0007 | 0.0001 | 0.0000 | | | |
| 5 | | 0.9999 | 0.9978 | 0.9389 | 0.8516 | 0.7216 | 0.4032 | 0.1509 | 0.0338 | 0.0037 | 0.0008 | 0.0001 | | | |
| 6 | | 1.0000 | 0.9997 | 0.9819 | 0.9434 | 0.8689 | 0.6098 | 0.3036 | 0.0950 | 0.0152 | 0.0042 | 0.0008 | | | |
| 7 | | | 1.0000 | 0.9958 | 0.9827 | 0.9500 | 0.7869 | 0.5000 | 0.2131 | 0.0500 | 0.0173 | 0.0042 | 0.0000 | | |
| 8 | | | | 0.9992 | 0.9958 | 0.9848 | 0.9050 | 0.6964 | 0.3902 | 0.1311 | 0.0566 | 0.0181 | 0.0003 | 0.0000 | |
| 9 | | | | 0.9999 | 0.9992 | 0.9963 | 0.9662 | 0.8491 | 0.5968 | 0.2784 | 0.1484 | 0.0611 | 0.0022 | 0.0001 | |
| 10 | | | | 1.0000 | 0.9999 | 0.9993 | 0.9907 | 0.9408 | 0.7827 | 0.4845 | 0.3135 | 0.1642 | 0.0127 | 0.0006 | |
| 11 | | | | | 1.0000 | 0.9999 | 0.9981 | 0.9824 | 0.9095 | 0.7031 | 0.5387 | 0.3518 | 0.0556 | 0.0055 | 0.0000 |
| 12 | | | | | | 1.0000 | 0.9997 | 0.9963 | 0.9729 | 0.8732 | 0.7639 | 0.6020 | 0.1841 | 0.0362 | 0.0004 |
| 13 | | | | | | | 1.0000 | 0.9995 | 0.9948 | 0.9647 | 0.9198 | 0.8329 | 0.4510 | 0.1710 | 0.0096 |
| 14 | | | | | | | | 1.0000 | 0.9995 | 0.9953 | 0.9866 | 0.9648 | 0.7941 | 0.5367 | 0.1399 |

**Table I: Binomial Distribution Cumulative Probabilities** (Continued)

$n = 20$

| $x$ \ $p$ | 0.01 | 0.05 | 0.10 | 0.20 | 0.25 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.75 | 0.80 | 0.90 | 0.95 | 0.99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.8179 | 0.3585 | 0.1216 | 0.0115 | 0.0032 | 0.0008 | 0.0000 | | | | | | | | |
| 1 | 0.9831 | 0.7358 | 0.3917 | 0.0692 | 0.0243 | 0.0076 | 0.0005 | 0.0000 | | | | | | | |
| 2 | 0.9990 | 0.9245 | 0.6769 | 0.2061 | 0.0913 | 0.0355 | 0.0036 | 0.0002 | | | | | | | |
| 3 | 1.0000 | 0.9841 | 0.8670 | 0.4114 | 0.2252 | 0.1071 | 0.0160 | 0.0013 | 0.0000 | | | | | | |
| 4 | | 0.9974 | 0.9568 | 0.6296 | 0.4148 | 0.2375 | 0.0510 | 0.0059 | 0.0003 | | | | | | |
| 5 | | 0.9997 | 0.9887 | 0.8042 | 0.6172 | 0.4164 | 0.1256 | 0.0207 | 0.0016 | 0.0000 | | | | | |
| 6 | | 1.0000 | 0.9976 | 0.9133 | 0.7858 | 0.6080 | 0.2500 | 0.0577 | 0.0065 | 0.0003 | 0.0000 | | | | |
| 7 | | | 0.9996 | 0.9679 | 0.8982 | 0.7723 | 0.4159 | 0.1316 | 0.0210 | 0.0013 | 0.0002 | 0.0000 | | | |
| 8 | | | 0.9999 | 0.9900 | 0.9591 | 0.8867 | 0.5956 | 0.2517 | 0.0565 | 0.0051 | 0.0009 | 0.0001 | | | |
| 9 | | | 1.0000 | 0.9974 | 0.9861 | 0.9520 | 0.7553 | 0.4119 | 0.1275 | 0.0171 | 0.0039 | 0.0006 | | | |
| 10 | | | | 0.9994 | 0.9961 | 0.9829 | 0.8725 | 0.5881 | 0.2447 | 0.0480 | 0.0139 | 0.0026 | 0.0000 | | |
| 11 | | | | 0.9999 | 0.9991 | 0.9949 | 0.9435 | 0.7483 | 0.4044 | 0.1133 | 0.0409 | 0.0100 | 0.0001 | | |
| 12 | | | | 1.0000 | 0.9998 | 0.9987 | 0.9790 | 0.8684 | 0.5841 | 0.2277 | 0.1018 | 0.0321 | 0.0004 | | |
| 13 | | | | | 1.0000 | 0.9997 | 0.9935 | 0.9423 | 0.7500 | 0.3920 | 0.2142 | 0.0867 | 0.0024 | 0.0000 | |
| 14 | | | | | | 1.0000 | 0.9984 | 0.9793 | 0.8744 | 0.5836 | 0.3828 | 0.1958 | 0.0113 | 0.0003 | |
| 15 | | | | | | | 0.9997 | 0.9941 | 0.9490 | 0.7625 | 0.5852 | 0.3704 | 0.0432 | 0.0026 | |
| 16 | | | | | | | 1.0000 | 0.9987 | 0.9840 | 0.8929 | 0.7748 | 0.5886 | 0.1330 | 0.0159 | 0.0000 |
| 17 | | | | | | | | 0.9998 | 0.9964 | 0.9645 | 0.9087 | 0.7939 | 0.3231 | 0.0755 | 0.0010 |
| 18 | | | | | | | | 1.0000 | 0.9995 | 0.9924 | 0.9757 | 0.9308 | 0.6083 | 0.2642 | 0.0169 |
| 19 | | | | | | | | | 1.0000 | 0.9992 | 0.9968 | 0.9885 | 0.8784 | 0.6415 | 0.1821 |

$n = 25$

| $x$ \ $p$ | 0.01 | 0.05 | 0.10 | 0.20 | 0.25 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.75 | 0.80 | 0.90 | 0.95 | 0.99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0.7778 | 0.2774 | 0.0718 | 0.0038 | 0.0008 | 0.0001 | 0.0000 | | | | | | | | |
| 1 | 0.9742 | 0.6424 | 0.2712 | 0.0274 | 0.0070 | 0.0016 | 0.0001 | | | | | | | | |
| 2 | 0.9980 | 0.8729 | 0.5371 | 0.0982 | 0.0321 | 0.0090 | 0.0004 | 0.0000 | | | | | | | |
| 3 | 0.9999 | 0.9659 | 0.7636 | 0.2340 | 0.0962 | 0.0332 | 0.0024 | 0.0001 | | | | | | | |
| 4 | 1.0000 | 0.9928 | 0.9020 | 0.4207 | 0.2137 | 0.0905 | 0.0095 | 0.0005 | 0.0000 | | | | | | |
| 5 | | 0.9988 | 0.9666 | 0.6167 | 0.3783 | 0.1935 | 0.0294 | 0.0020 | 0.0001 | | | | | | |
| 6 | | 0.9998 | 0.9905 | 0.7800 | 0.5611 | 0.3407 | 0.0736 | 0.0073 | 0.0003 | | | | | | |
| 7 | | 1.0000 | 0.9977 | 0.8909 | 0.7265 | 0.5118 | 0.1536 | 0.0216 | 0.0012 | 0.0000 | | | | | |
| 8 | | | 0.9995 | 0.9532 | 0.8506 | 0.6769 | 0.2735 | 0.0539 | 0.0043 | 0.0001 | | | | | |
| 9 | | | 0.9999 | 0.9827 | 0.9287 | 0.8106 | 0.4246 | 0.1148 | 0.0132 | 0.0005 | 0.0000 | | | | |
| 10 | | | 1.0000 | 0.9944 | 0.9703 | 0.9022 | 0.5858 | 0.2122 | 0.0344 | 0.0018 | 0.0002 | 0.0000 | | | |
| 11 | | | | 0.9985 | 0.9893 | 0.9558 | 0.7323 | 0.3450 | 0.0778 | 0.0060 | 0.0009 | 0.0001 | | | |
| 12 | | | | 0.9996 | 0.9966 | 0.9825 | 0.8462 | 0.5000 | 0.1538 | 0.0175 | 0.0034 | 0.0004 | | | |
| 13 | | | | 0.9999 | 0.9991 | 0.9940 | 0.9222 | 0.6550 | 0.2677 | 0.0442 | 0.0107 | 0.0015 | | | |
| 14 | | | | 1.0000 | 0.9998 | 0.9982 | 0.9656 | 0.7878 | 0.4142 | 0.0978 | 0.0297 | 0.0056 | 0.0000 | | |
| 15 | | | | | 1.0000 | 0.9995 | 0.9868 | 0.8852 | 0.5754 | 0.1894 | 0.0713 | 0.0173 | 0.0001 | | |
| 16 | | | | | | 0.9999 | 0.9957 | 0.9461 | 0.7265 | 0.3231 | 0.1494 | 0.0468 | 0.0005 | | |
| 17 | | | | | | 1.0000 | 0.9988 | 0.9784 | 0.8464 | 0.4882 | 0.2735 | 0.1091 | 0.0023 | 0.0000 | |
| 18 | | | | | | | 0.9997 | 0.9927 | 0.9264 | 0.6593 | 0.4389 | 0.2200 | 0.0095 | 0.0002 | |
| 19 | | | | | | | 0.9999 | 0.9980 | 0.9706 | 0.8065 | 0.6217 | 0.3833 | 0.0334 | 0.0012 | |
| 20 | | | | | | | 1.0000 | 0.9995 | 0.9905 | 0.9095 | 0.7863 | 0.5793 | 0.0980 | 0.0072 | 0.0000 |
| 21 | | | | | | | | 0.9999 | 0.9976 | 0.9668 | 0.9038 | 0.7660 | 0.2364 | 0.0341 | 0.0001 |
| 22 | | | | | | | | 1.0000 | 0.9996 | 0.9910 | 0.9679 | 0.9018 | 0.4629 | 0.1271 | 0.0020 |
| 23 | | | | | | | | | 0.9999 | 0.9984 | 0.9930 | 0.9726 | 0.7288 | 0.3576 | 0.0258 |
| 24 | | | | | | | | | 1.0000 | 0.9999 | 0.9992 | 0.9962 | 0.9282 | 0.7226 | 0.2222 |

**Table II**: **Poisson Distribution Cumulative Probabilities**

Let $X$ be a Poisson random variable with parameter $\lambda$. This table contains cumulative probabilities: $\mathrm{P}(X \leq x) = \sum_{k=0}^{x} \mathrm{P}(X = k) = \mathrm{P}(X = 0) + \mathrm{P}(X = 1) + \mathrm{P}(X = 2) + \cdots + \mathrm{P}(X = x)$.

| | | | | | $\lambda$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 |
| 0 | 0.9512 | 0.9048 | 0.8607 | 0.8187 | 0.7788 | 0.7408 | 0.7047 | 0.6703 | 0.6376 | 0.6065 |
| 1 | 0.9988 | 0.9953 | 0.9898 | 0.9825 | 0.9735 | 0.9631 | 0.9513 | 0.9384 | 0.9246 | 0.9098 |
| 2 | 1.0000 | 0.9998 | 0.9995 | 0.9989 | 0.9978 | 0.9964 | 0.9945 | 0.9921 | 0.9891 | 0.9856 |
| 3 | | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9995 | 0.9992 | 0.9988 | 0.9982 |
| 4 | | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 |
| 5 | | | | | | | | 1.0000 | 1.0000 | 1.0000 |

| | | | | | $\lambda$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 1.00 |
| 0 | 0.5769 | 0.5488 | 0.5220 | 0.4966 | 0.4724 | 0.4493 | 0.4274 | 0.4066 | 0.3867 | 0.3679 |
| 1 | 0.8943 | 0.8781 | 0.8614 | 0.8442 | 0.8266 | 0.8088 | 0.7907 | 0.7725 | 0.7541 | 0.7358 |
| 2 | 0.9815 | 0.9769 | 0.9717 | 0.9659 | 0.9595 | 0.9526 | 0.9451 | 0.9371 | 0.9287 | 0.9197 |
| 3 | 0.9975 | 0.9966 | 0.9956 | 0.9942 | 0.9927 | 0.9909 | 0.9889 | 0.9865 | 0.9839 | 0.9810 |
| 4 | 0.9997 | 0.9996 | 0.9994 | 0.9992 | 0.9989 | 0.9986 | 0.9982 | 0.9977 | 0.9971 | 0.9963 |
| 5 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9997 | 0.9995 | 0.9994 |
| 6 | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |
| 7 | | | | | | | | | 1.0000 | 1.0000 |

| | | | | | $\lambda$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 |
| 0 | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 | 0.2019 | 0.1827 | 0.1653 | 0.1496 | 0.1353 |
| 1 | 0.6990 | 0.6626 | 0.6268 | 0.5918 | 0.5578 | 0.5249 | 0.4932 | 0.4628 | 0.4337 | 0.4060 |
| 2 | 0.9004 | 0.8795 | 0.8571 | 0.8335 | 0.8088 | 0.7834 | 0.7572 | 0.7306 | 0.7037 | 0.6767 |
| 3 | 0.9743 | 0.9662 | 0.9569 | 0.9463 | 0.9344 | 0.9212 | 0.9068 | 0.8913 | 0.8747 | 0.8571 |
| 4 | 0.9946 | 0.9923 | 0.9893 | 0.9857 | 0.9814 | 0.9763 | 0.9704 | 0.9636 | 0.9559 | 0.9473 |
| 5 | 0.9990 | 0.9985 | 0.9978 | 0.9968 | 0.9955 | 0.9940 | 0.9920 | 0.9896 | 0.9868 | 0.9834 |
| 6 | 0.9999 | 0.9997 | 0.9996 | 0.9994 | 0.9991 | 0.9987 | 0.9981 | 0.9974 | 0.9966 | 0.9955 |
| 7 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9994 | 0.9992 | 0.9989 |
| 8 | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 9 | | | | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

**Table II**: **Poisson Distribution Cumulative Probabilities** (Continued)

| | | | | | $\lambda$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 | 3.0 |
| 0 | 0.1225 | 0.1108 | 0.1003 | 0.0907 | 0.0821 | 0.0743 | 0.0672 | 0.0608 | 0.0550 | 0.0498 |
| 1 | 0.3796 | 0.3546 | 0.3309 | 0.3084 | 0.2873 | 0.2674 | 0.2487 | 0.2311 | 0.2146 | 0.1991 |
| 2 | 0.6496 | 0.6227 | 0.5960 | 0.5697 | 0.5438 | 0.5184 | 0.4936 | 0.4695 | 0.4460 | 0.4232 |
| 3 | 0.8386 | 0.8194 | 0.7993 | 0.7787 | 0.7576 | 0.7360 | 0.7141 | 0.6919 | 0.6696 | 0.6472 |
| 4 | 0.9379 | 0.9275 | 0.9162 | 0.9041 | 0.8912 | 0.8774 | 0.8629 | 0.8477 | 0.8318 | 0.8153 |
| 5 | 0.9796 | 0.9751 | 0.9700 | 0.9643 | 0.9580 | 0.9510 | 0.9433 | 0.9349 | 0.9258 | 0.9161 |
| 6 | 0.9941 | 0.9925 | 0.9906 | 0.9884 | 0.9858 | 0.9828 | 0.9794 | 0.9756 | 0.9713 | 0.9665 |
| 7 | 0.9985 | 0.9980 | 0.9974 | 0.9967 | 0.9958 | 0.9947 | 0.9934 | 0.9919 | 0.9901 | 0.9881 |
| 8 | 0.9997 | 0.9995 | 0.9994 | 0.9991 | 0.9989 | 0.9985 | 0.9981 | 0.9976 | 0.9969 | 0.9962 |
| 9 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9991 | 0.9989 |
| 10 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 |
| 11 | | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |
| 12 | | | | | | | | | 1.0000 | 1.0000 |

| | | | | | $\lambda$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 3.6 | 3.7 | 3.8 | 3.9 | 4.0 |
| 0 | 0.0450 | 0.0408 | 0.0369 | 0.0334 | 0.0302 | 0.0273 | 0.0247 | 0.0224 | 0.0202 | 0.0183 |
| 1 | 0.1847 | 0.1712 | 0.1586 | 0.1468 | 0.1359 | 0.1257 | 0.1162 | 0.1074 | 0.0992 | 0.0916 |
| 2 | 0.4012 | 0.3799 | 0.3594 | 0.3397 | 0.3208 | 0.3027 | 0.2854 | 0.2689 | 0.2531 | 0.2381 |
| 3 | 0.6248 | 0.6025 | 0.5803 | 0.5584 | 0.5366 | 0.5152 | 0.4942 | 0.4735 | 0.4532 | 0.4335 |
| 4 | 0.7982 | 0.7806 | 0.7626 | 0.7442 | 0.7254 | 0.7064 | 0.6872 | 0.6678 | 0.6484 | 0.6288 |
| 5 | 0.9057 | 0.8946 | 0.8829 | 0.8705 | 0.8576 | 0.8441 | 0.8301 | 0.8156 | 0.8006 | 0.7851 |
| 6 | 0.9612 | 0.9554 | 0.9490 | 0.9421 | 0.9347 | 0.9267 | 0.9182 | 0.9091 | 0.8995 | 0.8893 |
| 7 | 0.9858 | 0.9832 | 0.9802 | 0.9769 | 0.9733 | 0.9692 | 0.9648 | 0.9599 | 0.9546 | 0.9489 |
| 8 | 0.9953 | 0.9943 | 0.9931 | 0.9917 | 0.9901 | 0.9883 | 0.9863 | 0.9840 | 0.9815 | 0.9786 |
| 9 | 0.9986 | 0.9982 | 0.9978 | 0.9973 | 0.9967 | 0.9960 | 0.9952 | 0.9942 | 0.9931 | 0.9919 |
| 10 | 0.9996 | 0.9995 | 0.9994 | 0.9992 | 0.9990 | 0.9987 | 0.9984 | 0.9981 | 0.9977 | 0.9972 |
| 11 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9994 | 0.9993 | 0.9991 |
| 12 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 |
| 13 | | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |
| 14 | | | | | | | | | 1.0000 | 1.0000 |

