Chapter 17. Inference About a Population Mean

Topics covered in this chapter:

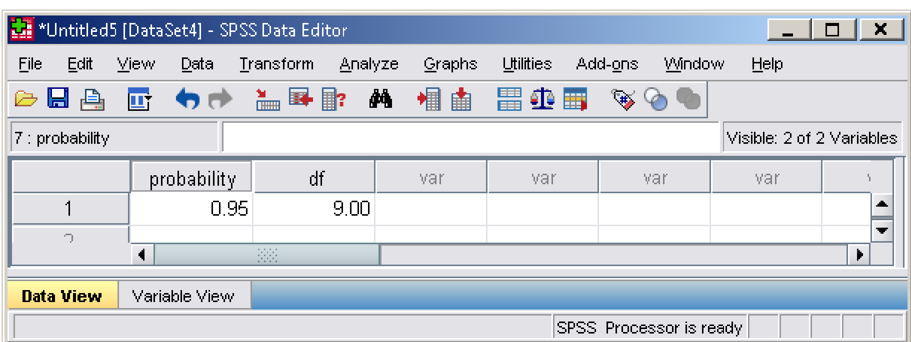
- **Finding t-Critical Values**
- **Confidence Interval**
- **One Sample t-Test**
- **Matched Pairs t Procedure**

Finding t-Critical Values

Example 17.1: t critical values

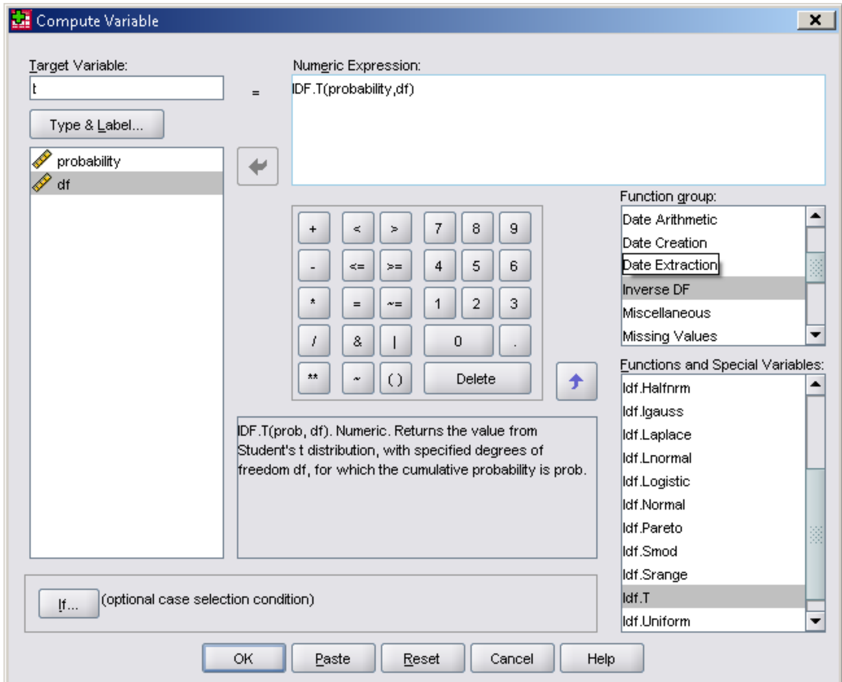
The Problem: Given a one-sided P-value of 0.05 and the degrees of freedom of 9, find the critical value using SPSS.

1. Find the critical value.
   a. Click on **Variable View**.
   b. Enter two variables, named ***probability*** and ***df***.
   c. Click on **Data View**.
   d. In the ***probability*** column in row 1, type “0.95”.
   e. In the ***df*** column in row 1, type “9”.



2. Compute the critical value.
   a. Click on **Transform**.
   b. Click on **Compute Variable**.
   c. Under **Target Variable**, type “t”, which corresponds to the t-critical value.
   d. Under **Function group**, click on **Inverse DF**.
   e. Choose and double-click on **Idf.T**.
   f. For the two bounds, replace (?,?) with (probability,df).

g. Click on **OK**.



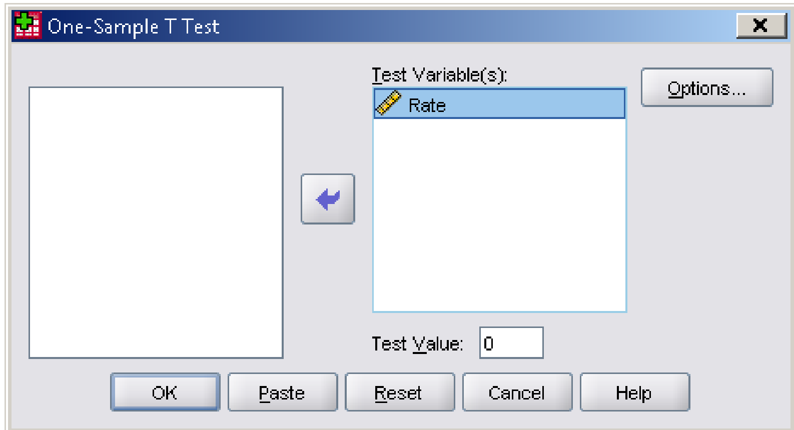
The new variable is created, and we can see the t-critical value is 1.83.

