Radiation Pressure

Classical Description

It was first pointed out by Maxwell in 1871 that electromagnetic (EM) radiation would exert pressure (force per unit area) on surfaces, a theoretical prediction verified experimentally by Lebedev in 1900 and by Nichols and Hull in 1901. The pressure results from the momentum carried by the EM wave, whose energy transport per unit time per unit area is given by the Poynting vector $\vec{S}$:

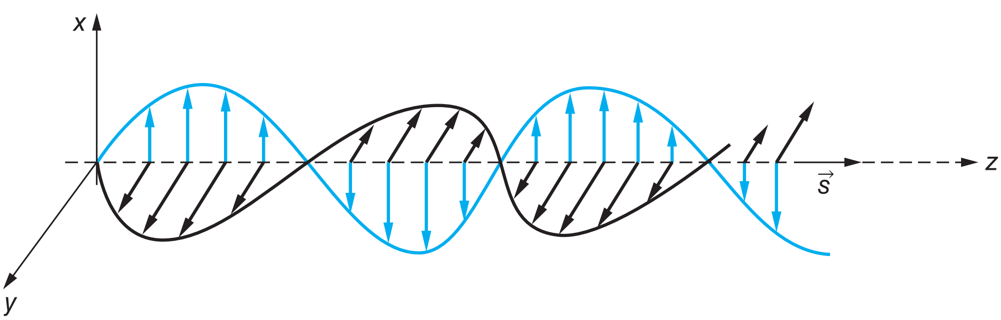
$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \qquad \textbf{RP-1}$$

where $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, respectively, and μ_0 is the permeability of vacuum (see Figure RP-1). The wave propagates in the direction of $\vec{S}$, whose SI units are W/m^2. Because the size of both $\vec{E}$ and $\vec{B}$ varies harmonically in time, the time average of the magnitude of the Poynting vector is

$$\langle S \rangle = \frac{1}{2\mu_0}E_{max}B_{max} \qquad \textbf{RP-2}$$

where E_{max} and B_{max} are the maximum magnitudes of the fields. In a vacuum the magnitudes of $\vec{E}$ and $\vec{B}$ are related by $E = cB$, so $\langle S \rangle \propto E^2$. Thus, although the values of $\vec{E}$ and $\vec{B}$ vary both positive and negative with time, the value of $\vec{S}$ is always positive or zero (refer to Figure RP-1 and use the right-hand rule for the cross product).

Because the EM wave transports energy, it carries momentum $\vec{p}$ and exerts a force $\vec{F}_{RP} = d\vec{p}/dt$ on any surface the wave encounters. The force $\vec{F}_{RP}$ that results from the radiation pressure acting on an area A depends on whether the wave is



RP-1 The electric field (blue arrows) and magnetic field (red arrows) are orthogonal transverse waves. Their cross product, the Poynting vector $\vec{S}$, is thus perpendicular to both $\vec{E}$ and $\vec{B}$. The EM wave travels in the direction of $\vec{S}$.

reflected or absorbed. Referring to Figure RP-2*a*, if the incident EM wave is completely absorbed, the force $\vec{F}_{RP}$ is in the direction of $\vec{S}$ and has magnitude

$$F_{RP} = \frac{\langle S \rangle A}{c} \cos \vartheta \quad (\text{absorption}) \qquad \textbf{RP-3}a$$

in which case the magnitude of the pressure due to radiation incident on the area A is

$$P_{RP} = \frac{F_{RP}}{A} = \frac{\langle S \rangle}{c} \cos \vartheta \qquad \textbf{RP-3}b$$

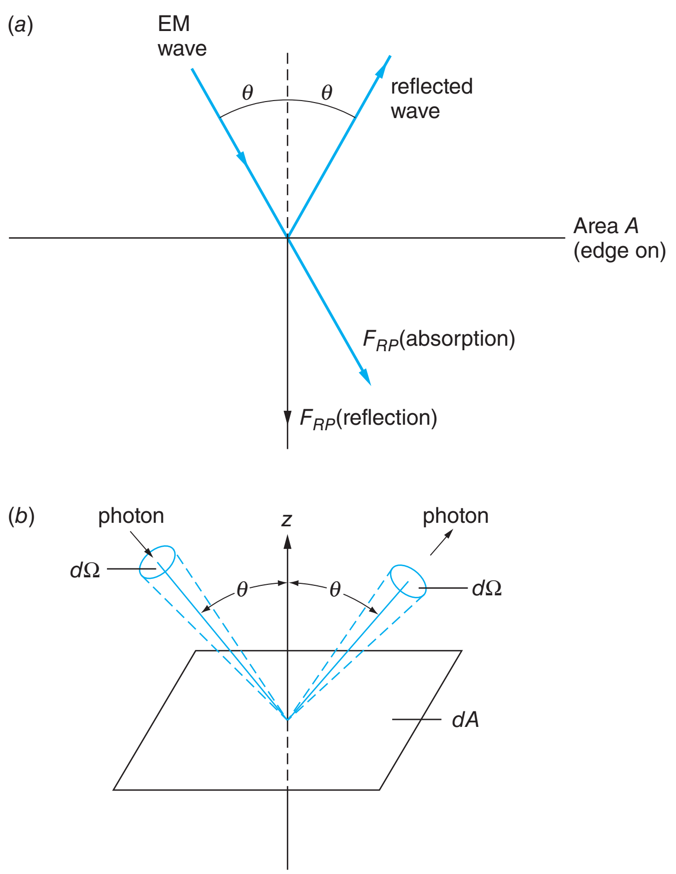
If the EM wave is totally reflected, $\vec{F}_{RP}$ must be perpendicular to the surface of the area A since in reflection there is no net force parallel to the surface. The magnitude of the force is then