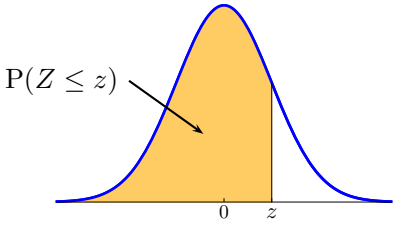
**Table II**: **Poisson Distribution Cumulative Probabilities** (Continued)

| | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 4.1 | 4.2 | 4.3 | 4.4 | 4.5 | 4.6 | 4.7 | 4.8 | 4.9 | 5.0 |
| 0 | 0.0166 | 0.0150 | 0.0136 | 0.0123 | 0.0111 | 0.0101 | 0.0091 | 0.0082 | 0.0074 | 0.0067 |
| 1 | 0.0845 | 0.0780 | 0.0719 | 0.0663 | 0.0611 | 0.0563 | 0.0518 | 0.0477 | 0.0439 | 0.0404 |
| 2 | 0.2238 | 0.2102 | 0.1974 | 0.1851 | 0.1736 | 0.1626 | 0.1523 | 0.1425 | 0.1333 | 0.1247 |
| 3 | 0.4142 | 0.3954 | 0.3772 | 0.3594 | 0.3423 | 0.3257 | 0.3097 | 0.2942 | 0.2793 | 0.2650 |
| 4 | 0.6093 | 0.5898 | 0.5704 | 0.5512 | 0.5321 | 0.5132 | 0.4946 | 0.4763 | 0.4582 | 0.4405 |
| 5 | 0.7693 | 0.7531 | 0.7367 | 0.7199 | 0.7029 | 0.6858 | 0.6684 | 0.6510 | 0.6335 | 0.6160 |
| 6 | 0.8786 | 0.8675 | 0.8558 | 0.8436 | 0.8311 | 0.8180 | 0.8046 | 0.7908 | 0.7767 | 0.7622 |
| 7 | 0.9427 | 0.9361 | 0.9290 | 0.9214 | 0.9134 | 0.9049 | 0.8960 | 0.8867 | 0.8769 | 0.8666 |
| 8 | 0.9755 | 0.9721 | 0.9683 | 0.9642 | 0.9597 | 0.9549 | 0.9497 | 0.9442 | 0.9382 | 0.9319 |
| 9 | 0.9905 | 0.9889 | 0.9871 | 0.9851 | 0.9829 | 0.9805 | 0.9778 | 0.9749 | 0.9717 | 0.9682 |
| 10 | 0.9966 | 0.9959 | 0.9952 | 0.9943 | 0.9933 | 0.9922 | 0.9910 | 0.9896 | 0.9880 | 0.9863 |
| 11 | 0.9989 | 0.9986 | 0.9983 | 0.9980 | 0.9976 | 0.9971 | 0.9966 | 0.9960 | 0.9953 | 0.9945 |
| 12 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9992 | 0.9990 | 0.9988 | 0.9986 | 0.9983 | 0.9980 |
| 14 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 | 0.9997 | 0.9996 | 0.9995 | 0.9994 | 0.9993 |
| 15 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 16 | | | | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |
| 17 | | | | | | | | | 1.0000 | 1.0000 |

| | $\lambda$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | 5.5 | 6.0 | 6.5 | 7.0 | 7.5 | 8.0 | 8.5 | 9.0 | 9.5 | 10.0 |
| 0 | 0.0041 | 0.0025 | 0.0015 | 0.0009 | 0.0006 | 0.0003 | 0.0002 | 0.0001 | 0.0001 | 0.0000 |
| 1 | 0.0266 | 0.0174 | 0.0113 | 0.0073 | 0.0047 | 0.0030 | 0.0019 | 0.0012 | 0.0008 | 0.0005 |
| 2 | 0.0884 | 0.0620 | 0.0430 | 0.0296 | 0.0203 | 0.0138 | 0.0093 | 0.0062 | 0.0042 | 0.0028 |
| 3 | 0.2017 | 0.1512 | 0.1118 | 0.0818 | 0.0591 | 0.0424 | 0.0301 | 0.0212 | 0.0149 | 0.0103 |
| 4 | 0.3575 | 0.2851 | 0.2237 | 0.1730 | 0.1321 | 0.0996 | 0.0744 | 0.0550 | 0.0403 | 0.0293 |
| 5 | 0.5289 | 0.4457 | 0.3690 | 0.3007 | 0.2414 | 0.1912 | 0.1496 | 0.1157 | 0.0885 | 0.0671 |
| 6 | 0.6860 | 0.6063 | 0.5265 | 0.4497 | 0.3782 | 0.3134 | 0.2562 | 0.2068 | 0.1649 | 0.1301 |
| 7 | 0.8095 | 0.7440 | 0.6728 | 0.5987 | 0.5246 | 0.4530 | 0.3856 | 0.3239 | 0.2687 | 0.2202 |
| 8 | 0.8944 | 0.8472 | 0.7916 | 0.7291 | 0.6620 | 0.5925 | 0.5231 | 0.4557 | 0.3918 | 0.3328 |
| 9 | 0.9462 | 0.9161 | 0.8774 | 0.8305 | 0.7764 | 0.7166 | 0.6530 | 0.5874 | 0.5218 | 0.4579 |
| 10 | 0.9747 | 0.9574 | 0.9332 | 0.9015 | 0.8622 | 0.8159 | 0.7634 | 0.7060 | 0.6453 | 0.5830 |
| 11 | 0.9890 | 0.9799 | 0.9661 | 0.9467 | 0.9208 | 0.8881 | 0.8487 | 0.8030 | 0.7520 | 0.6968 |
| 12 | 0.9955 | 0.9912 | 0.9840 | 0.9730 | 0.9573 | 0.9362 | 0.9091 | 0.8758 | 0.8364 | 0.7916 |
| 13 | 0.9983 | 0.9964 | 0.9929 | 0.9872 | 0.9784 | 0.9658 | 0.9486 | 0.9261 | 0.8981 | 0.8645 |
| 14 | 0.9994 | 0.9986 | 0.9970 | 0.9943 | 0.9897 | 0.9827 | 0.9726 | 0.9585 | 0.9400 | 0.9165 |
| 15 | 0.9998 | 0.9995 | 0.9988 | 0.9976 | 0.9954 | 0.9918 | 0.9862 | 0.9780 | 0.9665 | 0.9513 |
| 16 | 0.9999 | 0.9998 | 0.9996 | 0.9990 | 0.9980 | 0.9963 | 0.9934 | 0.9889 | 0.9823 | 0.9730 |
| 17 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9984 | 0.9970 | 0.9947 | 0.9911 | 0.9857 |
| 18 | | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9993 | 0.9987 | 0.9976 | 0.9957 | 0.9928 |
| 19 | | | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9995 | 0.9989 | 0.9980 | 0.9965 |
| 20 | | | | | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9991 | 0.9984 |
| 21 | | | | | | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9993 |
| 22 | | | | | | | 1.0000 | 0.9999 | 0.9999 | 0.9997 |
| 23 | | | | | | | | 1.0000 | 0.9999 | 0.9999 |
| 24 | | | | | | | | | 1.0000 | 1.0000 |

**Table III: Standard Normal Distribution Cumulative Probabilities**

Let $Z$ be a standard normal random variable: $\mu = 0$ and $\sigma = 1$.
This table contains cumulative probabilities: $\mathrm{P}(Z \leq z)$.




| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| −3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| −3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| −3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| −3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| −2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| −2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| −2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| −2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| −2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| −2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| −2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| −1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| −1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| −1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| −1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| −1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| −0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| −0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| −0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| −0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| −0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| −0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| −0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| −0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |

**Table III**: **Standard Normal Distribution Cumulative Probabilities** (Continued)

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

Special critical values: $\mathrm{P}(Z \geq z_\alpha) = \alpha$

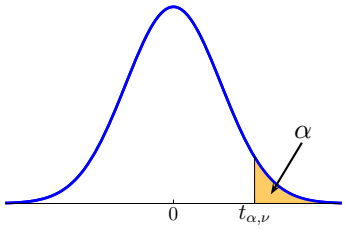
| $\alpha$ | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 | 0.0001 | |
|---|---|---|---|---|---|---|---|---|---|
| $z_\alpha$ | 1.2816 | 1.6449 | 1.9600 | 2.3263 | 2.5758 | 3.0902 | 3.2905 | 3.7190 | |
| $\alpha$ | 0.00009 | 0.00008 | 0.00007 | 0.00006 | 0.00005 | 0.00004 | 0.00003 | 0.00002 | 0.00001 |
| $z_\alpha$ | 3.7455 | 3.7750 | 3.8082 | 3.8461 | 3.8906 | 3.9444 | 4.0128 | 4.1075 | 4.2649 |

**Table IV: Standardized Normal Scores**

This table contains the standardized normal scores, $z_i$, for selected values of $n$.

| | $n$ | | | | | |
|---|---|---|---|---|---|---|
| $i$ | 10 | 20 | 25 | 30 | 40 | 50 |
| 1 | −1.55 | −1.87 | −1.96 | −2.04 | −2.16 | −2.24 |
| 2 | −1.00 | −1.40 | −1.52 | −1.61 | −1.75 | −1.85 |
| 3 | −0.66 | −1.13 | −1.26 | −1.36 | −1.51 | −1.62 |
| 4 | −0.38 | −0.92 | −1.06 | −1.18 | −1.34 | −1.46 |
| 5 | −0.12 | −0.74 | −0.90 | −1.02 | −1.20 | −1.33 |
| 6 | 0.12 | −0.59 | −0.76 | −0.89 | −1.08 | −1.22 |
| 7 | 0.38 | −0.45 | −0.64 | −0.78 | −0.98 | −1.12 |
| 8 | 0.66 | −0.31 | −0.52 | −0.67 | −0.88 | −1.03 |
| 9 | 1.00 | −0.19 | −0.41 | −0.57 | −0.79 | −0.95 |
| 10 | 1.55 | −0.06 | −0.30 | −0.47 | −0.71 | −0.87 |
| 11 | | 0.06 | −0.20 | −0.38 | −0.63 | −0.80 |
| 12 | | 0.19 | −0.10 | −0.29 | −0.56 | −0.73 |
| 13 | | 0.31 | 0.00 | −0.21 | −0.49 | −0.67 |
| 14 | | 0.45 | 0.10 | −0.12 | −0.42 | −0.61 |
| 15 | | 0.59 | 0.20 | −0.04 | −0.35 | −0.55 |
| 16 | | 0.74 | 0.30 | 0.04 | −0.28 | −0.49 |
| 17 | | 0.92 | 0.41 | 0.12 | −0.22 | −0.44 |
| 18 | | 1.13 | 0.52 | 0.21 | −0.16 | −0.38 |
| 19 | | 1.40 | 0.64 | 0.29 | −0.09 | −0.33 |
| 20 | | 1.87 | 0.76 | 0.38 | −0.03 | −0.28 |
| 21 | | | 0.90 | 0.47 | 0.03 | −0.23 |
| 22 | | | 1.06 | 0.57 | 0.09 | −0.18 |
| 23 | | | 1.26 | 0.67 | 0.16 | −0.13 |
| 24 | | | 1.52 | 0.78 | 0.22 | −0.07 |
| 25 | | | 1.96 | 0.89 | 0.28 | −0.02 |
| 26 | | | | 1.02 | 0.35 | 0.02 |
| 27 | | | | 1.18 | 0.42 | 0.07 |
| 28 | | | | 1.36 | 0.49 | 0.13 |
| 29 | | | | 1.61 | 0.56 | 0.18 |
| 30 | | | | 2.04 | 0.63 | 0.23 |
| 31 | | | | | 0.71 | 0.28 |
| 32 | | | | | 0.79 | 0.33 |
| 33 | | | | | 0.88 | 0.38 |
| 34 | | | | | 0.98 | 0.44 |
| 35 | | | | | 1.08 | 0.49 |
| 36 | | | | | 1.20 | 0.55 |
| 37 | | | | | 1.34 | 0.61 |
| 38 | | | | | 1.51 | 0.67 |
| 39 | | | | | 1.75 | 0.73 |
| 40 | | | | | 2.16 | 0.80 |
| 41 | | | | | | 0.87 |
| 42 | | | | | | 0.95 |
| 43 | | | | | | 1.03 |
| 44 | | | | | | 1.12 |
| 45 | | | | | | 1.22 |
| 46 | | | | | | 1.33 |
| 47 | | | | | | 1.46 |
| 48 | | | | | | 1.62 |
| 49 | | | | | | 1.85 |
| 50 | | | | | | 2.24 |

## Table V: Critical Values for the $t$ Distribution




This table contains critical values associated with the $t$ distribution, $t_{\alpha,\nu}$, defined by $\alpha$ and the degrees of freedom, $\nu$.

| | $\alpha$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | 0.20 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 | 0.0001 |
| 1 | 1.3764 | 3.0777 | 6.3138 | 12.7062 | 31.8205 | 63.6567 | 318.3088 | 636.6192 | 3183.0988 |
| 2 | 1.0607 | 1.8856 | 2.9200 | 4.3027 | 6.9646 | 9.9248 | 22.3271 | 31.5991 | 70.7001 |
| 3 | 0.9785 | 1.6377 | 2.3534 | 3.1824 | 4.5407 | 5.8409 | 10.2145 | 12.9240 | 22.2037 |
| 4 | 0.9410 | 1.5332 | 2.1318 | 2.7764 | 3.7469 | 4.6041 | 7.1732 | 8.6103 | 13.0337 |
| 5 | 0.9195 | 1.4759 | 2.0150 | 2.5706 | 3.3649 | 4.0321 | 5.8934 | 6.8688 | 9.6776 |
| 6 | 0.9057 | 1.4398 | 1.9432 | 2.4469 | 3.1427 | 3.7074 | 5.2076 | 5.9588 | 8.0248 |
| 7 | 0.8960 | 1.4149 | 1.8946 | 2.3646 | 2.9980 | 3.4995 | 4.7853 | 5.4079 | 7.0634 |
| 8 | 0.8889 | 1.3968 | 1.8595 | 2.3060 | 2.8965 | 3.3554 | 4.5008 | 5.0413 | 6.4420 |
| 9 | 0.8834 | 1.3830 | 1.8331 | 2.2622 | 2.8214 | 3.2498 | 4.2968 | 4.7809 | 6.0101 |
| 10 | 0.8791 | 1.3722 | 1.8125 | 2.2281 | 2.7638 | 3.1693 | 4.1437 | 4.5869 | 5.6938 |
| 11 | 0.8755 | 1.3634 | 1.7959 | 2.2010 | 2.7181 | 3.1058 | 4.0247 | 4.4370 | 5.4528 |
| 12 | 0.8726 | 1.3562 | 1.7823 | 2.1788 | 2.6810 | 3.0545 | 3.9296 | 4.3178 | 5.2633 |
| 13 | 0.8702 | 1.3502 | 1.7709 | 2.1604 | 2.6503 | 3.0123 | 3.8520 | 4.2208 | 5.1106 |
| 14 | 0.8681 | 1.3450 | 1.7613 | 2.1448 | 2.6245 | 2.9768 | 3.7874 | 4.1405 | 4.9850 |
| 15 | 0.8662 | 1.3406 | 1.7531 | 2.1314 | 2.6025 | 2.9467 | 3.7328 | 4.0728 | 4.8800 |
| 16 | 0.8647 | 1.3368 | 1.7459 | 2.1199 | 2.5835 | 2.9208 | 3.6862 | 4.0150 | 4.7909 |
| 17 | 0.8633 | 1.3334 | 1.7396 | 2.1098 | 2.5669 | 2.8982 | 3.6458 | 3.9651 | 4.7144 |
| 18 | 0.8620 | 1.3304 | 1.7341 | 2.1009 | 2.5524 | 2.8784 | 3.6105 | 3.9216 | 4.6480 |
| 19 | 0.8610 | 1.3277 | 1.7291 | 2.0930 | 2.5395 | 2.8609 | 3.5794 | 3.8834 | 4.5899 |
| 20 | 0.8600 | 1.3253 | 1.7247 | 2.0860 | 2.5280 | 2.8453 | 3.5518 | 3.8495 | 4.5385 |
| 21 | 0.8591 | 1.3232 | 1.7207 | 2.0796 | 2.5176 | 2.8314 | 3.5272 | 3.8193 | 4.4929 |
| 22 | 0.8583 | 1.3212 | 1.7171 | 2.0739 | 2.5083 | 2.8188 | 3.5050 | 3.7921 | 4.4520 |
| 23 | 0.8575 | 1.3195 | 1.7139 | 2.0687 | 2.4999 | 2.8073 | 3.4850 | 3.7676 | 4.4152 |
| 24 | 0.8569 | 1.3178 | 1.7109 | 2.0639 | 2.4922 | 2.7969 | 3.4668 | 3.7454 | 4.3819 |
| 25 | 0.8562 | 1.3163 | 1.7081 | 2.0595 | 2.4851 | 2.7874 | 3.4502 | 3.7251 | 4.3517 |
| 26 | 0.8557 | 1.3150 | 1.7056 | 2.0555 | 2.4786 | 2.7787 | 3.4350 | 3.7066 | 4.3240 |
| 27 | 0.8551 | 1.3137 | 1.7033 | 2.0518 | 2.4727 | 2.7707 | 3.4210 | 3.6896 | 4.2987 |
| 28 | 0.8546 | 1.3125 | 1.7011 | 2.0484 | 2.4671 | 2.7633 | 3.4082 | 3.6739 | 4.2754 |
| 29 | 0.8542 | 1.3114 | 1.6991 | 2.0452 | 2.4620 | 2.7564 | 3.3962 | 3.6594 | 4.2539 |
| 30 | 0.8538 | 1.3104 | 1.6973 | 2.0423 | 2.4573 | 2.7500 | 3.3852 | 3.6460 | 4.2340 |
| 40 | 0.8507 | 1.3031 | 1.6839 | 2.0211 | 2.4233 | 2.7045 | 3.3069 | 3.5510 | 4.0942 |
| 50 | 0.8489 | 1.2987 | 1.6759 | 2.0086 | 2.4033 | 2.6778 | 3.2614 | 3.4960 | 4.0140 |
| 60 | 0.8477 | 1.2958 | 1.6706 | 2.0003 | 2.3901 | 2.6603 | 3.2317 | 3.4602 | 3.9621 |
| 70 | 0.8468 | 1.2938 | 1.6669 | 1.9944 | 2.3808 | 2.6479 | 3.2108 | 3.4350 | 3.9257 |
| 80 | 0.8461 | 1.2922 | 1.6641 | 1.9901 | 2.3739 | 2.6387 | 3.1953 | 3.4163 | 3.8988 |
| 90 | 0.8456 | 1.2910 | 1.6620 | 1.9867 | 2.3685 | 2.6316 | 3.1833 | 3.4019 | 3.8780 |
| 100 | 0.8452 | 1.2901 | 1.6602 | 1.9840 | 2.3642 | 2.6259 | 3.1737 | 3.3905 | 3.8616 |
| 200 | 0.8434 | 1.2858 | 1.6525 | 1.9719 | 2.3451 | 2.6006 | 3.1315 | 3.3398 | 3.7891 |
| 500 | 0.8423 | 1.2832 | 1.6479 | 1.9647 | 2.3338 | 2.5857 | 3.1066 | 3.3101 | 3.7468 |
| $\infty$ | 0.8416 | 1.2816 | 1.6449 | 1.9600 | 2.3263 | 2.5758 | 3.0902 | 3.2905 | 3.7190 |