Confidence Interval

Example 17.2: Healing of skin wounds

The problem: This example revisits Example 14.3. A 95% confidence interval will be constructed using the given data and n - 1 degrees of freedom.

1. Construct the 95% confidence interval.
   a. Open the file *eg17-02.por*.
   b. Click on **Analyze**.
   c. Click on **Compare Means**.
   d. Click on **One-Sample T Test**.
   e. Choose ***Rate*** as the **Test Variable**.



f. Click on **OK**.

A new window, an SPSS Viewer will open in a separate window. The **One-Sample Statistics** and **One-Sample Test** are the output for this function. The 95% confidence interval can be found in the **One-Sample Test**, given as (21.53, 29.81) in micrometers per hour.

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Rate	18	25.67	8.324	1.962

One-Sample Test

	Test Value = 0					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Rate	13.081	17	.000	25.667	21.53	29.81

Look back at Chapter 14 to see the difference between the confidence interval in Example 14.3 using the Normal distribution versus this example using the t distribution with 17 degrees of freedom.

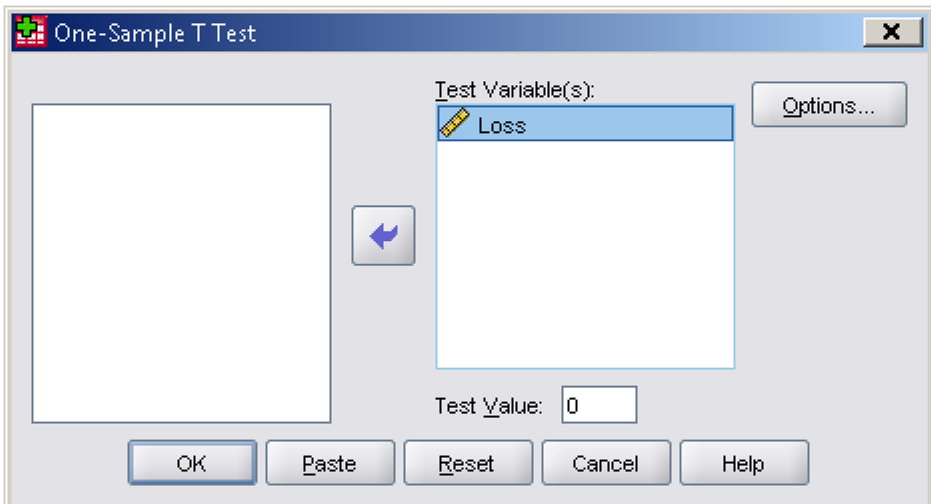
One Sample t-Test

Example 17.3: Sweetening Colas

The Problem: This example revisits the cola-sweetening example from chapter 14. We will test the following hypotheses about the average loss of sweetness:

H_0: $\mu = 0$
H_a: $\mu > 0$

1. Open the file *eg17-03.por*.
2. Click on **Analyze**.
3. Click on **Compare Means**.
4. Click on **One-Sample T Test**.
5. Choose ***Loss*** as the **Test Variable**.



6. Click **OK**.
7. The test statistic and the P-value will be found in the **One-Sample Test** output in the **SPSS Viewer** window.

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
Loss	10	1.0200	1.19610	.37824

One-Sample Test

	Test Value = 0					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Loss	2.697	9	.025	1.02000	.1644	1.8756

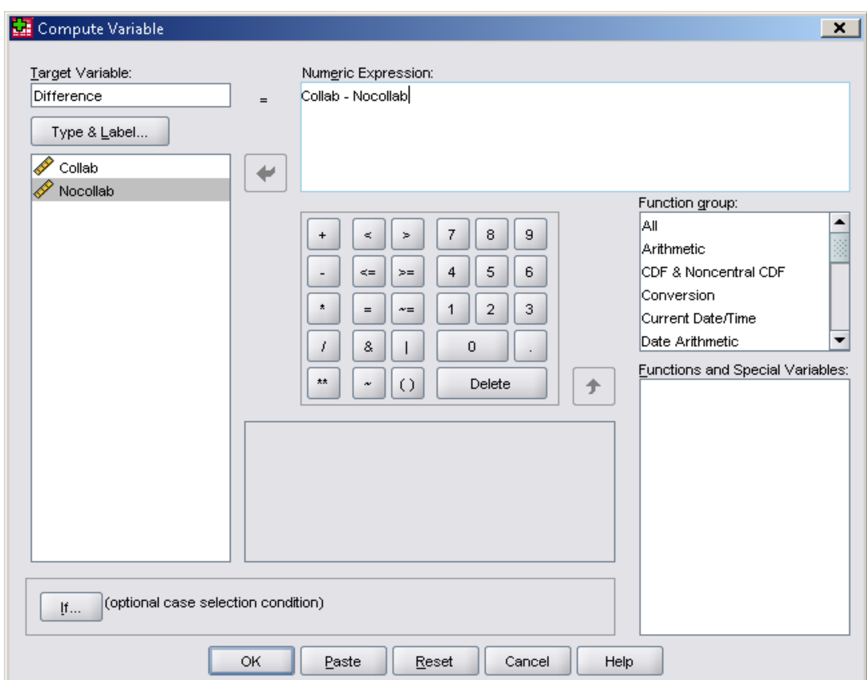
In this case the test statistic is t = 2.697. The p-value given is for the 2-tailed hypothesis test. Since our problem was only interested in the upper tail, we can divide the SPSS value by 2. The p-value = .025/2 = .0125.