**Table VI: Critical Values for the Chi-Square Distribution**

This table contains critical values associated with the chi-square distribution, $\chi^2_{\alpha,\nu}$, defined by $\alpha$ and the degrees of freedom, $\nu$.

| | $\alpha$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\nu$ | 0.9999 | 0.9995 | 0.999 | 0.995 | 0.99 | 0.975 | 0.95 | 0.90 |
| 1 | 1.57E-8 | 3.93E-7 | 1.57E-6 | 3.93E-5 | 0.0002 | 0.0010 | 0.0039 | 0.0158 |
| 2 | 0.0002 | 0.0010 | 0.0020 | 0.0100 | 0.0201 | 0.0506 | 0.1026 | 0.2107 |
| 3 | 0.0052 | 0.0153 | 0.0243 | 0.0717 | 0.1148 | 0.2158 | 0.3518 | 0.5844 |
| 4 | 0.0284 | 0.0639 | 0.0908 | 0.2070 | 0.2971 | 0.4844 | 0.7107 | 1.0636 |
| 5 | 0.0822 | 0.1581 | 0.2102 | 0.4117 | 0.5543 | 0.8312 | 1.1455 | 1.6103 |
| 6 | 0.1724 | 0.2994 | 0.3811 | 0.6757 | 0.8721 | 1.2373 | 1.6354 | 2.2041 |
| 7 | 0.3000 | 0.4849 | 0.5985 | 0.9893 | 1.2390 | 1.6899 | 2.1673 | 2.8331 |
| 8 | 0.4636 | 0.7104 | 0.8571 | 1.3444 | 1.6465 | 2.1797 | 2.7326 | 3.4895 |
| 9 | 0.6608 | 0.9717 | 1.1519 | 1.7349 | 2.0879 | 2.7004 | 3.3251 | 4.1682 |
| 10 | 0.8889 | 1.2650 | 1.4787 | 2.1559 | 2.5582 | 3.2470 | 3.9403 | 4.8652 |
| 11 | 1.1453 | 1.5868 | 1.8339 | 2.6032 | 3.0535 | 3.8157 | 4.5748 | 5.5778 |
| 12 | 1.4275 | 1.9344 | 2.2142 | 3.0738 | 3.5706 | 4.4038 | 5.2260 | 6.3038 |
| 13 | 1.7333 | 2.3051 | 2.6172 | 3.5650 | 4.1069 | 5.0088 | 5.8919 | 7.0415 |
| 14 | 2.0608 | 2.6967 | 3.0407 | 4.0747 | 4.6604 | 5.6287 | 6.5706 | 7.7895 |
| 15 | 2.4082 | 3.1075 | 3.4827 | 4.6009 | 5.2293 | 6.2621 | 7.2609 | 8.5468 |
| 16 | 2.7739 | 3.5358 | 3.9416 | 5.1422 | 5.8122 | 6.9077 | 7.9616 | 9.3122 |
| 17 | 3.1567 | 3.9802 | 4.4161 | 5.6972 | 6.4078 | 7.5642 | 8.6718 | 10.0852 |
| 18 | 3.5552 | 4.4394 | 4.9048 | 6.2648 | 7.0149 | 8.2307 | 9.3905 | 10.8649 |
| 19 | 3.9683 | 4.9123 | 5.4068 | 6.8440 | 7.6327 | 8.9065 | 10.1170 | 11.6509 |
| 20 | 4.3952 | 5.3981 | 5.9210 | 7.4338 | 8.2604 | 9.5908 | 10.8508 | 12.4426 |
| 21 | 4.8348 | 5.8957 | 6.4467 | 8.0337 | 8.8972 | 10.2829 | 11.5913 | 13.2396 |
| 22 | 5.2865 | 6.4045 | 6.9830 | 8.6427 | 9.5425 | 10.9823 | 12.3380 | 14.0415 |
| 23 | 5.7494 | 6.9237 | 7.5292 | 9.2604 | 10.1957 | 11.6886 | 13.0905 | 14.8480 |
| 24 | 6.2230 | 7.4527 | 8.0849 | 9.8862 | 10.8564 | 12.4012 | 13.8484 | 15.6587 |
| 25 | 6.7066 | 7.9910 | 8.6493 | 10.5197 | 11.5240 | 13.1197 | 14.6114 | 16.4734 |
| 26 | 7.1998 | 8.5379 | 9.2221 | 11.1602 | 12.1981 | 13.8439 | 15.3792 | 17.2919 |
| 27 | 7.7019 | 9.0932 | 9.8028 | 11.8076 | 12.8785 | 14.5734 | 16.1514 | 18.1139 |
| 28 | 8.2126 | 9.6563 | 10.3909 | 12.4613 | 13.5647 | 15.3079 | 16.9279 | 18.9392 |
| 29 | 8.7315 | 10.2268 | 10.9861 | 13.1211 | 14.2565 | 16.0471 | 17.7084 | 19.7677 |
| 30 | 9.2581 | 10.8044 | 11.5880 | 13.7867 | 14.9535 | 16.7908 | 18.4927 | 20.5992 |
| 31 | 9.7921 | 11.3887 | 12.1963 | 14.4578 | 15.6555 | 17.5387 | 19.2806 | 21.4336 |
| 32 | 10.3331 | 11.9794 | 12.8107 | 15.1340 | 16.3622 | 18.2908 | 20.0719 | 22.2706 |
| 33 | 10.8810 | 12.5763 | 13.4309 | 15.8153 | 17.0735 | 19.0467 | 20.8665 | 23.1102 |
| 34 | 11.4352 | 13.1791 | 14.0567 | 16.5013 | 17.7891 | 19.8063 | 21.6643 | 23.9523 |
| 35 | 11.9957 | 13.7875 | 14.6878 | 17.1918 | 18.5089 | 20.5694 | 22.4650 | 24.7967 |
| 36 | 12.5622 | 14.4012 | 15.3241 | 17.8867 | 19.2327 | 21.3359 | 23.2686 | 25.6433 |
| 37 | 13.1343 | 15.0202 | 15.9653 | 18.5858 | 19.9602 | 22.1056 | 24.0749 | 26.4921 |
| 38 | 13.7120 | 15.6441 | 16.6112 | 19.2889 | 20.6914 | 22.8785 | 24.8839 | 27.3430 |
| 39 | 14.2950 | 16.2729 | 17.2616 | 19.9959 | 21.4262 | 23.6543 | 25.6954 | 28.1958 |
| 40 | 14.8831 | 16.9062 | 17.9164 | 20.7065 | 22.1643 | 24.4330 | 26.5093 | 29.0505 |
| 50 | 21.0093 | 23.4610 | 24.6739 | 27.9907 | 29.7067 | 32.3574 | 34.7643 | 37.6886 |
| 60 | 27.4969 | 30.3405 | 31.7383 | 35.5345 | 37.4849 | 40.4817 | 43.1880 | 46.4589 |
| 70 | 34.2607 | 37.4674 | 39.0364 | 43.2752 | 45.4417 | 48.7576 | 51.7393 | 55.3289 |
| 80 | 41.2445 | 44.7910 | 46.5199 | 51.1719 | 53.5401 | 57.1532 | 60.3915 | 64.2778 |
| 90 | 48.4087 | 52.2758 | 54.1552 | 59.1963 | 61.7541 | 65.6466 | 69.1260 | 73.2911 |
| 100 | 55.7246 | 59.8957 | 61.9179 | 67.3276 | 70.0649 | 74.2219 | 77.9295 | 82.3581 |

**Table VI: Critical Values for the Chi-Square Distribution** (Continued)

| | $\alpha$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\nu$ | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 | 0.0001 |
| 1 | 2.7055 | 3.8415 | 5.0239 | 6.6349 | 7.8794 | 10.8276 | 12.1157 | 15.1367 |
| 2 | 4.6052 | 5.9915 | 7.3778 | 9.2103 | 10.5966 | 13.8155 | 15.2018 | 18.4207 |
| 3 | 6.2514 | 7.8147 | 9.3484 | 11.3449 | 12.8382 | 16.2662 | 17.7300 | 21.1075 |
| 4 | 7.7794 | 9.4877 | 11.1433 | 13.2767 | 14.8603 | 18.4668 | 19.9974 | 23.5127 |
| 5 | 9.2364 | 11.0705 | 12.8325 | 15.0863 | 16.7496 | 20.5150 | 22.1053 | 25.7448 |
| 6 | 10.6446 | 12.5916 | 14.4494 | 16.8119 | 18.5476 | 22.4577 | 24.1028 | 27.8563 |
| 7 | 12.0170 | 14.0671 | 16.0128 | 18.4753 | 20.2777 | 24.3219 | 26.0178 | 29.8775 |
| 8 | 13.3616 | 15.5073 | 17.5345 | 20.0902 | 21.9550 | 26.1245 | 27.8680 | 31.8276 |
| 9 | 14.6837 | 16.9190 | 19.0228 | 21.6660 | 23.5894 | 27.8772 | 29.6658 | 33.7199 |
| 10 | 15.9872 | 18.3070 | 20.4832 | 23.2093 | 25.1882 | 29.5883 | 31.4198 | 35.5640 |
| 11 | 17.2750 | 19.6751 | 21.9200 | 24.7250 | 26.7568 | 31.2641 | 33.1366 | 37.3670 |
| 12 | 18.5493 | 21.0261 | 23.3367 | 26.2170 | 28.2995 | 32.9095 | 34.8213 | 39.1344 |
| 13 | 19.8119 | 22.3620 | 24.7356 | 27.6882 | 29.8195 | 34.5282 | 36.4778 | 40.8707 |
| 14 | 21.0641 | 23.6848 | 26.1189 | 29.1412 | 31.3193 | 36.1233 | 38.1094 | 42.5793 |
| 15 | 22.3071 | 24.9958 | 27.4884 | 30.5779 | 32.8013 | 37.6973 | 39.7188 | 44.2632 |
| 16 | 23.5418 | 26.2962 | 28.8454 | 31.9999 | 34.2672 | 39.2524 | 41.3081 | 45.9249 |
| 17 | 24.7690 | 27.5871 | 30.1910 | 33.4087 | 35.7185 | 40.7902 | 42.8792 | 47.5664 |
| 18 | 25.9894 | 28.8693 | 31.5264 | 34.8053 | 37.1565 | 42.3124 | 44.4338 | 49.1894 |
| 19 | 27.2036 | 30.1435 | 32.8523 | 36.1909 | 38.5823 | 43.8202 | 45.9731 | 50.7955 |
| 20 | 28.4120 | 31.4104 | 34.1696 | 37.5662 | 39.9968 | 45.3147 | 47.4985 | 52.3860 |
| 21 | 29.6151 | 32.6706 | 35.4789 | 38.9322 | 41.4011 | 46.7970 | 49.0108 | 53.9620 |
| 22 | 30.8133 | 33.9244 | 36.7807 | 40.2894 | 42.7957 | 48.2679 | 50.5111 | 55.5246 |
| 23 | 32.0069 | 35.1725 | 38.0756 | 41.6384 | 44.1813 | 49.7282 | 52.0002 | 57.0746 |
| 24 | 33.1962 | 36.4150 | 39.3641 | 42.9798 | 45.5585 | 51.1786 | 53.4788 | 58.6130 |
| 25 | 34.3816 | 37.6525 | 40.6465 | 44.3141 | 46.9279 | 52.6197 | 54.9475 | 60.1403 |
| 26 | 35.5632 | 38.8851 | 41.9232 | 45.6417 | 48.2899 | 54.0520 | 56.4069 | 61.6573 |
| 27 | 36.7412 | 40.1133 | 43.1945 | 46.9629 | 49.6449 | 55.4760 | 57.8576 | 63.1645 |
| 28 | 37.9159 | 41.3371 | 44.4608 | 48.2782 | 50.9934 | 56.8923 | 59.3000 | 64.6624 |
| 29 | 39.0875 | 42.5570 | 45.7223 | 49.5879 | 52.3356 | 58.3012 | 60.7346 | 66.1517 |
| 30 | 40.2560 | 43.7730 | 46.9792 | 50.8922 | 53.6720 | 59.7031 | 62.1619 | 67.6326 |
| 31 | 41.4217 | 44.9853 | 48.2319 | 52.1914 | 55.0027 | 61.0983 | 63.5820 | 69.1057 |
| 32 | 42.5847 | 46.1943 | 49.4804 | 53.4858 | 56.3281 | 62.4872 | 64.9955 | 70.5712 |
| 33 | 43.7452 | 47.3999 | 50.7251 | 54.7755 | 57.6484 | 63.8701 | 66.4025 | 72.0296 |
| 34 | 44.9032 | 48.6024 | 51.9660 | 56.0609 | 58.9639 | 65.2472 | 67.8035 | 73.4812 |
| 35 | 46.0588 | 49.8018 | 53.2033 | 57.3421 | 60.2748 | 66.6188 | 69.1986 | 74.9262 |
| 36 | 47.2122 | 50.9985 | 54.4373 | 58.6192 | 61.5812 | 67.9852 | 70.5881 | 76.3650 |
| 37 | 48.3634 | 52.1923 | 55.6680 | 59.8925 | 62.8833 | 69.3465 | 71.9722 | 77.7977 |
| 38 | 49.5126 | 53.3835 | 56.8955 | 61.1621 | 64.1814 | 70.7029 | 73.3512 | 79.2247 |
| 39 | 50.6598 | 54.5722 | 58.1201 | 62.4281 | 65.4756 | 72.0547 | 74.7253 | 80.6462 |
| 40 | 51.8051 | 55.7585 | 59.3417 | 63.6907 | 66.7660 | 73.4020 | 76.0946 | 82.0623 |
| 50 | 63.1671 | 67.5048 | 71.4202 | 76.1539 | 79.4900 | 86.6608 | 89.5605 | 95.9687 |
| 60 | 74.3970 | 79.0819 | 83.2977 | 88.3794 | 91.9517 | 99.6072 | 102.6948 | 109.5029 |
| 70 | 85.5270 | 90.5312 | 95.0232 | 100.4252 | 104.2149 | 112.3169 | 115.5776 | 122.7547 |
| 80 | 96.5782 | 101.8795 | 106.6286 | 112.3288 | 116.3211 | 124.8392 | 128.2613 | 135.7825 |
| 90 | 107.5650 | 113.1453 | 118.1359 | 124.1163 | 128.2989 | 137.2084 | 140.7823 | 148.6273 |
| 100 | 118.4980 | 124.3421 | 129.5612 | 135.8067 | 140.1695 | 149.4493 | 153.1670 | 161.3187 |

**Table VII: Critical Values for the $F$ Distribution**

This table contains values associated with the $F$ distribution, $F_{\alpha,\nu_1,\nu_1}$, defined by $\alpha$ and the degrees of freedom $\nu_1$ and $\nu_2$.

$\alpha = 0.05$

| $\nu_2$ \ $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 30 | 40 | 50 | 60 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 245.95 | 248.01 | 250.10 | 251.14 | 251.77 | 252.20 | 253.04 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.43 | 19.45 | 19.46 | 19.47 | 19.48 | 19.48 | 19.49 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.70 | 8.66 | 8.62 | 8.59 | 8.58 | 8.57 | 8.55 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.86 | 5.80 | 5.75 | 5.72 | 5.70 | 5.69 | 5.66 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.62 | 4.56 | 4.50 | 4.46 | 4.44 | 4.43 | 4.41 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 3.94 | 3.87 | 3.81 | 3.77 | 3.75 | 3.74 | 3.71 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.51 | 3.44 | 3.38 | 3.34 | 3.32 | 3.30 | 3.27 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.22 | 3.15 | 3.08 | 3.04 | 3.02 | 3.01 | 2.97 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.01 | 2.94 | 2.86 | 2.83 | 2.80 | 2.79 | 2.76 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.85 | 2.77 | 2.70 | 2.66 | 2.64 | 2.62 | 2.59 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.72 | 2.65 | 2.57 | 2.53 | 2.51 | 2.49 | 2.46 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.62 | 2.54 | 2.47 | 2.43 | 2.40 | 2.38 | 2.35 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.53 | 2.46 | 2.38 | 2.34 | 2.31 | 2.30 | 2.26 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.46 | 2.39 | 2.31 | 2.27 | 2.24 | 2.22 | 2.19 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.40 | 2.33 | 2.25 | 2.20 | 2.18 | 2.16 | 2.12 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.35 | 2.28 | 2.19 | 2.15 | 2.12 | 2.11 | 2.07 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.31 | 2.23 | 2.15 | 2.10 | 2.08 | 2.06 | 2.02 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.27 | 2.19 | 2.11 | 2.06 | 2.04 | 2.02 | 1.98 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.23 | 2.16 | 2.07 | 2.03 | 2.00 | 1.98 | 1.94 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.20 | 2.12 | 2.04 | 1.99 | 1.97 | 1.95 | 1.91 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.18 | 2.10 | 2.01 | 1.96 | 1.94 | 1.92 | 1.88 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.15 | 2.07 | 1.98 | 1.94 | 1.91 | 1.89 | 1.85 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.13 | 2.05 | 1.96 | 1.91 | 1.88 | 1.86 | 1.82 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.11 | 2.03 | 1.94 | 1.89 | 1.86 | 1.84 | 1.80 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.09 | 2.01 | 1.92 | 1.87 | 1.84 | 1.82 | 1.78 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.01 | 1.93 | 1.84 | 1.79 | 1.76 | 1.74 | 1.70 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 1.92 | 1.84 | 1.74 | 1.69 | 1.66 | 1.64 | 1.59 |
| 50 | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 | 1.87 | 1.78 | 1.69 | 1.63 | 1.60 | 1.58 | 1.52 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.84 | 1.75 | 1.65 | 1.59 | 1.56 | 1.53 | 1.48 |
| 100 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.77 | 1.68 | 1.57 | 1.52 | 1.48 | 1.45 | 1.39 |

**Table VII: Critical Values for the $F$ Distribution (Continued)**

$\alpha = 0.01$

| $\nu_2$ \ $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 30 | 40 | 50 | 60 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.43 | 99.45 | 99.47 | 99.47 | 99.48 | 99.48 | 99.49 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 26.87 | 26.69 | 26.50 | 26.41 | 26.35 | 26.32 | 26.24 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.20 | 14.02 | 13.84 | 13.75 | 13.69 | 13.65 | 13.58 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.72 | 9.55 | 9.38 | 9.29 | 9.24 | 9.20 | 9.13 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.56 | 7.40 | 7.23 | 7.14 | 7.09 | 7.06 | 6.99 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.31 | 6.16 | 5.99 | 5.91 | 5.86 | 5.82 | 5.75 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.52 | 5.36 | 5.20 | 5.12 | 5.07 | 5.03 | 4.96 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 4.96 | 4.81 | 4.65 | 4.57 | 4.52 | 4.48 | 4.41 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.56 | 4.41 | 4.25 | 4.17 | 4.12 | 4.08 | 4.01 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.25 | 4.10 | 3.94 | 3.86 | 3.81 | 3.78 | 3.71 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.01 | 3.86 | 3.70 | 3.62 | 3.57 | 3.54 | 3.47 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.82 | 3.66 | 3.51 | 3.43 | 3.38 | 3.34 | 3.27 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.66 | 3.51 | 3.35 | 3.27 | 3.22 | 3.18 | 3.11 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.52 | 3.37 | 3.21 | 3.13 | 3.08 | 3.05 | 2.98 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.41 | 3.26 | 3.10 | 3.02 | 2.97 | 2.93 | 2.86 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.31 | 3.16 | 3.00 | 2.92 | 2.87 | 2.83 | 2.76 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.23 | 3.08 | 2.92 | 2.84 | 2.78 | 2.75 | 2.68 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.15 | 3.00 | 2.84 | 2.76 | 2.71 | 2.67 | 2.60 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.09 | 2.94 | 2.78 | 2.69 | 2.64 | 2.61 | 2.54 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.03 | 2.88 | 2.72 | 2.64 | 2.58 | 2.55 | 2.48 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 2.98 | 2.83 | 2.67 | 2.58 | 2.53 | 2.50 | 2.42 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 2.93 | 2.78 | 2.62 | 2.54 | 2.48 | 2.45 | 2.37 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 2.89 | 2.74 | 2.58 | 2.49 | 2.44 | 2.40 | 2.33 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.85 | 2.70 | 2.54 | 2.45 | 2.40 | 2.36 | 2.29 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.70 | 2.55 | 2.39 | 2.30 | 2.25 | 2.21 | 2.13 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.52 | 2.37 | 2.20 | 2.11 | 2.06 | 2.02 | 1.94 |
| 50 | 7.17 | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 | 2.70 | 2.42 | 2.27 | 2.10 | 2.01 | 1.95 | 1.91 | 1.82 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.35 | 2.20 | 2.03 | 1.94 | 1.88 | 1.84 | 1.75 |
| 100 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.22 | 2.07 | 1.89 | 1.80 | 1.74 | 1.69 | 1.60 |

**Table VII: Critical Values for the $F$ Distribution (Continued)**

$\alpha = 0.01$

| $\nu_2$ \ $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 | 30 | 40 | 50 | 60 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 998.50 | 999.00 | 999.17 | 999.25 | 999.30 | 999.33 | 999.36 | 999.37 | 999.39 | 999.40 | 999.43 | 999.45 | 999.47 | 999.47 | 999.48 | 999.48 | 999.49 |
| 3 | 167.03 | 148.50 | 141.11 | 137.10 | 134.58 | 132.85 | 131.58 | 130.62 | 129.86 | 129.25 | 127.37 | 126.42 | 125.45 | 124.96 | 124.66 | 124.47 | 124.07 |
| 4 | 74.14 | 61.25 | 56.18 | 53.44 | 51.71 | 50.53 | 49.66 | 49.00 | 48.47 | 48.05 | 46.76 | 46.10 | 45.43 | 45.09 | 44.88 | 44.75 | 44.47 |
| 5 | 47.18 | 37.12 | 33.20 | 31.09 | 29.75 | 28.83 | 28.16 | 27.65 | 27.24 | 26.92 | 25.91 | 25.39 | 24.87 | 24.60 | 24.44 | 24.33 | 24.12 |
| 6 | 35.51 | 27.00 | 23.70 | 21.92 | 20.80 | 20.03 | 19.46 | 19.03 | 18.69 | 18.41 | 17.56 | 17.12 | 16.67 | 16.44 | 16.31 | 16.21 | 16.03 |
| 7 | 29.25 | 21.69 | 18.77 | 17.20 | 16.21 | 15.52 | 15.02 | 14.63 | 14.33 | 14.08 | 13.32 | 12.93 | 12.53 | 12.33 | 12.20 | 12.12 | 11.95 |
| 8 | 25.41 | 18.49 | 15.83 | 14.39 | 13.48 | 12.86 | 12.40 | 12.05 | 11.77 | 11.54 | 10.84 | 10.48 | 10.11 | 9.92 | 9.80 | 9.73 | 9.57 |
| 9 | 22.86 | 16.39 | 13.90 | 12.56 | 11.71 | 11.13 | 10.70 | 10.37 | 10.11 | 9.89 | 9.24 | 8.90 | 8.55 | 8.37 | 8.26 | 8.19 | 8.04 |
| 10 | 21.04 | 14.91 | 12.55 | 11.28 | 10.48 | 9.93 | 9.52 | 9.20 | 8.96 | 8.75 | 8.13 | 7.80 | 7.47 | 7.30 | 7.19 | 7.12 | 6.98 |
| 11 | 19.69 | 13.81 | 11.56 | 10.35 | 9.58 | 9.05 | 8.66 | 8.35 | 8.12 | 7.92 | 7.32 | 7.01 | 6.68 | 6.52 | 6.42 | 6.35 | 6.21 |
| 12 | 18.64 | 12.97 | 10.80 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 | 7.29 | 6.71 | 6.40 | 6.09 | 5.93 | 5.83 | 5.76 | 5.63 |
| 13 | 17.82 | 12.31 | 10.21 | 9.07 | 8.35 | 7.86 | 7.49 | 7.21 | 6.98 | 6.80 | 6.23 | 5.93 | 5.63 | 5.47 | 5.37 | 5.30 | 5.17 |
| 14 | 17.14 | 11.78 | 9.73 | 8.62 | 7.92 | 7.44 | 7.08 | 6.80 | 6.58 | 6.40 | 5.85 | 5.56 | 5.25 | 5.10 | 5.00 | 4.94 | 4.81 |
| 15 | 16.59 | 11.34 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 | 6.08 | 5.54 | 5.25 | 4.95 | 4.80 | 4.70 | 4.64 | 4.51 |
| 16 | 16.12 | 10.97 | 9.01 | 7.94 | 7.27 | 6.80 | 6.46 | 6.19 | 5.98 | 5.81 | 5.27 | 4.99 | 4.70 | 4.54 | 4.45 | 4.39 | 4.26 |
| 17 | 15.72 | 10.66 | 8.73 | 7.68 | 7.02 | 6.56 | 6.22 | 5.96 | 5.75 | 5.58 | 5.05 | 4.78 | 4.48 | 4.33 | 4.24 | 4.18 | 4.05 |
| 18 | 15.38 | 10.39 | 8.49 | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 | 5.39 | 4.87 | 4.59 | 4.30 | 4.15 | 4.06 | 4.00 | 3.87 |
| 19 | 15.08 | 10.16 | 8.28 | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 | 5.22 | 4.70 | 4.43 | 4.14 | 3.99 | 3.90 | 3.84 | 3.71 |
| 20 | 14.82 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 | 5.08 | 4.56 | 4.29 | 4.00 | 3.86 | 3.77 | 3.70 | 3.58 |
| 21 | 14.59 | 9.77 | 7.94 | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 | 4.95 | 4.44 | 4.17 | 3.88 | 3.74 | 3.64 | 3.58 | 3.46 |
| 22 | 14.38 | 9.61 | 7.80 | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 | 4.83 | 4.33 | 4.06 | 3.78 | 3.63 | 3.54 | 3.48 | 3.35 |
| 23 | 14.20 | 9.47 | 7.67 | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 | 4.73 | 4.23 | 3.96 | 3.68 | 3.53 | 3.44 | 3.38 | 3.25 |
| 24 | 14.03 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 | 4.64 | 4.14 | 3.87 | 3.59 | 3.45 | 3.36 | 3.29 | 3.17 |
| 25 | 13.88 | 9.22 | 7.45 | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 | 4.56 | 4.06 | 3.79 | 3.52 | 3.37 | 3.28 | 3.22 | 3.09 |
| 30 | 13.29 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 | 4.24 | 3.75 | 3.49 | 3.22 | 3.07 | 2.98 | 2.92 | 2.79 |
| 40 | 12.61 | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 | 3.87 | 3.40 | 3.14 | 2.87 | 2.73 | 2.64 | 2.57 | 2.44 |
| 50 | 12.22 | 7.96 | 6.34 | 5.46 | 4.90 | 4.51 | 4.22 | 4.00 | 3.82 | 3.67 | 3.20 | 2.95 | 2.68 | 2.53 | 2.44 | 2.38 | 2.25 |
| 60 | 11.97 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 | 3.54 | 3.08 | 2.83 | 2.55 | 2.41 | 2.32 | 2.25 | 2.12 |
| 100 | 11.50 | 7.41 | 5.86 | 5.02 | 4.48 | 4.11 | 3.83 | 3.61 | 3.44 | 3.30 | 2.84 | 2.59 | 2.32 | 2.17 | 2.08 | 2.01 | 1.87 |

## Table VIII: Critical Values for the Studentized Range Distribution

This table contains critical values associated with the Studentized Range Distribution, $Q_{\alpha,k,\nu}$, defined by $\alpha$, and the degrees of freedom $k$ and $\nu$, where $k$ is the number of degrees of freedom in the numerator (the number of treatment groups) and $\nu$ is the number of degrees of freedom in the denominator.

| $\alpha = 0.05$ | $k$ | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 2 | 6.085 | 8.331 | 9.798 | 10.881 | 11.734 | 12.434 | 13.027 | 13.538 | 13.987 | 14.387 | 14.747 | 15.076 | 15.375 | 15.650 | 15.905 | 16.143 | 16.365 | 16.573 | 16.769 |
| 3 | 4.501 | 5.910 | 6.825 | 7.502 | 8.037 | 8.478 | 8.852 | 9.177 | 9.462 | 9.717 | 9.946 | 10.155 | 10.346 | 10.522 | 10.686 | 10.838 | 10.980 | 11.114 | 11.240 |
| 4 | 3.926 | 5.040 | 5.757 | 6.287 | 6.706 | 7.053 | 7.347 | 7.602 | 7.826 | 8.027 | 8.208 | 8.373 | 8.524 | 8.664 | 8.793 | 8.914 | 9.027 | 9.133 | 9.233 |
| 5 | 3.635 | 4.602 | 5.218 | 5.673 | 6.033 | 6.330 | 6.582 | 6.801 | 6.995 | 7.167 | 7.324 | 7.465 | 7.596 | 7.716 | 7.828 | 7.932 | 8.030 | 8.122 | 8.208 |
| 6 | 3.460 | 4.339 | 4.896 | 5.305 | 5.629 | 5.895 | 6.122 | 6.319 | 6.493 | 6.649 | 6.789 | 6.917 | 7.034 | 7.143 | 7.244 | 7.338 | 7.426 | 7.509 | 7.587 |
| 7 | 3.344 | 4.165 | 4.681 | 5.060 | 5.359 | 5.606 | 5.815 | 5.997 | 6.158 | 6.302 | 6.431 | 6.550 | 6.658 | 6.759 | 6.852 | 6.939 | 7.020 | 7.097 | 7.169 |
| 8 | 3.261 | 4.041 | 4.529 | 4.886 | 5.167 | 5.399 | 5.596 | 5.767 | 5.918 | 6.053 | 6.175 | 6.287 | 6.389 | 6.483 | 6.571 | 6.653 | 6.729 | 6.801 | 6.870 |
| 9 | 3.199 | 3.948 | 4.415 | 4.755 | 5.024 | 5.244 | 5.432 | 5.595 | 5.738 | 5.867 | 5.983 | 6.089 | 6.186 | 6.276 | 6.359 | 6.437 | 6.510 | 6.579 | 6.644 |
| 10 | 3.151 | 3.877 | 4.327 | 4.654 | 4.912 | 5.124 | 5.304 | 5.460 | 5.598 | 5.722 | 5.833 | 5.935 | 6.028 | 6.114 | 6.194 | 6.269 | 6.339 | 6.405 | 6.467 |
| 11 | 3.113 | 3.820 | 4.256 | 4.574 | 4.823 | 5.028 | 5.202 | 5.353 | 5.486 | 5.605 | 5.713 | 5.811 | 5.901 | 5.984 | 6.062 | 6.134 | 6.202 | 6.265 | 6.325 |
| 12 | 3.081 | 3.773 | 4.199 | 4.508 | 4.750 | 4.950 | 5.119 | 5.265 | 5.395 | 5.510 | 5.615 | 5.710 | 5.797 | 5.878 | 5.953 | 6.023 | 6.089 | 6.151 | 6.209 |
| 14 | 3.033 | 3.701 | 4.111 | 4.407 | 4.639 | 4.829 | 4.990 | 5.130 | 5.253 | 5.363 | 5.463 | 5.554 | 5.637 | 5.714 | 5.785 | 5.852 | 5.915 | 5.973 | 6.029 |
| 15 | 3.014 | 3.673 | 4.076 | 4.367 | 4.595 | 4.782 | 4.940 | 5.077 | 5.198 | 5.306 | 5.403 | 5.492 | 5.574 | 5.649 | 5.719 | 5.785 | 5.846 | 5.904 | 5.958 |
| 16 | 2.998 | 3.649 | 4.046 | 4.333 | 4.557 | 4.741 | 4.896 | 5.031 | 5.150 | 5.256 | 5.352 | 5.439 | 5.519 | 5.593 | 5.662 | 5.726 | 5.786 | 5.843 | 5.896 |
| 17 | 2.984 | 3.628 | 4.020 | 4.303 | 4.524 | 4.705 | 4.858 | 4.991 | 5.108 | 5.212 | 5.306 | 5.392 | 5.471 | 5.544 | 5.612 | 5.675 | 5.734 | 5.790 | 5.842 |
| 18 | 2.971 | 3.609 | 3.997 | 4.276 | 4.494 | 4.673 | 4.824 | 4.955 | 5.071 | 5.173 | 5.266 | 5.351 | 5.429 | 5.501 | 5.567 | 5.629 | 5.688 | 5.743 | 5.794 |
| 19 | 2.960 | 3.593 | 3.977 | 4.253 | 4.468 | 4.645 | 4.794 | 4.924 | 5.037 | 5.139 | 5.231 | 5.314 | 5.391 | 5.462 | 5.528 | 5.589 | 5.647 | 5.701 | 5.752 |
| 20 | 2.950 | 3.578 | 3.958 | 4.232 | 4.445 | 4.620 | 4.768 | 4.895 | 5.008 | 5.108 | 5.199 | 5.282 | 5.357 | 5.427 | 5.492 | 5.553 | 5.610 | 5.663 | 5.714 |
| 25 | 2.913 | 3.523 | 3.890 | 4.153 | 4.358 | 4.526 | 4.667 | 4.789 | 4.897 | 4.993 | 5.079 | 5.158 | 5.230 | 5.297 | 5.359 | 5.417 | 5.471 | 5.522 | 5.570 |
| 30 | 2.888 | 3.487 | 3.845 | 4.102 | 4.301 | 4.464 | 4.601 | 4.720 | 4.824 | 4.917 | 5.001 | 5.077 | 5.147 | 5.211 | 5.271 | 5.327 | 5.379 | 5.429 | 5.475 |
| 40 | 2.858 | 3.442 | 3.791 | 4.039 | 4.232 | 4.388 | 4.521 | 4.634 | 4.735 | 4.824 | 4.904 | 4.977 | 5.044 | 5.106 | 5.163 | 5.216 | 5.266 | 5.313 | 5.358 |
| 50 | 2.841 | 3.416 | 3.758 | 4.002 | 4.190 | 4.344 | 4.473 | 4.584 | 4.681 | 4.768 | 4.847 | 4.918 | 4.983 | 5.043 | 5.098 | 5.150 | 5.199 | 5.245 | 5.288 |
| 100 | 2.806 | 3.365 | 3.695 | 3.929 | 4.109 | 4.256 | 4.379 | 4.484 | 4.577 | 4.659 | 4.733 | 4.800 | 4.862 | 4.918 | 4.971 | 5.020 | 5.066 | 5.108 | 5.149 |
| 200 | 2.789 | 3.339 | 3.664 | 3.893 | 4.069 | 4.212 | 4.332 | 4.435 | 4.525 | 4.605 | 4.677 | 4.742 | 4.802 | 4.857 | 4.908 | 4.955 | 4.999 | 5.041 | 5.080 |
| 300 | 2.783 | 3.331 | 3.654 | 3.881 | 4.056 | 4.198 | 4.317 | 4.419 | 4.508 | 4.587 | 4.659 | 4.723 | 4.782 | 4.837 | 4.887 | 4.934 | 4.978 | 5.019 | 5.057 |
| 400 | 2.780 | 3.327 | 3.649 | 3.875 | 4.050 | 4.191 | 4.309 | 4.411 | 4.500 | 4.578 | 4.649 | 4.714 | 4.772 | 4.826 | 4.876 | 4.923 | 4.967 | 5.007 | 5.046 |
| 500 | 2.779 | 3.324 | 3.645 | 3.872 | 4.046 | 4.187 | 4.305 | 4.406 | 4.494 | 4.573 | 4.644 | 4.708 | 4.766 | 4.820 | 4.870 | 4.917 | 4.960 | 5.001 | 5.039 |