Matched Pairs t Procedure

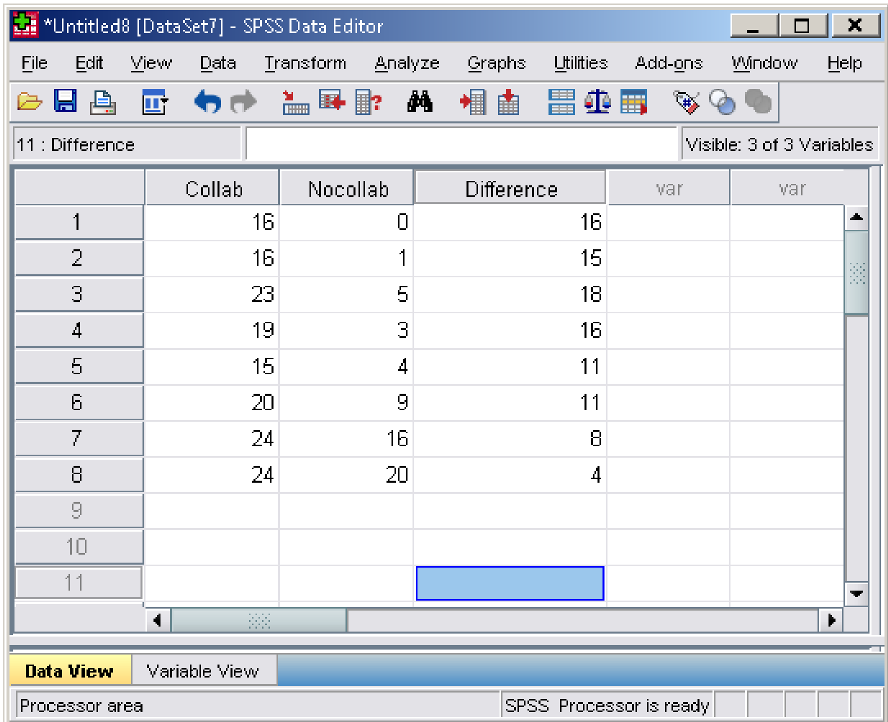
Example 17.4: Do Chimpanzees Collaborate?

The Problem: Humans often collaborate to solve problems. Will chimpanzees recruit another chimp when solving a problem requires collaboration? Researchers presented chimpanzee subjects with food outside their cage that they could bring within reach by pulling two ropes, one attached to each end of the food tray. If a chimp pulled only one rope, the rope came loose and the food was lost. Another chimp was available as a partner, but only if the subject unlocked a door joining two cages. The same 8 chimpanzee subjects faced this problem in two versions: the two ropes were close enough together that one chimp could pull both ropes (no collaboration needed) or the two ropes were too far apart for one chimp to pull both (collaboration needed). Is there evidence that chimpanzees recruit partners more often when a problem requires collaboration?

1. Open the data set *ta17-01.por.*
2. Compute the mean of the difference between collaboration and no collaboration.
   a. Click on **Transform**.
   b. Click on **Compute Variable**.
   c. Under **Target Variable**, type "Difference".
   d. For the **Numeric Expression**, type "Collab – Nocollab".



e. Click **OK**.
f. A third column containing the difference between the number of trials where the chimps collaborated and those they did not collaborate is shown in the **Data Editor**.



Now you may calculate a confidence interval for the ***Difference*** or test a hypothesis about the difference using the techniques above for the one-sample t test and the confidence interval. Just use the ***Difference*** column as your data set and follow the given procedures.

Chapter 17 Exercises

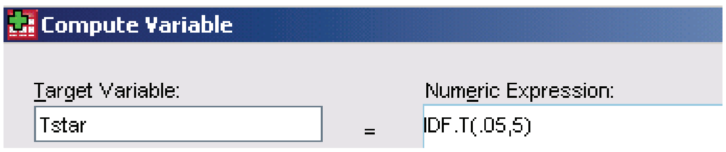
17.3 Critical values.
17.7 Ancient air.
17.9 Is it significant?
17.11 The brain responds to sound.
17.13 Diamonds.
17.25 Read carefully.
17.27 Reading scores in Atlanta.
17.29 The placebo effect.
17.31 Learning Blissymbols.

17.33 An outlier's effect.
17.35 Genetic engineering for cancer treatment.
17.37 Growing trees faster.
17.39 Weeds among the corn.
17.43 How much better does nature heal?
17.45 Right versus left.
17.47 Practical significance.

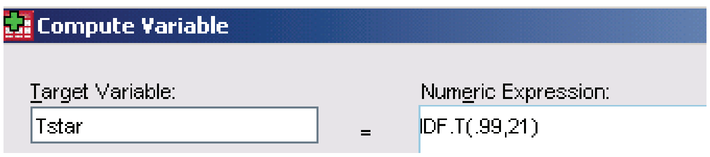
Chapter 17 SPSS Solutions

NOTE: SPSS does not perform inference on variables that are already summarized. If you really want to use SPSS for these problems or chapters, follow the instructions below (you'll be basically using **<u>Transform</u>, **<u>Compute Variable</u>** as a calculator) or use another technology (such as a graphing calculator or another statistics program like Minitab or Crunchit.)