**Table VIII: Critical Values for the Studentized Range Distribution (Continued)**

| $\alpha = 0.01$ | $k$ | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 2 | 14.035 | 19.019 | 22.293 | 24.717 | 26.628 | 28.199 | 29.528 | 30.677 | 31.687 | 32.585 | 33.395 | 34.129 | 34.802 | 35.421 | 35.995 | 36.529 | 37.028 | 37.496 | 37.937 |
| 3 | 8.260 | 10.616 | 12.169 | 13.324 | 14.240 | 14.997 | 15.640 | 16.198 | 16.689 | 17.128 | 17.524 | 17.884 | 18.214 | 18.519 | 18.802 | 19.065 | 19.311 | 19.543 | 19.761 |
| 4 | 6.511 | 8.118 | 9.173 | 9.958 | 10.582 | 11.099 | 11.539 | 11.925 | 12.264 | 12.566 | 12.840 | 13.089 | 13.318 | 13.530 | 13.726 | 13.909 | 14.081 | 14.242 | 14.394 |
| 5 | 5.702 | 6.976 | 7.806 | 8.421 | 8.913 | 9.321 | 9.669 | 9.971 | 10.239 | 10.479 | 10.695 | 10.893 | 11.075 | 11.243 | 11.399 | 11.544 | 11.681 | 11.809 | 11.930 |
| 6 | 5.243 | 6.331 | 7.033 | 7.556 | 7.974 | 8.318 | 8.611 | 8.869 | 9.097 | 9.300 | 9.485 | 9.653 | 9.808 | 9.951 | 10.084 | 10.208 | 10.325 | 10.434 | 10.538 |
| 7 | 4.948 | 5.919 | 6.543 | 7.006 | 7.373 | 7.678 | 7.940 | 8.167 | 8.368 | 8.548 | 8.711 | 8.859 | 8.996 | 9.124 | 9.242 | 9.353 | 9.456 | 9.553 | 9.645 |
| 8 | 4.745 | 5.635 | 6.204 | 6.625 | 6.960 | 7.238 | 7.475 | 7.681 | 7.864 | 8.028 | 8.177 | 8.312 | 8.437 | 8.552 | 8.659 | 8.760 | 8.854 | 8.942 | 9.026 |
| 9 | 4.595 | 5.428 | 5.957 | 6.347 | 6.658 | 6.915 | 7.134 | 7.326 | 7.495 | 7.647 | 7.785 | 7.910 | 8.026 | 8.133 | 8.233 | 8.326 | 8.413 | 8.495 | 8.573 |
| 10 | 4.482 | 5.270 | 5.769 | 6.136 | 6.428 | 6.669 | 6.875 | 7.055 | 7.214 | 7.356 | 7.485 | 7.603 | 7.712 | 7.813 | 7.906 | 7.994 | 8.076 | 8.153 | 8.226 |
| 11 | 4.392 | 5.146 | 5.621 | 5.970 | 6.247 | 6.476 | 6.671 | 6.842 | 6.992 | 7.127 | 7.250 | 7.362 | 7.465 | 7.560 | 7.649 | 7.732 | 7.810 | 7.883 | 7.952 |
| 12 | 4.320 | 5.046 | 5.502 | 5.836 | 6.101 | 6.321 | 6.507 | 6.670 | 6.814 | 6.943 | 7.060 | 7.167 | 7.265 | 7.356 | 7.441 | 7.520 | 7.594 | 7.664 | 7.731 |
| 13 | 4.261 | 4.964 | 5.404 | 5.727 | 5.981 | 6.192 | 6.372 | 6.528 | 6.666 | 6.791 | 6.903 | 7.006 | 7.100 | 7.188 | 7.269 | 7.345 | 7.417 | 7.484 | 7.548 |
| 14 | 4.210 | 4.895 | 5.322 | 5.634 | 5.881 | 6.085 | 6.258 | 6.409 | 6.543 | 6.664 | 6.772 | 6.871 | 6.962 | 7.047 | 7.125 | 7.199 | 7.268 | 7.333 | 7.394 |
| 15 | 4.167 | 4.836 | 5.252 | 5.556 | 5.796 | 5.994 | 6.162 | 6.309 | 6.438 | 6.555 | 6.660 | 6.757 | 6.845 | 6.927 | 7.003 | 7.074 | 7.141 | 7.204 | 7.264 |
| 16 | 4.131 | 4.786 | 5.192 | 5.488 | 5.722 | 5.915 | 6.079 | 6.222 | 6.348 | 6.461 | 6.564 | 6.658 | 6.743 | 6.824 | 6.897 | 6.967 | 7.032 | 7.093 | 7.151 |
| 17 | 4.099 | 4.742 | 5.140 | 5.430 | 5.659 | 5.847 | 6.007 | 6.147 | 6.270 | 6.380 | 6.480 | 6.572 | 6.656 | 6.733 | 6.806 | 6.873 | 6.937 | 6.997 | 7.053 |
| 18 | 4.071 | 4.703 | 5.094 | 5.379 | 5.603 | 5.787 | 5.944 | 6.081 | 6.201 | 6.309 | 6.407 | 6.496 | 6.579 | 6.655 | 6.725 | 6.791 | 6.854 | 6.912 | 6.967 |
| 19 | 4.046 | 4.669 | 5.054 | 5.333 | 5.553 | 5.735 | 5.888 | 6.022 | 6.141 | 6.246 | 6.342 | 6.430 | 6.510 | 6.585 | 6.654 | 6.719 | 6.780 | 6.837 | 6.891 |
| 20 | 4.024 | 4.639 | 5.018 | 5.293 | 5.509 | 5.687 | 5.839 | 5.970 | 6.086 | 6.190 | 6.285 | 6.370 | 6.449 | 6.523 | 6.591 | 6.654 | 6.714 | 6.770 | 6.823 |
| 25 | 3.942 | 4.527 | 4.884 | 5.143 | 5.346 | 5.513 | 5.654 | 5.777 | 5.885 | 5.982 | 6.070 | 6.150 | 6.223 | 6.291 | 6.355 | 6.414 | 6.469 | 6.521 | 6.571 |
| 30 | 3.889 | 4.454 | 4.799 | 5.048 | 5.242 | 5.401 | 5.536 | 5.653 | 5.756 | 5.848 | 5.932 | 6.008 | 6.078 | 6.142 | 6.202 | 6.258 | 6.311 | 6.360 | 6.407 |
| 40 | 3.825 | 4.367 | 4.695 | 4.931 | 5.114 | 5.265 | 5.392 | 5.502 | 5.599 | 5.685 | 5.764 | 5.835 | 5.900 | 5.961 | 6.017 | 6.069 | 6.118 | 6.165 | 6.208 |
| 50 | 3.787 | 4.316 | 4.634 | 4.863 | 5.040 | 5.185 | 5.308 | 5.414 | 5.507 | 5.590 | 5.665 | 5.734 | 5.796 | 5.854 | 5.908 | 5.958 | 6.005 | 6.050 | 6.092 |
| 100 | 3.714 | 4.216 | 4.516 | 4.730 | 4.896 | 5.031 | 5.144 | 5.242 | 5.328 | 5.405 | 5.474 | 5.537 | 5.594 | 5.648 | 5.697 | 5.743 | 5.786 | 5.826 | 5.864 |
| 200 | 3.714 | 4.216 | 4.516 | 4.730 | 4.896 | 5.031 | 5.144 | 5.242 | 5.328 | 5.405 | 5.474 | 5.537 | 5.594 | 5.648 | 5.697 | 5.743 | 5.786 | 5.826 | 5.864 |
| 300 | 3.666 | 4.152 | 4.440 | 4.645 | 4.803 | 4.931 | 5.039 | 5.132 | 5.213 | 5.286 | 5.351 | 5.410 | 5.464 | 5.514 | 5.560 | 5.603 | 5.644 | 5.682 | 5.717 |
| 400 | 3.661 | 4.144 | 4.431 | 4.634 | 4.791 | 4.919 | 5.026 | 5.118 | 5.199 | 5.271 | 5.335 | 5.394 | 5.448 | 5.498 | 5.543 | 5.586 | 5.626 | 5.664 | 5.699 |
| 500 | 3.657 | 4.139 | 4.425 | 4.628 | 4.784 | 4.911 | 5.018 | 5.110 | 5.190 | 5.262 | 5.327 | 5.385 | 5.438 | 5.488 | 5.533 | 5.576 | 5.616 | 5.653 | 5.688 |

**Table VIII: Critical Values for the Studentized Range Distribution (Continued)**

| $\alpha = 0.001$ | $k$ | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 2 | 44.666 | 60.323 | 70.586 | 78.162 | 84.127 | 89.022 | 93.650 | 97.285 | 100.480 | 103.325 | 105.886 | 108.211 | 110.340 | 112.300 | 114.115 | 115.805 | 117.385 | 118.867 | 120.263 |
| 3 | 18.275 | 23.298 | 26.609 | 29.075 | 31.030 | 32.645 | 34.016 | 35.327 | 36.389 | 37.338 | 38.194 | 38.974 | 39.688 | 40.347 | 40.959 | 41.529 | 42.062 | 42.564 | 43.036 |
| 4 | 12.174 | 14.965 | 16.798 | 18.225 | 19.333 | 20.253 | 21.037 | 21.719 | 22.323 | 22.862 | 23.350 | 23.795 | 24.204 | 24.581 | 24.932 | 25.259 | 25.566 | 25.854 | 26.126 |
| 5 | 9.710 | 11.671 | 12.959 | 13.924 | 14.695 | 15.335 | 15.882 | 16.358 | 16.780 | 17.158 | 17.500 | 17.811 | 18.098 | 18.402 | 18.651 | 18.884 | 19.102 | 19.307 | 19.501 |
| 6 | 8.431 | 9.955 | 10.965 | 11.719 | 12.322 | 12.824 | 13.254 | 13.629 | 13.961 | 14.260 | 14.530 | 14.777 | 15.004 | 15.215 | 15.411 | 15.593 | 15.765 | 15.927 | 16.079 |
| 7 | 7.649 | 8.933 | 9.761 | 10.388 | 10.883 | 11.316 | 11.674 | 11.988 | 12.265 | 12.515 | 12.742 | 12.949 | 13.139 | 13.316 | 13.480 | 13.634 | 13.778 | 13.914 | 14.043 |
| 8 | 7.130 | 8.252 | 8.980 | 9.523 | 9.948 | 10.317 | 10.625 | 10.894 | 11.133 | 11.347 | 11.559 | 11.740 | 11.906 | 12.060 | 12.203 | 12.337 | 12.463 | 12.582 | 12.694 |
| 9 | 7.130 | 8.252 | 8.980 | 9.523 | 9.948 | 10.317 | 10.625 | 10.894 | 11.133 | 11.347 | 11.559 | 11.740 | 11.906 | 12.060 | 12.203 | 12.337 | 12.463 | 12.582 | 12.694 |
| 10 | 6.486 | 7.411 | 8.007 | 8.451 | 8.805 | 9.100 | 9.353 | 9.574 | 9.770 | 9.954 | 10.106 | 10.245 | 10.387 | 10.512 | 10.629 | 10.737 | 10.840 | 10.936 | 11.027 |
| 11 | 6.274 | 7.137 | 7.688 | 8.099 | 8.427 | 8.700 | 8.934 | 9.138 | 9.320 | 9.483 | 9.631 | 9.767 | 9.892 | 10.017 | 10.121 | 10.218 | 10.309 | 10.394 | 10.475 |
| 12 | 6.106 | 6.917 | 7.442 | 7.820 | 8.128 | 8.383 | 8.602 | 8.793 | 8.963 | 9.116 | 9.254 | 9.381 | 9.498 | 9.607 | 9.708 | 9.803 | 9.892 | 9.976 | 10.055 |
| 13 | 5.969 | 6.740 | 7.234 | 7.595 | 7.885 | 8.126 | 8.333 | 8.513 | 8.674 | 8.818 | 8.949 | 9.068 | 9.179 | 9.281 | 9.377 | 9.466 | 9.550 | 9.630 | 9.705 |
| 14 | 5.855 | 6.593 | 7.070 | 7.410 | 7.692 | 7.914 | 8.111 | 8.282 | 8.434 | 8.571 | 8.696 | 8.810 | 8.915 | 9.012 | 9.103 | 9.188 | 9.268 | 9.343 | 9.414 |
| 15 | 5.760 | 6.470 | 6.920 | 7.257 | 7.517 | 7.742 | 7.924 | 8.088 | 8.234 | 8.365 | 8.483 | 8.592 | 8.693 | 8.786 | 8.873 | 8.954 | 9.030 | 9.102 | 9.170 |
| 16 | 5.678 | 6.365 | 6.799 | 7.125 | 7.377 | 7.585 | 7.769 | 7.923 | 8.063 | 8.189 | 8.303 | 8.407 | 8.504 | 8.593 | 8.676 | 8.754 | 8.828 | 8.897 | 8.963 |
| 17 | 5.614 | 6.274 | 6.695 | 7.010 | 7.254 | 7.457 | 7.629 | 7.783 | 7.921 | 8.037 | 8.147 | 8.248 | 8.341 | 8.427 | 8.508 | 8.583 | 8.654 | 8.720 | 8.783 |
| 18 | 5.550 | 6.201 | 6.609 | 6.909 | 7.147 | 7.343 | 7.511 | 7.658 | 7.781 | 7.908 | 8.017 | 8.116 | 8.199 | 8.283 | 8.361 | 8.433 | 8.502 | 8.566 | 8.628 |
| 19 | 5.493 | 6.129 | 6.527 | 6.820 | 7.051 | 7.243 | 7.407 | 7.550 | 7.676 | 7.790 | 7.894 | 7.990 | 8.079 | 8.162 | 8.238 | 8.302 | 8.369 | 8.431 | 8.491 |
| 20 | 5.444 | 6.065 | 6.455 | 6.741 | 6.967 | 7.154 | 7.314 | 7.453 | 7.577 | 7.687 | 7.788 | 7.880 | 7.966 | 8.046 | 8.121 | 8.190 | 8.256 | 8.318 | 8.376 |
| 25 | 5.264 | 5.840 | 6.196 | 6.456 | 6.662 | 6.831 | 6.976 | 7.102 | 7.213 | 7.314 | 7.404 | 7.487 | 7.558 | 7.629 | 7.696 | 7.758 | 7.816 | 7.871 | 7.924 |
| 30 | 5.154 | 5.698 | 6.033 | 6.277 | 6.469 | 6.628 | 6.763 | 6.880 | 6.984 | 7.077 | 7.161 | 7.238 | 7.309 | 7.375 | 7.436 | 7.494 | 7.548 | 7.598 | 7.646 |
| 40 | 5.022 | 5.527 | 5.837 | 6.062 | 6.239 | 6.385 | 6.508 | 6.615 | 6.710 | 6.795 | 6.872 | 6.942 | 7.006 | 7.066 | 7.121 | 7.173 | 7.222 | 7.268 | 7.312 |
| 50 | 4.946 | 5.426 | 5.725 | 5.939 | 6.107 | 6.245 | 6.361 | 6.463 | 6.552 | 6.632 | 6.705 | 6.771 | 6.832 | 6.888 | 6.940 | 6.989 | 7.035 | 7.078 | 7.119 |
| 100 | 4.795 | 5.244 | 5.512 | 5.706 | 5.855 | 5.978 | 6.083 | 6.173 | 6.252 | 6.323 | 6.387 | 6.445 | 6.499 | 6.548 | 6.594 | 6.637 | 6.678 | 6.715 | 6.751 |
| 200 | 4.723 | 5.151 | 5.408 | 5.596 | 5.738 | 5.854 | 5.952 | 6.038 | 6.110 | 6.178 | 6.237 | 6.292 | 6.342 | 6.388 | 6.431 | 6.471 | 6.509 | 6.544 | 6.577 |
| 300 | 4.700 | 5.122 | 5.375 | 5.556 | 5.696 | 5.814 | 5.910 | 5.993 | 6.066 | 6.131 | 6.189 | 6.244 | 6.291 | 6.335 | 6.379 | 6.418 | 6.455 | 6.489 | 6.522 |
| 400 | 4.688 | 5.107 | 5.358 | 5.538 | 5.677 | 5.791 | 5.890 | 5.972 | 6.044 | 6.108 | 6.166 | 6.219 | 6.267 | 6.312 | 6.355 | 6.393 | 6.427 | 6.460 | 6.494 |
| 500 | 4.681 | 5.098 | 5.348 | 5.527 | 5.665 | 5.778 | 5.874 | 5.959 | 6.031 | 6.095 | 6.152 | 6.205 | 6.253 | 6.297 | 6.338 | 6.376 | 6.412 | 6.448 | 6.479 |

**Table IX: Critical Values for the Wilcoxon Signed-Rank Statistic**

This table contains critical values and probabilities for the Wilcoxon signed-rank statistic $T_+$: $n$ is the sample size, $c_1$ and $c_2$ are defined by $\mathrm{P}(T_+ \leq c1) = \alpha$ and $\mathrm{P}(T_+ \geq c_2) = \alpha$.