17.3 We use **<u>IDF.T</u>** from the Inverse DF Function group to find the critical values.

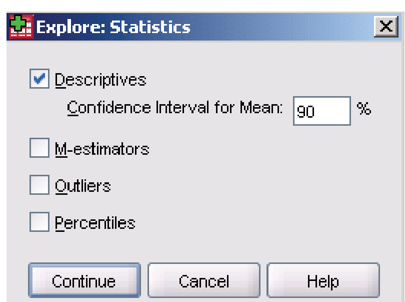


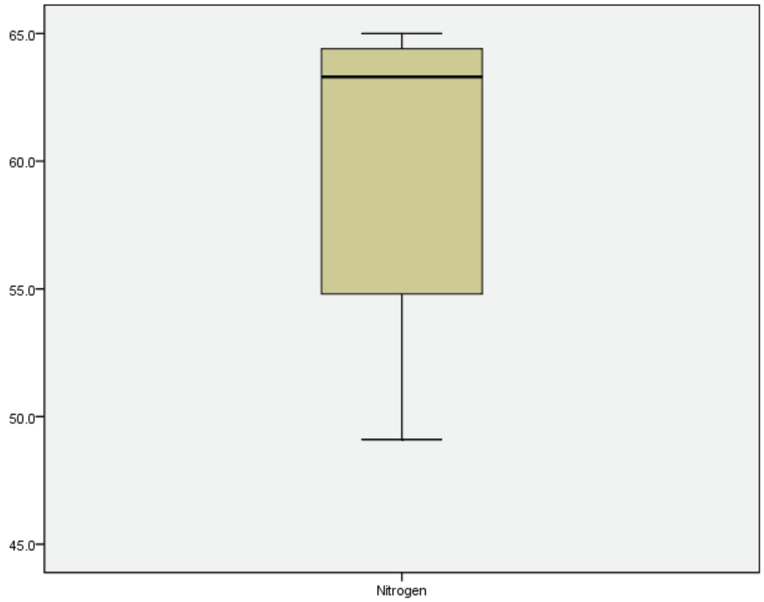
Tstar
-2.015



Tstar
2.518

17.7 The data have been entered in a column we named **Nitrogen**. If μ is the mean nitrogen content of Cretaceous era air, we'd like a 90% confidence interval estimate. First, check the conditions: we're assuming our data come from a SRS; can we believe these data came from an (approximately) Normal distribution? With only 9 data values, a histogram will not show the distribution very well. We'll use **<u>Analyze</u>**, **<u>Descriptive Statistics</u>**, **<u>Explore</u>** to create the confidence interval and a boxplot of the data. Click to enter the variable name in the Dependent list, then click **<u>Statistics</u>**. Change the confidence level to 90%, then **<u>Continue</u>** and **<u>OK</u>**.





There are no outliers, but the distribution is definitely skewed; observe the median far to the high end in the box. Use of *t* procedures might not be valid, we can only proceed with caution.

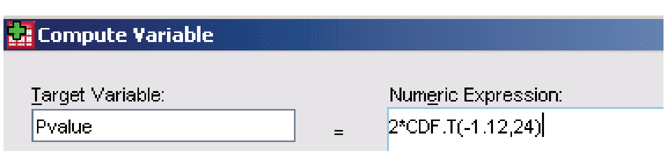
The confidence interval is included in the Descriptives table, as shown below (some of the table has been omitted).

Descriptives

			Statistic	Std. Error
Nitrogen	Mean		59.589	2.0851
	90% Confidence Interval for Mean	Lower Bound	55.712	
		Upper Bound	63.466	

Based on these samples, we estimate that Cretaceous era air had between 55.7% and 63.5% nitrogen, with 90% confidence (assuming the distribution is really approximately Normal).

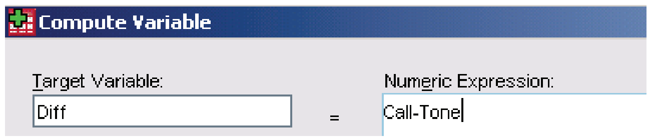
17.9 With n = 25, there are 25 – 1 = 24 degrees of freedom. Using technology, we'll find an exact *P*-value for this test. Since the test is two-tailed, we'll use the symmetry of the distribution and double the area to the *left* of $t = -1.12$. Use **Transform**, **Compute Variable** and **CDF.T** from the **CDF and Noncentral CDF** function group. As always, if you want more decimal places than the default 2, increase them on the **Variable View**.