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 1 | 0 | 1 | 0.5000 |
| 2 | 0 | 3 | 0.2500 |
| 3 | 0 | 6 | 0.1250 |
| 4 | 0 | 10 | 0.0625 |
| | 1 | 9 | 0.1250 |
| 5 | 0 | 15 | 0.0313 |
| | 1 | 14 | 0.0625 |
| | 2 | 13 | 0.0938 |
| | 3 | 12 | 0.1563 |
| 6 | 0 | 21 | 0.0156 |
| | 1 | 20 | 0.0313 |
| | 2 | 19 | 0.0469 |
| | 3 | 18 | 0.0781 |
| | 4 | 17 | 0.1094 |
| | 5 | 16 | 0.1563 |
| 7 | 0 | 28 | 0.0078 |
| | 1 | 27 | 0.0156 |
| | 2 | 26 | 0.0234 |
| | 3 | 25 | 0.0391 |
| | 4 | 24 | 0.0547 |
| | 5 | 23 | 0.0781 |
| | 6 | 22 | 0.1094 |
| | 7 | 21 | 0.1484 |
| 8 | 0 | 36 | 0.0039 |
| | 1 | 35 | 0.0078 |
| | 2 | 34 | 0.0117 |
| | 3 | 33 | 0.0195 |
| | 4 | 32 | 0.0273 |
| | 5 | 31 | 0.0391 |
| | 6 | 30 | 0.0547 |
| | 7 | 29 | 0.0742 |
| | 8 | 28 | 0.0977 |
| | 9 | 27 | 0.1250 |
| 9 | 0 | 45 | 0.0020 |
| | 1 | 44 | 0.0039 |
| | 2 | 43 | 0.0059 |
| | 3 | 42 | 0.0098 |
| | 4 | 41 | 0.0137 |
| | 5 | 40 | 0.0195 |
| | 6 | 39 | 0.0273 |
| | 7 | 38 | 0.0371 |
| | 8 | 37 | 0.0488 |
| | 9 | 36 | 0.0645 |
| | 10 | 35 | 0.0820 |
| | 11 | 34 | 0.1016 |
| | 12 | 33 | 0.1250 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 10 | 0 | 55 | 0.0010 |
| | 1 | 54 | 0.0020 |
| | 2 | 53 | 0.0029 |
| | 3 | 52 | 0.0049 |
| | 4 | 51 | 0.0068 |
| | 5 | 50 | 0.0098 |
| | 6 | 49 | 0.0137 |
| | 7 | 48 | 0.0186 |
| | 8 | 47 | 0.0244 |
| | 9 | 46 | 0.0322 |
| | 10 | 45 | 0.0420 |
| | 11 | 44 | 0.0527 |
| | 12 | 43 | 0.0654 |
| | 13 | 42 | 0.0801 |
| | 14 | 41 | 0.0967 |
| | 15 | 40 | 0.1162 |
| | 16 | 39 | 0.1377 |
| 11 | 0 | 66 | 0.0005 |
| | 1 | 65 | 0.0010 |
| | 2 | 64 | 0.0015 |
| | 3 | 63 | 0.0024 |
| | 4 | 62 | 0.0034 |
| | 5 | 61 | 0.0049 |
| | 6 | 60 | 0.0068 |
| | 7 | 59 | 0.0093 |
| | 8 | 58 | 0.0122 |
| | 9 | 57 | 0.0161 |
| | 10 | 56 | 0.0210 |
| | 11 | 55 | 0.0269 |
| | 12 | 54 | 0.0337 |
| | 13 | 53 | 0.0415 |
| | 14 | 52 | 0.0508 |
| | 15 | 51 | 0.0615 |
| | 16 | 50 | 0.0737 |
| | 17 | 49 | 0.0874 |
| | 18 | 48 | 0.1030 |
| | 19 | 47 | 0.1201 |
| | 20 | 46 | 0.1392 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 12 | 0 | 78 | 0.0002 |
| | 1 | 77 | 0.0005 |
| | 2 | 76 | 0.0007 |
| | 3 | 75 | 0.0012 |
| | 4 | 74 | 0.0017 |
| | 5 | 73 | 0.0024 |
| | 6 | 72 | 0.0034 |
| | 7 | 71 | 0.0046 |
| | 8 | 70 | 0.0061 |
| | 9 | 69 | 0.0081 |
| | 10 | 68 | 0.0105 |
| | 11 | 67 | 0.0134 |
| | 12 | 66 | 0.0171 |
| | 13 | 65 | 0.0212 |
| | 14 | 64 | 0.0261 |
| | 15 | 63 | 0.0320 |
| | 16 | 62 | 0.0386 |
| | 17 | 61 | 0.0461 |
| | 18 | 60 | 0.0549 |
| | 19 | 59 | 0.0647 |
| | 20 | 58 | 0.0757 |
| | 21 | 57 | 0.0881 |
| | 22 | 56 | 0.1018 |
| | 23 | 55 | 0.1167 |
| | 24 | 54 | 0.1331 |
| | 25 | 53 | 0.1506 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 13 | 0 | 91 | 0.0001 |
| | 1 | 90 | 0.0002 |
| | 2 | 89 | 0.0004 |
| | 3 | 88 | 0.0006 |
| | 4 | 87 | 0.0009 |
| | 5 | 86 | 0.0012 |
| | 6 | 85 | 0.0017 |
| | 7 | 84 | 0.0023 |
| | 8 | 83 | 0.0031 |
| | 9 | 82 | 0.0040 |
| | 10 | 81 | 0.0052 |
| | 11 | 80 | 0.0067 |
| | 12 | 79 | 0.0085 |
| | 13 | 78 | 0.0107 |
| | 14 | 77 | 0.0133 |
| | 15 | 76 | 0.0164 |
| | 16 | 75 | 0.0199 |
| | 17 | 74 | 0.0239 |
| | 18 | 73 | 0.0287 |
| | 19 | 72 | 0.0341 |
| | 20 | 71 | 0.0402 |
| | 21 | 70 | 0.0471 |
| | 22 | 69 | 0.0549 |
| | 23 | 68 | 0.0636 |
| | 24 | 67 | 0.0732 |
| | 25 | 66 | 0.0839 |
| | 26 | 65 | 0.0955 |
| | 27 | 64 | 0.1082 |
| | 28 | 63 | 0.1219 |
| | 29 | 62 | 0.1367 |
| | 30 | 61 | 0.1527 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 14 | 0 | 105 | 0.0001 |
| | 1 | 104 | 0.0001 |
| | 2 | 103 | 0.0002 |
| | 3 | 102 | 0.0003 |
| | 4 | 101 | 0.0004 |
| | 5 | 100 | 0.0006 |
| | 6 | 99 | 0.0009 |
| | 7 | 98 | 0.0012 |
| | 8 | 97 | 0.0015 |
| | 9 | 96 | 0.0020 |
| | 10 | 95 | 0.0026 |
| | 11 | 94 | 0.0034 |
| | 12 | 93 | 0.0043 |
| | 13 | 92 | 0.0054 |
| | 14 | 91 | 0.0067 |
| | 15 | 90 | 0.0083 |
| | 16 | 89 | 0.0101 |
| | 17 | 88 | 0.0123 |
| | 18 | 87 | 0.0148 |
| | 19 | 86 | 0.0176 |
| | 20 | 85 | 0.0209 |
| | 21 | 84 | 0.0247 |
| | 22 | 83 | 0.0290 |
| | 23 | 82 | 0.0338 |
| | 24 | 81 | 0.0392 |
| | 25 | 80 | 0.0453 |
| | 26 | 79 | 0.0520 |
| | 27 | 78 | 0.0594 |
| | 28 | 77 | 0.0676 |
| | 29 | 76 | 0.0765 |
| | 30 | 75 | 0.0863 |
| | 31 | 74 | 0.0969 |
| | 32 | 73 | 0.1083 |
| | 33 | 72 | 0.1206 |
| | 34 | 71 | 0.1338 |
| | 35 | 70 | 0.1479 |
| | 36 | 69 | 0.1629 |

**Table IX: Critical Values for the Wilcoxon Signed-Rank Statistic (Continued)**

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 15 | 0 | 120 | 0.0000 |
| | 1 | 119 | 0.0001 |
| | 2 | 118 | 0.0001 |
| | 3 | 117 | 0.0002 |
| | 4 | 116 | 0.0002 |
| | 5 | 115 | 0.0003 |
| | 6 | 114 | 0.0004 |
| | 7 | 113 | 0.0006 |
| | 8 | 112 | 0.0008 |
| | 9 | 111 | 0.0010 |
| | 10 | 110 | 0.0013 |
| | 11 | 109 | 0.0017 |
| | 12 | 108 | 0.0021 |
| | 13 | 107 | 0.0027 |
| | 14 | 106 | 0.0034 |
| | 15 | 105 | 0.0042 |
| | 16 | 104 | 0.0051 |
| | 17 | 103 | 0.0062 |
| | 18 | 102 | 0.0075 |
| | 19 | 101 | 0.0090 |
| | 20 | 100 | 0.0108 |
| | 21 | 99 | 0.0128 |
| | 22 | 98 | 0.0151 |
| | 23 | 97 | 0.0177 |
| | 24 | 96 | 0.0206 |
| | 25 | 95 | 0.0240 |
| | 26 | 94 | 0.0277 |
| | 27 | 93 | 0.0319 |
| | 28 | 92 | 0.0365 |
| | 29 | 91 | 0.0416 |
| | 30 | 90 | 0.0473 |
| | 31 | 89 | 0.0535 |
| | 32 | 88 | 0.0603 |
| | 33 | 87 | 0.0677 |
| | 34 | 86 | 0.0757 |
| | 35 | 85 | 0.0844 |
| | 36 | 84 | 0.0938 |
| | 37 | 83 | 0.1039 |
| | 38 | 82 | 0.1147 |
| | 39 | 81 | 0.1262 |
| | 40 | 80 | 0.1384 |
| | 41 | 79 | 0.1514 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 16 | 0 | 136 | 0.0000 |
| | 1 | 135 | 0.0000 |
| | 2 | 134 | 0.0000 |
| | 3 | 133 | 0.0001 |
| | 4 | 132 | 0.0001 |
| | 5 | 131 | 0.0002 |
| | 6 | 130 | 0.0002 |
| | 7 | 129 | 0.0003 |
| | 8 | 128 | 0.0004 |
| | 9 | 127 | 0.0005 |
| | 10 | 126 | 0.0007 |
| | 11 | 125 | 0.0008 |
| | 12 | 124 | 0.0011 |
| | 13 | 123 | 0.0013 |
| | 14 | 122 | 0.0017 |
| | 15 | 121 | 0.0021 |
| | 16 | 120 | 0.0026 |
| | 17 | 119 | 0.0031 |
| | 18 | 118 | 0.0038 |
| | 19 | 117 | 0.0046 |
| | 20 | 116 | 0.0055 |
| | 21 | 115 | 0.0065 |
| | 22 | 114 | 0.0078 |
| | 23 | 113 | 0.0091 |
| | 24 | 112 | 0.0107 |
| | 25 | 111 | 0.0125 |
| | 26 | 110 | 0.0145 |
| | 27 | 109 | 0.0168 |
| | 28 | 108 | 0.0193 |
| | 29 | 107 | 0.0222 |
| | 30 | 106 | 0.0253 |
| | 31 | 105 | 0.0288 |
| | 32 | 104 | 0.0327 |
| | 33 | 103 | 0.0370 |
| | 34 | 102 | 0.0416 |
| | 35 | 101 | 0.0467 |
| | 36 | 100 | 0.0523 |
| | 37 | 99 | 0.0583 |
| | 38 | 98 | 0.0649 |
| | 39 | 97 | 0.0719 |
| | 40 | 96 | 0.0795 |
| | 41 | 95 | 0.0877 |
| | 42 | 94 | 0.0964 |
| | 43 | 93 | 0.1057 |
| | 44 | 92 | 0.1156 |
| | 45 | 91 | 0.1261 |
| | 46 | 90 | 0.1372 |
| | 47 | 89 | 0.1489 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 17 | 0 | 153 | 0.0000 |
| | 1 | 152 | 0.0000 |
| | 2 | 151 | 0.0000 |
| | 3 | 150 | 0.0000 |
| | 4 | 149 | 0.0001 |
| | 5 | 148 | 0.0001 |
| | 6 | 147 | 0.0001 |
| | 7 | 146 | 0.0001 |
| | 8 | 145 | 0.0002 |
| | 9 | 144 | 0.0003 |
| | 10 | 143 | 0.0003 |
| | 11 | 142 | 0.0004 |
| | 12 | 141 | 0.0005 |
| | 13 | 140 | 0.0007 |
| | 14 | 139 | 0.0008 |
| | 15 | 138 | 0.0010 |
| | 16 | 137 | 0.0013 |
| | 17 | 136 | 0.0016 |
| | 18 | 135 | 0.0019 |
| | 19 | 134 | 0.0023 |
| | 20 | 133 | 0.0028 |
| | 21 | 132 | 0.0033 |
| | 22 | 131 | 0.0040 |
| | 23 | 130 | 0.0047 |
| | 24 | 129 | 0.0055 |
| | 25 | 128 | 0.0064 |
| | 26 | 127 | 0.0075 |
| | 27 | 126 | 0.0087 |
| | 28 | 125 | 0.0101 |
| | 29 | 124 | 0.0116 |
| | 30 | 123 | 0.0133 |
| | 31 | 122 | 0.0153 |
| | 32 | 121 | 0.0174 |
| | 33 | 120 | 0.0198 |
| | 34 | 119 | 0.0224 |
| | 35 | 118 | 0.0253 |
| | 36 | 117 | 0.0284 |
| | 37 | 116 | 0.0319 |
| | 38 | 115 | 0.0357 |
| | 39 | 114 | 0.0398 |
| | 40 | 113 | 0.0443 |
| | 41 | 112 | 0.0492 |
| | 42 | 111 | 0.0544 |
| | 43 | 110 | 0.0601 |
| | 44 | 109 | 0.0662 |
| | 45 | 108 | 0.0727 |
| | 46 | 107 | 0.0797 |
| | 47 | 106 | 0.0871 |
| | 48 | 105 | 0.0950 |
| | 49 | 104 | 0.1034 |
| | 50 | 103 | 0.1123 |
| | 51 | 102 | 0.1217 |
| | 52 | 101 | 0.1317 |
| | 53 | 100 | 0.1421 |
| | 54 | 99 | 0.1530 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 18 | 0 | 171 | 0.0000 |
| | 1 | 170 | 0.0000 |
| | 2 | 169 | 0.0000 |
| | 3 | 168 | 0.0000 |
| | 4 | 167 | 0.0000 |
| | 5 | 166 | 0.0000 |
| | 6 | 165 | 0.0001 |
| | 7 | 164 | 0.0001 |
| | 8 | 163 | 0.0001 |
| | 9 | 162 | 0.0001 |
| | 10 | 161 | 0.0002 |
| | 11 | 160 | 0.0002 |
| | 12 | 159 | 0.0003 |
| | 13 | 158 | 0.0003 |
| | 14 | 157 | 0.0004 |
| | 15 | 156 | 0.0005 |
| | 16 | 155 | 0.0006 |
| | 17 | 154 | 0.0008 |
| | 18 | 153 | 0.0010 |
| | 19 | 152 | 0.0012 |
| | 20 | 151 | 0.0014 |
| | 21 | 150 | 0.0017 |
| | 22 | 149 | 0.0020 |
| | 23 | 148 | 0.0024 |
| | 24 | 147 | 0.0028 |
| | 25 | 146 | 0.0033 |
| | 26 | 145 | 0.0038 |
| | 27 | 144 | 0.0045 |
| | 28 | 143 | 0.0052 |
| | 29 | 142 | 0.0060 |
| | 30 | 141 | 0.0069 |
| | 31 | 140 | 0.0080 |
| | 32 | 139 | 0.0091 |
| | 33 | 138 | 0.0104 |
| | 34 | 137 | 0.0118 |
| | 35 | 136 | 0.0134 |
| | 36 | 135 | 0.0152 |
| | 37 | 134 | 0.0171 |
| | 38 | 133 | 0.0192 |
| | 39 | 132 | 0.0216 |
| | 40 | 131 | 0.0241 |
| | 41 | 130 | 0.0269 |
| | 42 | 129 | 0.0300 |
| | 43 | 128 | 0.0333 |
| | 44 | 127 | 0.0368 |
| | 45 | 126 | 0.0407 |
| | 46 | 125 | 0.0449 |
| | 47 | 124 | 0.0494 |
| | 48 | 123 | 0.0542 |
| | 49 | 122 | 0.0594 |
| | 50 | 121 | 0.0649 |
| | 51 | 120 | 0.0708 |
| | 52 | 119 | 0.0770 |
| | 53 | 118 | 0.0837 |
| | 54 | 117 | 0.0907 |
| | 55 | 116 | 0.0982 |
| | 56 | 115 | 0.1061 |
| | 57 | 114 | 0.1144 |
| | 58 | 113 | 0.1231 |
| | 59 | 112 | 0.1323 |
| | 60 | 111 | 0.1419 |
| | 61 | 110 | 0.1519 |

**Table IX: Critical Values for the Wilcoxon Signed-Rank Statistic (Continued)**

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 19 | 0 | 190 | 0.0000 |
| | 1 | 189 | 0.0000 |
| | 2 | 188 | 0.0000 |
| | 3 | 187 | 0.0000 |
| | 4 | 186 | 0.0000 |
| | 5 | 185 | 0.0000 |
| | 6 | 184 | 0.0000 |
| | 7 | 183 | 0.0000 |
| | 8 | 182 | 0.0000 |
| | 9 | 181 | 0.0001 |
| | 10 | 180 | 0.0001 |
| | 11 | 179 | 0.0001 |
| | 12 | 178 | 0.0001 |
| | 13 | 177 | 0.0002 |
| | 14 | 176 | 0.0002 |
| | 15 | 175 | 0.0003 |
| | 16 | 174 | 0.0003 |
| | 17 | 173 | 0.0004 |
| | 18 | 172 | 0.0005 |
| | 19 | 171 | 0.0006 |
| | 20 | 170 | 0.0007 |
| | 21 | 169 | 0.0008 |
| | 22 | 168 | 0.0010 |
| | 23 | 167 | 0.0012 |
| | 24 | 166 | 0.0014 |
| | 25 | 165 | 0.0017 |
| | 26 | 164 | 0.0020 |
| | 27 | 163 | 0.0023 |
| | 28 | 162 | 0.0027 |
| | 29 | 161 | 0.0031 |
| | 30 | 160 | 0.0036 |
| | 31 | 159 | 0.0041 |
| | 32 | 158 | 0.0047 |
| | 33 | 157 | 0.0054 |
| | 34 | 156 | 0.0062 |
| | 35 | 155 | 0.0070 |
| | 36 | 154 | 0.0080 |
| | 37 | 153 | 0.0090 |
| | 38 | 152 | 0.0102 |
| | 39 | 151 | 0.0115 |
| | 40 | 150 | 0.0129 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 19 | 41 | 149 | 0.0145 |
| | 42 | 148 | 0.0162 |
| | 43 | 147 | 0.0180 |
| | 44 | 146 | 0.0201 |
| | 45 | 145 | 0.0223 |
| | 46 | 144 | 0.0247 |
| | 47 | 143 | 0.0273 |
| | 48 | 142 | 0.0301 |
| | 49 | 141 | 0.0331 |
| | 50 | 140 | 0.0364 |
| | 51 | 139 | 0.0399 |
| | 52 | 138 | 0.0437 |
| | 53 | 137 | 0.0478 |
| | 54 | 136 | 0.0521 |
| | 55 | 135 | 0.0567 |
| | 56 | 134 | 0.0616 |
| | 57 | 133 | 0.0668 |
| | 58 | 132 | 0.0723 |
| | 59 | 131 | 0.0782 |
| | 60 | 130 | 0.0844 |
| | 61 | 129 | 0.0909 |
| | 62 | 128 | 0.0978 |
| | 63 | 127 | 0.1051 |
| | 64 | 126 | 0.1127 |
| | 65 | 125 | 0.1206 |
| | 66 | 124 | 0.1290 |
| | 67 | 123 | 0.1377 |
| | 68 | 122 | 0.1467 |
| | 69 | 121 | 0.1562 |
| | 70 | 120 | 0.1660 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 20 | 0 | 210 | 0.0000 |
| | 1 | 209 | 0.0000 |
| | 2 | 208 | 0.0000 |
| | 3 | 207 | 0.0000 |
| | 4 | 206 | 0.0000 |
| | 5 | 205 | 0.0000 |
| | 6 | 204 | 0.0000 |
| | 7 | 203 | 0.0000 |
| | 8 | 202 | 0.0000 |
| | 9 | 201 | 0.0000 |
| | 10 | 200 | 0.0000 |
| | 11 | 199 | 0.0001 |
| | 12 | 198 | 0.0001 |
| | 13 | 197 | 0.0001 |
| | 14 | 196 | 0.0001 |
| | 15 | 195 | 0.0001 |
| | 16 | 194 | 0.0002 |
| | 17 | 193 | 0.0002 |
| | 18 | 192 | 0.0002 |
| | 19 | 191 | 0.0003 |
| | 20 | 190 | 0.0004 |
| | 21 | 189 | 0.0004 |
| | 22 | 188 | 0.0005 |
| | 23 | 187 | 0.0006 |
| | 24 | 186 | 0.0007 |
| | 25 | 185 | 0.0008 |
| | 26 | 184 | 0.0010 |
| | 27 | 183 | 0.0012 |
| | 28 | 182 | 0.0014 |
| | 29 | 181 | 0.0016 |
| | 30 | 180 | 0.0018 |
| | 31 | 179 | 0.0021 |
| | 32 | 178 | 0.0024 |
| | 33 | 177 | 0.0028 |
| | 34 | 176 | 0.0032 |
| | 35 | 175 | 0.0036 |
| | 36 | 174 | 0.0042 |
| | 37 | 173 | 0.0047 |
| | 38 | 172 | 0.0053 |
| | 39 | 171 | 0.0060 |
| | 40 | 170 | 0.0068 |

| $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|
| 20 | 41 | 169 | 0.0077 |
| | 42 | 168 | 0.0086 |
| | 43 | 167 | 0.0096 |
| | 44 | 166 | 0.0107 |
| | 45 | 165 | 0.0120 |
| | 46 | 164 | 0.0133 |
| | 47 | 163 | 0.0148 |
| | 48 | 162 | 0.0164 |
| | 49 | 161 | 0.0181 |
| | 50 | 160 | 0.0200 |
| | 51 | 159 | 0.0220 |
| | 52 | 158 | 0.0242 |
| | 53 | 157 | 0.0266 |
| | 54 | 156 | 0.0291 |
| | 55 | 155 | 0.0319 |
| | 56 | 154 | 0.0348 |
| | 57 | 153 | 0.0379 |
| | 58 | 152 | 0.0413 |
| | 59 | 151 | 0.0448 |
| | 60 | 150 | 0.0487 |
| | 61 | 149 | 0.0527 |
| | 62 | 148 | 0.0570 |
| | 63 | 147 | 0.0615 |
| | 64 | 146 | 0.0664 |
| | 65 | 145 | 0.0715 |
| | 66 | 144 | 0.0768 |
| | 67 | 143 | 0.0825 |
| | 68 | 142 | 0.0884 |
| | 69 | 141 | 0.0947 |
| | 70 | 140 | 0.1012 |
| | 71 | 139 | 0.1081 |
| | 72 | 138 | 0.1153 |
| | 73 | 137 | 0.1227 |
| | 74 | 136 | 0.1305 |
| | 75 | 135 | 0.1387 |
| | 76 | 134 | 0.1471 |
| | 77 | 133 | 0.1559 |

### Table X: Critical Values for the Wilcoxon Rank-Sum Statistic

This table contains critical values and probabilities for the Wilcoxon rank-sum statistic $W$ = the sum of the ranks of the $m$ observations in the smaller sample: $m$ and $n$ are the sample sizes, $c_1$ and $c_2$ are defined by $P(W \le c_1) = \alpha$ and $P(W \ge c_2) = \alpha$.