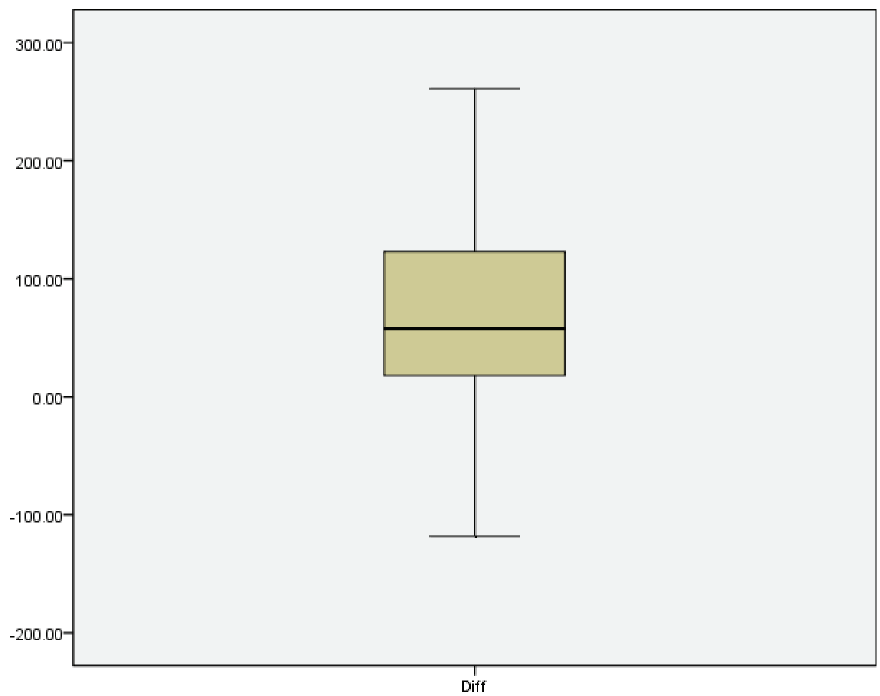
Pvalue
0.2738

Our P-value is 0.2738. This test is not significant at any normal alpha level.

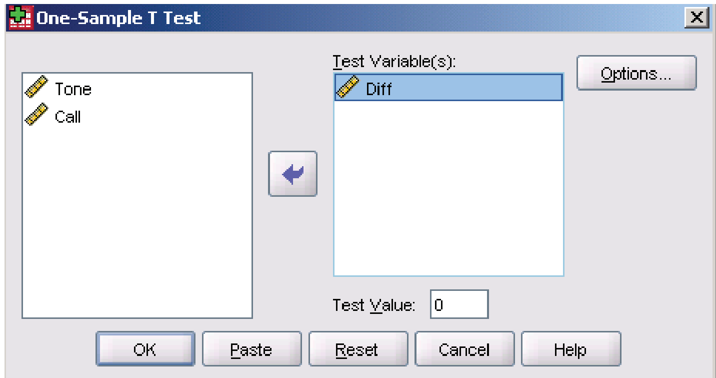
17.11 This is a matched pairs situation (we have the response to both tone and call for each neuron). If μ_D is the mean of the differences (call – tone), we'll have hypotheses $H_0: \mu_D = 0$ and $H_a: \mu_D > 0$. SPSS has a built-in matched pairs t test procedure, but it doesn't create any graphs to check assumptions (such as the data come from an approximately Normal distribution). Use **Transform**, **Compute Variable** to create a variable named **Diff** as shown below.



We'll now create a boxplot of these differences using **Graphs**, **Legacy Dialogs**, **Boxplot**. Use the **Summaries of Separate Variables** option for a **Simple** Boxplot.



Our boxplot looks symmetric (with no outliers), so *t* procedures are justified. At this point, we have formed the differences, so we can use **Analyze**, **Compare Means**, **1-Sample T Test** to perform our test.



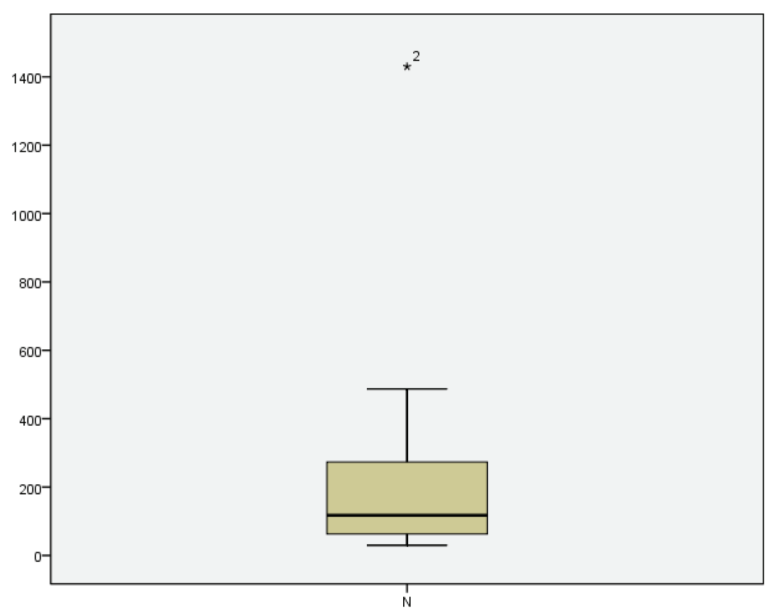
Since our null hypothesis is that the mean difference is 0, the Test Value is set to 0 (the default).

One-Sample Test

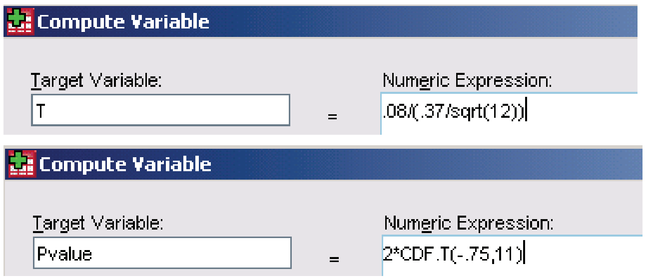
	Test Value = 0					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
Diff	4.840	36	.000	70.37838	40.8888	99.8679

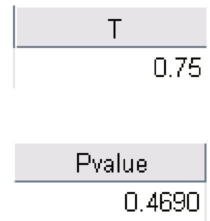
SPSS gives only two-sided *P*-values (in the Sig column). To find the one-sided P-value, divide by 2; however .000/2 = .000, so this test rejects the null hypothesis. With a test statistic of $t = –4.84$ and *P*-value of 0.000, this is extremely strong evidence that the call response is stronger than the tone response, on average, in macaque monkeys.

17.13 We use **<u>Graphs</u>**, **<u>Legacy Dialogs</u>**, **<u>Boxplot</u>** to create a boxplot of the Nitrogen data (labeled N in data file *ta17-03*). We are looking for skewness and outliers. The graph shows this data set has an extreme outlier; further, the main portion seems rather skewed right (toward the high end). We can't trust *t* procedures for these data.

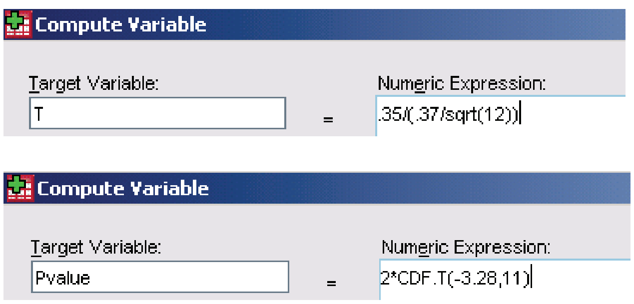


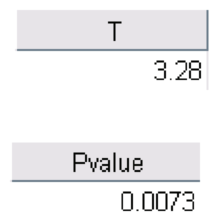
17.25 We'll recompute the t statistics and find the correct *P*-values using **<u>Transform</u>**, **<u>Compute Variable</u>**. We also use the symmetry of the *t* distributions; the area to the right of a positive value is the same as the area to the left of the negative of that value. For the student group, we have



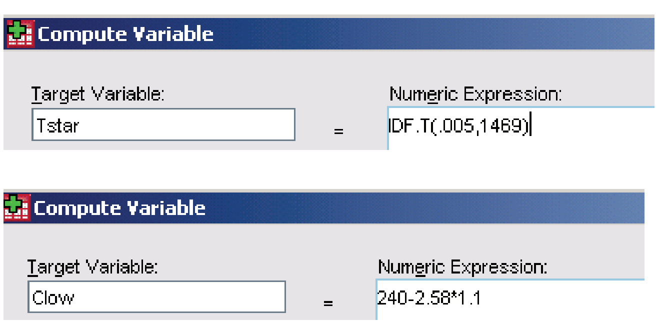


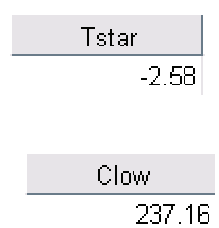
For the student group, we have t = 0.75 with *P*-value 0.469 (the conclusion was correct, however, there is no significant effect here). Repeating for the non-student group, we find have t = 3.28 with *P*-value 0.0073; there is a significant effect in this group.

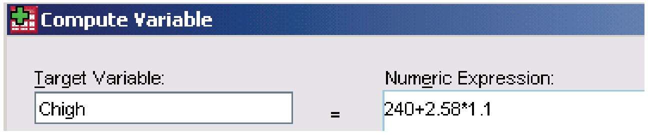


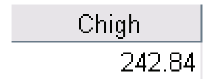


17.27 With a sample size of n = 1470, this is certainly large enough to appeal to the Central Limit Theorem, and call $\bar{x}$ approximately Normal. To find the confidence interval, we'll first find the critical value t^* using **<u>IDF.T</u>** and then find the confidence interval as $\bar{x} \pm t^* SE$.



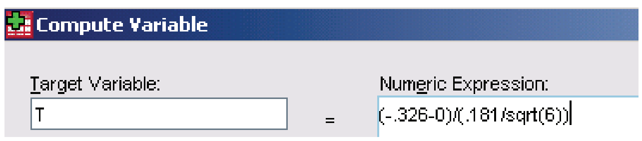


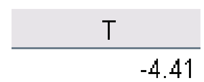


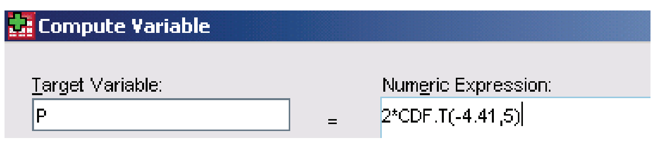


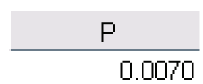
Based on this information, we're 99% confident Atlanta eighth-graders should have a mean TUDA score between 237.2 and 242.8. Since the high end of this interval is below 243 (basic), our indications are that Atlanta eighth-graders, on average, perform below this level.

17.29 Since each patient was given both treatments, we use a matched pairs t test to compare the treatments to control variability among the subjects. We are given the summary statistics, so we'll use SPSS as a calculator to compute the test statistic and P-value. With a test statistic of –4.41 and a P-value of 0.0070, there is evidence of a significant difference between treatment and control.









17.31 We'll use **Analyze**, **Descriptive Statistics**, **Explore** to create the stemplot and the confidence interval. Be sure to check Statistics for the correct confidence level (90%).