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 2 | 3 | 3 | 9 | 0.1000 |
| 2 | 4 | 3 | 11 | 0.0667 |
| | | 4 | 10 | 0.1333 |
| 2 | 5 | 3 | 13 | 0.0476 |
| | | 4 | 12 | 0.0952 |
| 2 | 6 | 3 | 15 | 0.0357 |
| | | 4 | 14 | 0.0714 |
| | | 5 | 13 | 0.1429 |
| 2 | 7 | 3 | 17 | 0.0278 |
| | | 4 | 16 | 0.0556 |
| | | 5 | 15 | 0.1111 |
| 2 | 8 | 3 | 19 | 0.0222 |
| | | 4 | 18 | 0.0444 |
| | | 5 | 17 | 0.0889 |
| | | 6 | 16 | 0.1333 |
| 2 | 9 | 3 | 21 | 0.0182 |
| | | 4 | 20 | 0.0364 |
| | | 5 | 19 | 0.0727 |
| | | 6 | 18 | 0.1091 |
| 2 | 10 | 3 | 23 | 0.0152 |
| | | 4 | 22 | 0.0303 |
| | | 5 | 21 | 0.0606 |
| | | 6 | 20 | 0.0909 |
| | | 7 | 19 | 0.1364 |
| 3 | 3 | 6 | 15 | 0.0500 |
| | | 7 | 14 | 0.1000 |
| 3 | 4 | 6 | 18 | 0.0286 |
| | | 7 | 17 | 0.0571 |
| | | 8 | 16 | 0.1143 |
| 3 | 5 | 6 | 21 | 0.0179 |
| | | 7 | 20 | 0.0357 |
| | | 8 | 19 | 0.0714 |
| | | 9 | 18 | 0.1250 |
| 3 | 6 | 6 | 24 | 0.0119 |
| | | 7 | 23 | 0.0238 |
| | | 8 | 22 | 0.0476 |
| | | 9 | 21 | 0.0833 |
| | | 10 | 20 | 0.1310 |
| 3 | 7 | 6 | 27 | 0.0083 |
| | | 7 | 26 | 0.0167 |
| | | 8 | 25 | 0.0333 |
| | | 9 | 24 | 0.0583 |
| | | 10 | 23 | 0.0917 |
| | | 11 | 22 | 0.1333 |
| 3 | 8 | 6 | 30 | 0.0061 |
| | | 7 | 29 | 0.0121 |
| | | 8 | 28 | 0.0242 |
| | | 9 | 27 | 0.0424 |
| | | 10 | 26 | 0.0667 |
| | | 11 | 25 | 0.0970 |
| | | 12 | 24 | 0.1394 |
| 3 | 9 | 6 | 33 | 0.0045 |
| | | 7 | 32 | 0.0091 |
| | | 8 | 31 | 0.0182 |
| | | 9 | 30 | 0.0318 |
| | | 10 | 29 | 0.0500 |
| | | 11 | 28 | 0.0727 |
| | | 12 | 27 | 0.1045 |
| | | 13 | 26 | 0.1409 |
| 3 | 10 | 6 | 36 | 0.0035 |
| | | 7 | 35 | 0.0070 |
| | | 8 | 34 | 0.0140 |
| | | 9 | 33 | 0.0245 |
| | | 10 | 32 | 0.0385 |
| | | 11 | 31 | 0.0559 |
| | | 12 | 30 | 0.0804 |
| | | 13 | 29 | 0.1084 |
| | | 14 | 28 | 0.1434 |
| 4 | 4 | 10 | 26 | 0.0143 |
| | | 11 | 25 | 0.0286 |
| | | 12 | 24 | 0.0571 |
| | | 13 | 23 | 0.1000 |
| 4 | 5 | 10 | 30 | 0.0079 |
| | | 11 | 29 | 0.0159 |
| | | 12 | 28 | 0.0317 |
| | | 13 | 27 | 0.0556 |
| | | 14 | 26 | 0.0952 |
| | | 15 | 25 | 0.1429 |
| 4 | 6 | 10 | 34 | 0.0048 |
| | | 11 | 33 | 0.0095 |
| | | 12 | 32 | 0.0190 |
| | | 13 | 31 | 0.0333 |
| | | 14 | 30 | 0.0571 |
| | | 15 | 29 | 0.0857 |
| | | 16 | 28 | 0.1286 |
| 4 | 7 | 10 | 38 | 0.0030 |
| | | 11 | 37 | 0.0061 |
| | | 12 | 36 | 0.0121 |
| | | 13 | 35 | 0.0212 |
| | | 14 | 34 | 0.0364 |
| | | 15 | 33 | 0.0545 |
| | | 16 | 32 | 0.0818 |
| | | 17 | 31 | 0.1152 |
| | | 18 | 30 | 0.1576 |
| 4 | 8 | 10 | 42 | 0.0020 |
| | | 11 | 41 | 0.0040 |
| | | 12 | 40 | 0.0081 |
| | | 13 | 39 | 0.0141 |
| | | 14 | 38 | 0.0242 |
| | | 15 | 37 | 0.0364 |
| | | 16 | 36 | 0.0545 |
| | | 17 | 35 | 0.0768 |
| | | 18 | 34 | 0.1071 |
| | | 19 | 33 | 0.1414 |
| 4 | 9 | 10 | 46 | 0.0014 |
| | | 11 | 45 | 0.0028 |
| | | 12 | 44 | 0.0056 |
| | | 13 | 43 | 0.0098 |
| | | 14 | 42 | 0.0168 |
| | | 15 | 41 | 0.0252 |
| | | 16 | 40 | 0.0378 |
| | | 17 | 39 | 0.0531 |
| | | 18 | 38 | 0.0741 |
| | | 19 | 37 | 0.0993 |
| | | 20 | 36 | 0.1301 |
| 4 | 10 | 10 | 50 | 0.0010 |
| | | 11 | 49 | 0.0020 |
| | | 12 | 48 | 0.0040 |
| | | 13 | 47 | 0.0070 |
| | | 14 | 46 | 0.0120 |
| | | 15 | 45 | 0.0180 |
| | | 16 | 44 | 0.0270 |
| | | 17 | 43 | 0.0380 |
| | | 18 | 42 | 0.0529 |
| | | 19 | 41 | 0.0709 |
| | | 20 | 40 | 0.0939 |
| | | 21 | 39 | 0.1199 |
| | | 22 | 38 | 0.1518 |
| 5 | 5 | 15 | 40 | 0.0040 |
| | | 16 | 39 | 0.0079 |
| | | 17 | 38 | 0.0159 |
| | | 18 | 37 | 0.0278 |
| | | 19 | 36 | 0.0476 |
| | | 20 | 35 | 0.0754 |
| | | 21 | 34 | 0.1111 |
| | | 22 | 33 | 0.1548 |
| 5 | 6 | 15 | 45 | 0.0022 |
| | | 16 | 44 | 0.0043 |
| | | 17 | 43 | 0.0087 |
| | | 18 | 42 | 0.0152 |
| | | 19 | 41 | 0.0260 |
| | | 20 | 40 | 0.0411 |
| | | 21 | 39 | 0.0628 |
| | | 22 | 38 | 0.0887 |
| | | 23 | 37 | 0.1234 |
| 5 | 7 | 15 | 50 | 0.0013 |
| | | 16 | 49 | 0.0025 |
| | | 17 | 48 | 0.0051 |
| | | 18 | 47 | 0.0088 |
| | | 19 | 46 | 0.0152 |
| | | 20 | 45 | 0.0240 |
| | | 21 | 44 | 0.0366 |
| | | 22 | 43 | 0.0530 |
| | | 23 | 42 | 0.0745 |
| | | 24 | 41 | 0.1010 |
| | | 25 | 40 | 0.1338 |
| 5 | 8 | 15 | 55 | 0.0008 |
| | | 16 | 54 | 0.0016 |
| | | 17 | 53 | 0.0031 |
| | | 18 | 52 | 0.0054 |
| | | 19 | 51 | 0.0093 |
| | | 20 | 50 | 0.0148 |
| | | 21 | 49 | 0.0225 |
| | | 22 | 48 | 0.0326 |
| | | 23 | 47 | 0.0466 |
| | | 24 | 46 | 0.0637 |
| | | 25 | 45 | 0.0855 |
| | | 26 | 44 | 0.1111 |
| | | 27 | 43 | 0.1422 |

**Table X: Critical Values for the Wilcoxon Rank-Sum Statistic (Continued)**

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 5 | 9 | 15 | 60 | 0.0005 |
| | | 16 | 59 | 0.0010 |
| | | 17 | 58 | 0.0020 |
| | | 18 | 57 | 0.0035 |
| | | 19 | 56 | 0.0060 |
| | | 20 | 55 | 0.0095 |
| | | 21 | 54 | 0.0145 |
| | | 22 | 53 | 0.0210 |
| | | 23 | 52 | 0.0300 |
| | | 24 | 51 | 0.0415 |
| | | 25 | 50 | 0.0559 |
| | | 26 | 49 | 0.0734 |
| | | 27 | 48 | 0.0949 |
| | | 28 | 47 | 0.1199 |
| | | 29 | 46 | 0.1489 |
| 5 | 10 | 15 | 65 | 0.0003 |
| | | 16 | 64 | 0.0007 |
| | | 17 | 63 | 0.0013 |
| | | 18 | 62 | 0.0023 |
| | | 19 | 61 | 0.0040 |
| | | 20 | 60 | 0.0063 |
| | | 21 | 59 | 0.0097 |
| | | 22 | 58 | 0.0140 |
| | | 23 | 57 | 0.0200 |
| | | 24 | 56 | 0.0276 |
| | | 25 | 55 | 0.0376 |
| | | 26 | 54 | 0.0496 |
| | | 27 | 53 | 0.0646 |
| | | 28 | 52 | 0.0823 |
| | | 29 | 51 | 0.1032 |
| | | 30 | 50 | 0.1272 |
| | | 31 | 49 | 0.1548 |
| 6 | 6 | 21 | 57 | 0.0011 |
| | | 22 | 56 | 0.0022 |
| | | 23 | 55 | 0.0043 |
| | | 24 | 54 | 0.0076 |
| | | 25 | 53 | 0.0130 |
| | | 26 | 52 | 0.0206 |
| | | 27 | 51 | 0.0325 |
| | | 28 | 50 | 0.0465 |
| | | 29 | 49 | 0.0660 |
| | | 30 | 48 | 0.0898 |
| | | 31 | 47 | 0.1201 |
| | | 32 | 46 | 0.1548 |

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 6 | 7 | 21 | 63 | 0.0006 |
| | | 22 | 62 | 0.0012 |
| | | 23 | 61 | 0.0023 |
| | | 24 | 60 | 0.0041 |
| | | 25 | 59 | 0.0070 |
| | | 26 | 58 | 0.0111 |
| | | 27 | 57 | 0.0175 |
| | | 28 | 56 | 0.0256 |
| | | 29 | 55 | 0.0367 |
| | | 30 | 54 | 0.0507 |
| | | 31 | 53 | 0.0688 |
| | | 32 | 52 | 0.0903 |
| | | 33 | 51 | 0.1171 |
| | | 34 | 50 | 0.1474 |
| 6 | 8 | 21 | 69 | 0.0003 |
| | | 22 | 68 | 0.0007 |
| | | 23 | 67 | 0.0013 |
| | | 24 | 66 | 0.0023 |
| | | 25 | 65 | 0.0040 |
| | | 26 | 64 | 0.0063 |
| | | 27 | 63 | 0.0100 |
| | | 28 | 62 | 0.0147 |
| | | 29 | 61 | 0.0213 |
| | | 30 | 60 | 0.0296 |
| | | 31 | 59 | 0.0406 |
| | | 32 | 58 | 0.0539 |
| | | 33 | 57 | 0.0709 |
| | | 34 | 56 | 0.0906 |
| | | 35 | 55 | 0.1142 |
| | | 36 | 54 | 0.1412 |
| 6 | 9 | 21 | 75 | 0.0002 |
| | | 22 | 74 | 0.0004 |
| | | 23 | 73 | 0.0008 |
| | | 24 | 72 | 0.0014 |
| | | 25 | 71 | 0.0024 |
| | | 26 | 70 | 0.0038 |
| | | 27 | 69 | 0.0060 |
| | | 28 | 68 | 0.0088 |
| | | 29 | 67 | 0.0128 |
| | | 30 | 66 | 0.0180 |
| | | 31 | 65 | 0.0248 |
| | | 32 | 64 | 0.0332 |
| | | 33 | 63 | 0.0440 |
| | | 34 | 62 | 0.0567 |
| | | 35 | 61 | 0.0723 |
| | | 36 | 60 | 0.0905 |
| | | 37 | 59 | 0.1119 |
| | | 38 | 58 | 0.1361 |

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 6 | 10 | 21 | 81 | 0.0001 |
| | | 22 | 80 | 0.0002 |
| | | 23 | 79 | 0.0005 |
| | | 24 | 78 | 0.0009 |
| | | 25 | 77 | 0.0015 |
| | | 26 | 76 | 0.0024 |
| | | 27 | 75 | 0.0037 |
| | | 28 | 74 | 0.0055 |
| | | 29 | 73 | 0.0080 |
| | | 30 | 72 | 0.0112 |
| | | 31 | 71 | 0.0156 |
| | | 32 | 70 | 0.0210 |
| | | 33 | 69 | 0.0280 |
| | | 34 | 68 | 0.0363 |
| | | 35 | 67 | 0.0467 |
| | | 36 | 66 | 0.0589 |
| | | 37 | 65 | 0.0736 |
| | | 38 | 64 | 0.0903 |
| | | 39 | 63 | 0.1099 |
| | | 40 | 62 | 0.1317 |
| | | 41 | 61 | 0.1566 |
| 7 | 7 | 28 | 77 | 0.0003 |
| | | 29 | 76 | 0.0006 |
| | | 30 | 75 | 0.0012 |
| | | 31 | 74 | 0.0020 |
| | | 32 | 73 | 0.0035 |
| | | 33 | 72 | 0.0055 |
| | | 34 | 71 | 0.0087 |
| | | 35 | 70 | 0.0131 |
| | | 36 | 69 | 0.0189 |
| | | 37 | 68 | 0.0265 |
| | | 38 | 67 | 0.0364 |
| | | 39 | 66 | 0.0487 |
| | | 40 | 65 | 0.0641 |
| | | 41 | 64 | 0.0825 |
| | | 42 | 63 | 0.1043 |
| | | 43 | 62 | 0.1297 |
| | | 44 | 61 | 0.1588 |

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 7 | 8 | 28 | 84 | 0.0002 |
| | | 29 | 83 | 0.0003 |
| | | 30 | 82 | 0.0006 |
| | | 31 | 81 | 0.0011 |
| | | 32 | 80 | 0.0019 |
| | | 33 | 79 | 0.0030 |
| | | 34 | 78 | 0.0047 |
| | | 35 | 77 | 0.0070 |
| | | 36 | 76 | 0.0103 |
| | | 37 | 75 | 0.0145 |
| | | 38 | 74 | 0.0200 |
| | | 39 | 73 | 0.0270 |
| | | 40 | 72 | 0.0361 |
| | | 41 | 71 | 0.0469 |
| | | 42 | 70 | 0.0603 |
| | | 43 | 69 | 0.0760 |
| | | 44 | 68 | 0.0946 |
| | | 45 | 67 | 0.1159 |
| | | 46 | 66 | 0.1405 |
| 7 | 9 | 28 | 91 | 0.0001 |
| | | 29 | 90 | 0.0002 |
| | | 30 | 89 | 0.0003 |
| | | 31 | 88 | 0.0006 |
| | | 32 | 87 | 0.0010 |
| | | 33 | 86 | 0.0017 |
| | | 34 | 85 | 0.0026 |
| | | 35 | 84 | 0.0039 |
| | | 36 | 83 | 0.0058 |
| | | 37 | 82 | 0.0082 |
| | | 38 | 81 | 0.0115 |
| | | 39 | 80 | 0.0156 |
| | | 40 | 79 | 0.0209 |
| | | 41 | 78 | 0.0274 |
| | | 42 | 77 | 0.0356 |
| | | 43 | 76 | 0.0454 |
| | | 44 | 75 | 0.0571 |
| | | 45 | 74 | 0.0708 |
| | | 46 | 73 | 0.0869 |
| | | 47 | 72 | 0.1052 |
| | | 48 | 71 | 0.1261 |
| | | 49 | 70 | 0.1496 |

**Table X: Critical Values for the Wilcoxon Rank-Sum Statistic (Continued)**

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 7 | 10 | 28 | 98 | 0.0001 |
| | | 29 | 97 | 0.0001 |
| | | 30 | 96 | 0.0002 |
| | | 31 | 95 | 0.0004 |
| | | 32 | 94 | 0.0006 |
| | | 33 | 93 | 0.0010 |
| | | 34 | 92 | 0.0015 |
| | | 35 | 91 | 0.0023 |
| | | 36 | 90 | 0.0034 |
| | | 37 | 89 | 0.0048 |
| | | 38 | 88 | 0.0068 |
| | | 39 | 87 | 0.0093 |
| | | 40 | 86 | 0.0125 |
| | | 41 | 85 | 0.0165 |
| | | 42 | 84 | 0.0215 |
| | | 43 | 83 | 0.0277 |
| | | 44 | 82 | 0.0351 |
| | | 45 | 81 | 0.0439 |
| | | 46 | 80 | 0.0544 |
| | | 47 | 79 | 0.0665 |
| | | 48 | 78 | 0.0806 |
| | | 49 | 77 | 0.0966 |
| | | 50 | 76 | 0.1148 |
| | | 51 | 75 | 0.1349 |
| | | 52 | 74 | 0.1574 |
| 8 | 8 | 36 | 100 | 0.0001 |
| | | 37 | 99 | 0.0002 |
| | | 38 | 98 | 0.0003 |
| | | 39 | 97 | 0.0005 |
| | | 40 | 96 | 0.0009 |
| | | 41 | 95 | 0.0015 |
| | | 42 | 94 | 0.0023 |
| | | 43 | 93 | 0.0035 |
| | | 44 | 92 | 0.0052 |
| | | 45 | 91 | 0.0074 |
| | | 46 | 90 | 0.0103 |
| | | 47 | 89 | 0.0141 |
| | | 48 | 88 | 0.0190 |
| | | 49 | 87 | 0.0249 |
| | | 50 | 86 | 0.0325 |
| | | 51 | 85 | 0.0415 |
| | | 52 | 84 | 0.0524 |
| | | 53 | 83 | 0.0652 |
| | | 54 | 82 | 0.0803 |
| | | 55 | 81 | 0.0974 |
| | | 56 | 80 | 0.1172 |
| | | 57 | 79 | 0.1393 |

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 8 | 9 | 36 | 108 | 0.0000 |
| | | 37 | 107 | 0.0001 |
| | | 38 | 106 | 0.0002 |
| | | 39 | 105 | 0.0003 |
| | | 40 | 104 | 0.0005 |
| | | 41 | 103 | 0.0008 |
| | | 42 | 102 | 0.0012 |
| | | 43 | 101 | 0.0019 |
| | | 44 | 100 | 0.0028 |
| | | 45 | 99 | 0.0039 |
| | | 46 | 98 | 0.0056 |
| | | 47 | 97 | 0.0076 |
| | | 48 | 96 | 0.0103 |
| | | 49 | 95 | 0.0137 |
| | | 50 | 94 | 0.0180 |
| | | 51 | 93 | 0.0232 |
| | | 52 | 92 | 0.0296 |
| | | 53 | 91 | 0.0372 |
| | | 54 | 90 | 0.0464 |
| | | 55 | 89 | 0.0570 |
| | | 56 | 88 | 0.0694 |
| | | 57 | 87 | 0.0836 |
| | | 58 | 86 | 0.0998 |
| | | 59 | 85 | 0.1179 |
| | | 60 | 84 | 0.1383 |
| 8 | 10 | 36 | 116 | 0.0000 |
| | | 37 | 115 | 0.0000 |
| | | 38 | 114 | 0.0001 |
| | | 39 | 113 | 0.0002 |
| | | 40 | 112 | 0.0003 |
| | | 41 | 111 | 0.0004 |
| | | 42 | 110 | 0.0007 |
| | | 43 | 109 | 0.0010 |
| | | 44 | 108 | 0.0015 |
| | | 45 | 107 | 0.0022 |
| | | 46 | 106 | 0.0031 |
| | | 47 | 105 | 0.0043 |
| | | 48 | 104 | 0.0058 |
| | | 49 | 103 | 0.0078 |
| | | 50 | 102 | 0.0103 |
| | | 51 | 101 | 0.0133 |
| | | 52 | 100 | 0.0171 |
| | | 53 | 99 | 0.0217 |
| | | 54 | 98 | 0.0273 |
| | | 55 | 97 | 0.0338 |
| | | 56 | 96 | 0.0416 |
| | | 57 | 95 | 0.0506 |
| | | 58 | 94 | 0.0610 |
| | | 59 | 93 | 0.0729 |
| | | 60 | 92 | 0.0864 |
| | | 61 | 91 | 0.1015 |
| | | 62 | 90 | 0.1185 |
| | | 63 | 89 | 0.1371 |
| | | 64 | 88 | 0.1577 |

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 9 | 9 | 45 | 126 | 0.0000 |
| | | 46 | 125 | 0.0000 |
| | | 47 | 124 | 0.0001 |
| | | 48 | 123 | 0.0001 |
| | | 49 | 122 | 0.0002 |
| | | 50 | 121 | 0.0004 |
| | | 51 | 120 | 0.0006 |
| | | 52 | 119 | 0.0009 |
| | | 53 | 118 | 0.0014 |
| | | 54 | 117 | 0.0020 |
| | | 55 | 116 | 0.0028 |
| | | 56 | 115 | 0.0039 |
| | | 57 | 114 | 0.0053 |
| | | 58 | 113 | 0.0071 |
| | | 59 | 112 | 0.0094 |
| | | 60 | 111 | 0.0122 |
| | | 61 | 110 | 0.0157 |
| | | 62 | 109 | 0.0200 |
| | | 63 | 108 | 0.0252 |
| | | 64 | 107 | 0.0313 |
| | | 65 | 106 | 0.0385 |
| | | 66 | 105 | 0.0470 |
| | | 67 | 104 | 0.0567 |
| | | 68 | 103 | 0.0680 |
| | | 69 | 102 | 0.0807 |
| | | 70 | 101 | 0.0951 |
| | | 71 | 100 | 0.1112 |
| | | 72 | 99 | 0.1290 |
| 9 | 10 | 45 | 135 | 0.0000 |
| | | 46 | 134 | 0.0000 |
| | | 47 | 133 | 0.0000 |
| | | 48 | 132 | 0.0001 |
| | | 49 | 131 | 0.0001 |
| | | 50 | 130 | 0.0002 |
| | | 51 | 129 | 0.0003 |
| | | 52 | 128 | 0.0005 |
| | | 53 | 127 | 0.0007 |
| | | 54 | 126 | 0.0011 |
| | | 55 | 125 | 0.0015 |
| | | 56 | 124 | 0.0021 |
| | | 57 | 123 | 0.0028 |
| | | 58 | 122 | 0.0038 |
| | | 59 | 121 | 0.0051 |
| | | 60 | 120 | 0.0066 |
| | | 61 | 119 | 0.0086 |
| | | 62 | 118 | 0.0110 |
| | | 63 | 117 | 0.0140 |
| | | 64 | 116 | 0.0175 |
| | | 65 | 115 | 0.0217 |
| | | 66 | 114 | 0.0267 |
| | | 67 | 113 | 0.0326 |
| | | 68 | 112 | 0.0394 |
| | | 69 | 111 | 0.0474 |
| | | 70 | 110 | 0.0564 |
| | | 71 | 109 | 0.0667 |
| | | 72 | 108 | 0.0782 |
| | | 73 | 107 | 0.0912 |
| | | 74 | 106 | 0.1055 |
| | | 75 | 105 | 0.1214 |
| | | 76 | 104 | 0.1388 |

| $m$ | $n$ | $c_1$ | $c_2$ | $\alpha$ |
|---|---|---|---|---|
| 10 | 10 | 55 | 155 | 0.0000 |
| | | 56 | 154 | 0.0000 |
| | | 57 | 153 | 0.0000 |
| | | 58 | 152 | 0.0000 |
| | | 59 | 151 | 0.0001 |
| | | 60 | 150 | 0.0001 |
| | | 61 | 149 | 0.0002 |
| | | 62 | 148 | 0.0002 |
| | | 63 | 147 | 0.0004 |
| | | 64 | 146 | 0.0005 |
| | | 65 | 145 | 0.0008 |
| | | 66 | 144 | 0.0010 |
| | | 67 | 143 | 0.0014 |
| | | 68 | 142 | 0.0019 |
| | | 69 | 141 | 0.0026 |
| | | 70 | 140 | 0.0034 |
| | | 71 | 139 | 0.0045 |
| | | 72 | 138 | 0.0057 |
| | | 73 | 137 | 0.0073 |
| | | 74 | 136 | 0.0093 |
| | | 75 | 135 | 0.0116 |
| | | 76 | 134 | 0.0144 |
| | | 77 | 133 | 0.0177 |
| | | 78 | 132 | 0.0216 |
| | | 79 | 131 | 0.0262 |
| | | 80 | 130 | 0.0315 |
| | | 81 | 129 | 0.0376 |
| | | 82 | 128 | 0.0446 |
| | | 83 | 127 | 0.0526 |
| | | 84 | 126 | 0.0615 |
| | | 85 | 125 | 0.0716 |
| | | 86 | 124 | 0.0827 |
| | | 87 | 123 | 0.0952 |
| | | 88 | 122 | 0.1088 |
| | | 89 | 121 | 0.1237 |
| | | 90 | 120 | 0.1399 |
| | | 91 | 119 | 0.1575 |

**Table XI: Critical Values for the Runs Test**

This table contains cumulative probabilities associated with the runs test. Let $m$ be the number of observations in one category and $n$ the number of observations in the other category ($m \leq n$), and $V$ be the number of runs. The values in this table are the probabilities $\mathrm{P}(V \leq v)$ if the order of observations is random.

| | | $v$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $m$ | $n$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 2 | 0.3333 | 0.6667 | 1.0000 | | | | | |
| 2 | 3 | 0.2000 | 0.5000 | 0.9000 | 1.0000 | | | | |
| 2 | 4 | 0.1333 | 0.4000 | 0.8000 | 1.0000 | | | | |
| 2 | 5 | 0.0952 | 0.3333 | 0.7143 | 1.0000 | | | | |
| 2 | 6 | 0.0714 | 0.2857 | 0.6429 | 1.0000 | | | | |
| 2 | 7 | 0.0556 | 0.2500 | 0.5833 | 1.0000 | | | | |
| 2 | 8 | 0.0444 | 0.2222 | 0.5333 | 1.0000 | | | | |
| 2 | 9 | 0.0364 | 0.2000 | 0.4909 | 1.0000 | | | | |
| 2 | 10 | 0.0303 | 0.1818 | 0.4545 | 1.0000 | | | | |
| 3 | 3 | 0.1000 | 0.3000 | 0.7000 | 0.9000 | 1.0000 | | | |
| 3 | 4 | 0.0571 | 0.2000 | 0.5429 | 0.8000 | 0.9714 | 1.0000 | | |
| 3 | 5 | 0.0357 | 0.1429 | 0.4286 | 0.7143 | 0.9286 | 1.0000 | | |
| 3 | 6 | 0.0238 | 0.1071 | 0.3452 | 0.6429 | 0.8810 | 1.0000 | | |
| 3 | 7 | 0.0167 | 0.0833 | 0.2833 | 0.5833 | 0.8333 | 1.0000 | | |
| 3 | 8 | 0.0121 | 0.0667 | 0.2364 | 0.5333 | 0.7879 | 1.0000 | | |
| 3 | 9 | 0.0091 | 0.0545 | 0.2000 | 0.4909 | 0.7455 | 1.0000 | | |
| 3 | 10 | 0.0070 | 0.0455 | 0.1713 | 0.4545 | 0.7063 | 1.0000 | | |
| 4 | 4 | 0.0286 | 0.1143 | 0.3714 | 0.6286 | 0.8857 | 0.9714 | 1.0000 | |
| 4 | 5 | 0.0159 | 0.0714 | 0.2619 | 0.5000 | 0.7857 | 0.9286 | 0.9921 | 1.0000 |
| 4 | 6 | 0.0095 | 0.0476 | 0.1905 | 0.4048 | 0.6905 | 0.8810 | 0.9762 | 1.0000 |
| 4 | 7 | 0.0061 | 0.0333 | 0.1424 | 0.3333 | 0.6061 | 0.8333 | 0.9545 | 1.0000 |
| 4 | 8 | 0.0040 | 0.0242 | 0.1091 | 0.2788 | 0.5333 | 0.7879 | 0.9293 | 1.0000 |
| 4 | 9 | 0.0028 | 0.0182 | 0.0853 | 0.2364 | 0.4713 | 0.7455 | 0.9021 | 1.0000 |
| 4 | 10 | 0.0020 | 0.0140 | 0.0679 | 0.2028 | 0.4186 | 0.7063 | 0.8741 | 1.0000 |

**Table XI: Critical Values for the Runs Test (Continued)**

| | | $v$ | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m$ | $n$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 5 | 5 | 0.0079 | 0.0397 | 0.1667 | 0.3571 | 0.6429 | 0.8333 | 0.9603 | 0.9921 | 1.0000 | | | | | | | | | | |
| 5 | 6 | 0.0043 | 0.0238 | 0.1104 | 0.2619 | 0.5216 | 0.7381 | 0.9113 | 0.9762 | 0.9978 | 1.0000 | | | | | | | | | |
| 5 | 7 | 0.0025 | 0.0152 | 0.0758 | 0.1970 | 0.4242 | 0.6515 | 0.8535 | 0.9545 | 0.9924 | 1.0000 | | | | | | | | | |
| 5 | 8 | 0.0016 | 0.0101 | 0.0536 | 0.1515 | 0.3473 | 0.5758 | 0.7933 | 0.9293 | 0.9837 | 1.0000 | | | | | | | | | |
| 5 | 9 | 0.0010 | 0.0070 | 0.0390 | 0.1189 | 0.2867 | 0.5105 | 0.7343 | 0.9021 | 0.9720 | 1.0000 | | | | | | | | | |
| 5 | 10 | 0.0007 | 0.0050 | 0.0290 | 0.0949 | 0.2388 | 0.4545 | 0.6783 | 0.8741 | 0.9580 | 1.0000 | | | | | | | | | |
| 6 | 6 | 0.0022 | 0.0130 | 0.0671 | 0.1753 | 0.3918 | 0.6082 | 0.8247 | 0.9329 | 0.9870 | 0.9978 | 1.0000 | | | | | | | | |
| 6 | 7 | 0.0012 | 0.0076 | 0.0425 | 0.1212 | 0.2960 | 0.5000 | 0.7331 | 0.8788 | 0.9662 | 0.9924 | 0.9994 | 1.0000 | | | | | | | |
| 6 | 8 | 0.0007 | 0.0047 | 0.0280 | 0.0862 | 0.2261 | 0.4126 | 0.6457 | 0.8205 | 0.9371 | 0.9837 | 0.9977 | 1.0000 | | | | | | | |
| 6 | 9 | 0.0004 | 0.0030 | 0.0190 | 0.0629 | 0.1748 | 0.3427 | 0.5664 | 0.7622 | 0.9021 | 0.9720 | 0.9944 | 1.0000 | | | | | | | |
| 6 | 10 | 0.0002 | 0.0020 | 0.0132 | 0.0470 | 0.1369 | 0.2867 | 0.4965 | 0.7063 | 0.8636 | 0.9580 | 0.9895 | 1.0000 | | | | | | | |
| 7 | 7 | 0.0006 | 0.0041 | 0.0251 | 0.0775 | 0.2086 | 0.3834 | 0.6166 | 0.7914 | 0.9225 | 0.9749 | 0.9959 | 0.9994 | 1.0000 | | | | | | |
| 7 | 8 | 0.0003 | 0.0023 | 0.0154 | 0.0513 | 0.1492 | 0.2960 | 0.5136 | 0.7040 | 0.8671 | 0.9487 | 0.9879 | 0.9977 | 0.9998 | 1.0000 | | | | | |
| 7 | 9 | 0.0002 | 0.0014 | 0.0098 | 0.0350 | 0.1084 | 0.2308 | 0.4266 | 0.6224 | 0.8059 | 0.9161 | 0.9748 | 0.9944 | 0.9993 | 1.0000 | | | | | |
| 7 | 10 | 0.0001 | 0.0009 | 0.0064 | 0.0245 | 0.0800 | 0.1818 | 0.3546 | 0.5490 | 0.7433 | 0.8794 | 0.9571 | 0.9895 | 0.9981 | 1.0000 | | | | | |
| 8 | 8 | 0.0002 | 0.0012 | 0.0089 | 0.0317 | 0.1002 | 0.2145 | 0.4048 | 0.5952 | 0.7855 | 0.8998 | 0.9683 | 0.9911 | 0.9988 | 0.9998 | 1.0000 | | | | |
| 8 | 9 | 0.0001 | 0.0007 | 0.0053 | 0.0203 | 0.0687 | 0.1573 | 0.3186 | 0.5000 | 0.7016 | 0.8427 | 0.9394 | 0.9797 | 0.9958 | 0.9993 | 1.0000 | 1.0000 | | | |
| 8 | 10 | 0.0000 | 0.0004 | 0.0033 | 0.0134 | 0.0479 | 0.1170 | 0.2514 | 0.4194 | 0.6209 | 0.7822 | 0.9031 | 0.9636 | 0.9905 | 0.9981 | 0.9998 | 1.0000 | | | |
| 9 | 9 | 0.0000 | 0.0004 | 0.0030 | 0.0122 | 0.0445 | 0.1090 | 0.2380 | 0.3992 | 0.6008 | 0.7620 | 0.8910 | 0.9555 | 0.9878 | 0.9970 | 0.9996 | 1.0000 | 1.0000 | | |
| 9 | 10 | 0.0000 | 0.0002 | 0.0018 | 0.0076 | 0.0294 | 0.0767 | 0.1786 | 0.3186 | 0.5095 | 0.6814 | 0.8342 | 0.9233 | 0.9742 | 0.9924 | 0.9986 | 0.9998 | 1.0000 | 1.0000 | |
| 10 | 10 | 0.0000 | 0.0001 | 0.0010 | 0.0045 | 0.0185 | 0.0513 | 0.1276 | 0.2422 | 0.4141 | 0.5859 | 0.7578 | 0.8724 | 0.9487 | 0.9815 | 0.9955 | 0.9990 | 0.9999 | 1.0000 | 1.0000 |