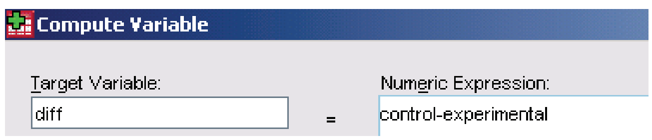
```
Count Stem-and-Leaf Plot

 Frequency     Stem &  Leaf

     3.00        0 .  699
     5.00        1 .  01124
     2.00        1 .  55
     2.00        2 .  02

 Stem width:   10
 Each leaf:        1 case(s)
```

The histogram shows no overt skewness nor any outliers, so we'll check the confidence interval in the Descriptives table.

Descriptives				
			Statistic	Std. Error
Count	Mean		12.83	1.342
	90% Confidence Interval for Mean	Lower Bound	10.42	
		Upper Bound	15.24	

We're 90% confident the average count of correct Blissymbols among children using this program will be between 10.4 and 15.2.

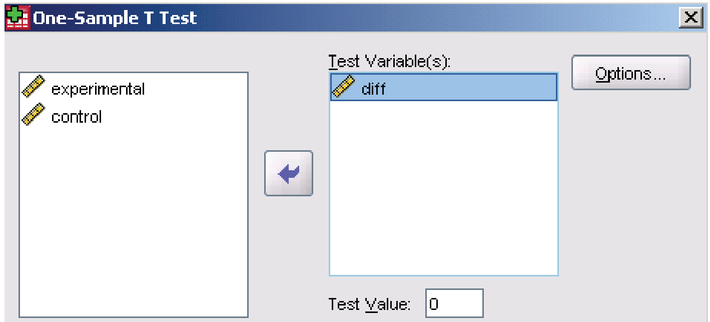
17.33 The parameter of interest is μ_D, the mean difference between the experimental and control limbs. We want to know whether the electrical field slows healing, so forming differences as Experimental – Control, we have hypotheses $H_0: \mu_D = 0$ and $H_a: \mu_D > 0$. Open data file *ta17_03.por*. Compute the differences using **Transform**, **Compute Variable**.



To create the stemplot of the differences, use **Analyze**, **Descriptive Statistics**, **Explore**.

```
diff Stem-and-Leaf Plot

 Frequency     Stem &  Leaf

     1.00 Extremes    (=<-13)
     1.00       -0 .  6
      .00       -0 .
     2.00        0 .  12
     4.00        0 .  5789
     3.00        1 .  012
     1.00 Extremes    (>=31)

 Stem width:     10.00
 Each leaf:       1 case(s)
```

Since we've actually computed the differences, we use **Analyze**, **Compare Means**, **One-Sample T Test** using **diff** as our variable. The test value is 0 (no difference between the control and experimental limbs).



One-Sample Test

	Test Value = 0					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
diff	2.076	11	.062	6.41667	-.3859	13.2192

With a one-sided *P*-value of 0.0310 (divide the SPSS two-tailed by 2), we conclude at the 5% significance level that the electrical field does slow healing, on average. Now, delete the high outlier (31) from **diff** and recalculate the test.

One-Sample Test

	Test Value = 0					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
diff	1.788	10	.104	4.18182	-1.0291	9.3927

We now have a *P*-value of 0.0520; this is insufficient evidence at the 0.05 level that the electrical field (or lack thereof) made a difference in average healing rates.

17.35 We'll use **Analyze**, **Descriptive Statistics**, **Explore** to create the stemplot and the confidence interval. Be sure to check Statistics for the correct confidence level (90%).

```
Doubling Stem-and-Leaf Plot

 Frequency     Stem &  Leaf

     4.00        0 .  6789
     5.00        1 .  00334
     1.00        1 .  9
     1.00        2 .  0

 Stem width:        1.0
 Each leaf:       1 case(s)
```

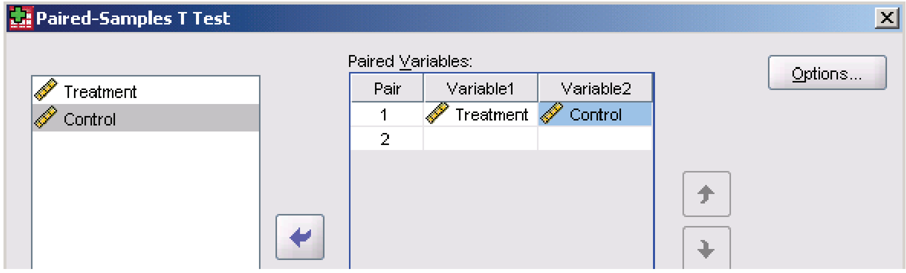
The distribution is rather skewed right (the high hand side on the boxplot is twice as long as the low from the median out), but there are no outliers; further, all the data values are reasonably close to one another.

Descriptives

			Statistic	Std. Error
Doubling	Mean		1.173	.1389
	90% Confidence Interval for Mean	Lower Bound	.921	
		Upper Bound	1.424	
	Kurtosis		-.357	1.279

Based on this sample, the mean doubling time is between 0.92 and 1.42 days (with 90% confidence); however, based on the shape of the data distribution, we'd hesitate to use this for inference about all possible similar patients.

17.37 Because the investigators believed that extra CO_2 would cause the trees to grow faster, the hypotheses are $H_0: \mu_D = 0$ and $H_a: \mu_D > 0$, where μ_D is the mean difference, treatment – control. We enter the Treatment values in a column and the Control in another, then use **Analyze**, **Compare Means**, **Paired Samples t test** to perform the test.



Paired Samples Test

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference: Lower	Upper			
Pair 1 Treatment - Control	1.916333	1.050494	.606503	-.693238	4.525905	3.160	2	.087

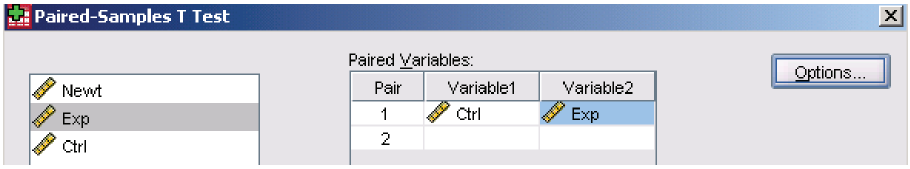
The test statistic is $t = 3.16$ with P-value $0.087/2 = 0.0435$. While significant at the 5% level, a sample of only $n = 3$ is not very convincing, and risky because we do not have a good idea of the real variation that might occur.

17.39 We can use Analyze, Descriptive Statistics, Explore to examine a stemplot (and boxplot) for these data.

```
Seeds Stem-and-Leaf Plot

 Frequency     Stem &   Leaf

     4.00         0 .   1123
     3.00         0 .   788
     4.00         1 .   0011
     7.00         1 .   5677899
     6.00         2 .   011124
     2.00         2 .   58
     2.00 Extremes      (>=5973)

 Stem width:   1000
 Each leaf:        1 case(s)
```

This stemplot indicates 2 high extremes (5973 and larger). With two high outliers, these data are not suitable for t procedures.

17.43 If you haven't done Exercise 17.42, you can find the confidence interval for the difference using **Analyze**, **Compare Means**, **Paired Samples T Test**. If you did Exercise 17.42, the confidence interval is given on the output from the test. Use **Options** to change the confidence level to 90%.

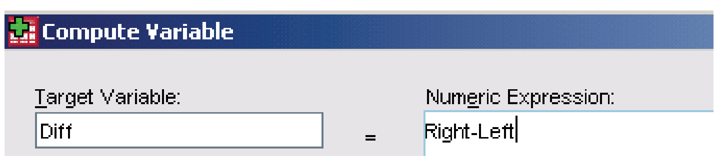


Paired Samples Test

	Paired Differences					t	df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	90% Confidence Interval of the Difference: Lower	90% Confidence Interval of the Difference: Upper			
Pair 1 Ctrl - Exp	5.714	10.564	2.823	.714	10.714	2.024	13	.064

Based on these data, we are 90% confident the difference in mean healing rate will be between 0.71 and 10.71. This indicates the control limb has the faster healing rate.

17.45 Since all subjects will use both instruments, we'll flip a coin for each to see which hand is used first. So that we can examine the shape of the distribution of differences for skewness and outliers we'll actually compute them using **<u>Transform</u>**, **<u>Compute Variable</u>**.



We now examine a stemplot of the differences using **<u>Analyze</u>**, **<u>Descriptive Statistics</u>**, **<u>Explore</u>**.

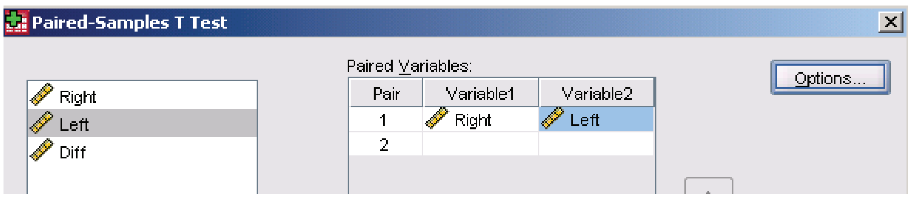
```
Diff Stem-and-Leaf Plot

 Frequency     Stem &  Leaf

     1.00        -5 .  2
     3.00        -4 .  358
     3.00        -3 .  115
     2.00        -2 .  49
     5.00        -1 .  12666
     5.00        -0 .  13347
     2.00         0 .  02
     1.00         1 .  1
     2.00         2 .  03
     1.00         3 .  8

 Stem width:      10.00
 Each leaf:        1 case(s)
```

This distribution is symmetric and shows no outliers (so does the boxplot). Since we believe the right hand times should be faster, we will test

$$H_0 : \mu_D = 0$$

$$H_a : \mu_D < 0$$

(if you subtract the other way, the direction of the alternate hypothesis will change). We'll use **Analyze**, **Compare Means**, **Paired Samples T Test**. Since we want the confidence interval in Exercise 17.47, use Options to change the confidence level to 90%.



Paired Samples Test

		Paired Differences						
	Mean	Std. Deviation	Std. Error Mean	90% Confidence Interval of the Difference: Lower	90% Confidence Interval of the Difference: Upper	t	df	Sig. (2-tailed)
Pair 1 Right - Left	-13.320	22.936	4.587	-21.168	-5.472	-2.904	24	.008

With a test statistic of t = –2.90 and P-value 0.008/2 = 0.004, we reject the null hypothesis of no difference. This experiment does show that people find right-hand threads easier to use.

17.47 If you followed our instructions in the solution to Exercise 17.45, you already have the confidence interval for the difference. We are 90% confident that the right-hand thread will save between 5.5 and 21.2 seconds. This could be of great importance if a task were performed over and over – a minute might be saved for every three repetitions.