$$F_{RP} = \frac{2\langle S \rangle A}{c} \cos^2 \vartheta \quad (\text{reflection}) \qquad \textbf{RP-4}a$$

and the magnitude of the pressure due to radiation falling on the area A is

$$P_{RP} = \frac{F_{RP}}{A} = \frac{2\langle S \rangle}{c} \cos^2 \vartheta \qquad \textbf{RP-4}b$$

(To order to confirm that the units on $\langle S \rangle / c$ are those of pressure, pascals, note that $\frac{\text{J}}{\text{s} \cdot \text{m}^2} \times \frac{1}{\text{m/s}} = \frac{\text{N} \cdot \text{m}}{\text{s} \cdot \text{m}^2} \times \frac{\text{s}}{\text{m}} = \frac{\text{N}}{\text{m}^2} = \text{Pa}$).



RP-2 (a) Forces $\vec{F}_{RP}$ due to radiation pressure on the area A. $\vec{F}_{RP}$ (green vector) results from total absorption of the EM wave, $\vec{F}_{RP}$ (red vector) from total reflection. (b) Photons impinging on a perfectly reflecting surface dA at an angle of incidence ϑ within a solid angle cone $d\Omega$. The photons reflect at the same angle ϑ into the same solid angle cone $d\Omega$.

Quantum Description

As was pointed out in the discussion following Equation 2-31, although photons are massless, they carry momentum of magnitude $p = E/c$. Thus, photons with wavelengths between λ and $\lambda + d\lambda$ that are incident on a perfectly reflecting surface of area dA within a solid angle cone $d\Omega$ at an angle ϑ (see Figure RP-2*b*) experience a change in the z component of their momentum given by

$$\begin{aligned} dp_\lambda\, d\lambda &= (p_{\text{ref},z} - p_{\text{inc},z})\, d\lambda \\ &= \left[\frac{E_\lambda \cos\vartheta}{c} - \left(-\frac{E_\lambda \cos\vartheta}{c}\right)\right] d\lambda \\ &= \frac{2E_\lambda \cos\vartheta}{c} d\lambda \\ &= \frac{2}{c} u(\lambda)\, d\lambda\, dA\, dt \cos^2\vartheta\, d\Omega \end{aligned} \qquad \textbf{RP-5}$$

where $u(\lambda)\,d\lambda$ is given by Planck's law, Equation 3-18. Dividing dp_λ by dA and dt gives

$$\frac{dp_\lambda}{dt\,dA} = (dp_\lambda/dt)/dA = dF_\lambda/dA \qquad \textbf{RP-6}$$

where $dp_\lambda/dt = dF_\lambda$ is the force exerted on the photons by the area dA. Newton's third law reaction to that force, $-dp_\lambda/dt = -dF_\lambda$, is the force exerted on the area dA by the photons; the minus sign simply indicates that the force is in the $-z$ direction. For our discussion here we can ignore the minus sign.

The magnitude of the pressure P_λ exerted on the area dA by the photons with wavelengths between λ and $\lambda + d\lambda$ incident at all angles with $z > 0$ is then

$$\begin{aligned} P(\lambda)\, d\lambda &= \int_{z>0} \frac{dF_\lambda}{dA} d\lambda = \int_{z>0} \frac{2}{c} u(\lambda)\, d\lambda \cos^2\vartheta\, d\Omega \\ &= \frac{2}{c} \int_0^{2\pi} \int_0^{\pi/2} u(\lambda)\, d\lambda \cos^2\vartheta \sin\vartheta\, d\vartheta\, d\varphi \\ &= \frac{4\pi}{3c} u(\lambda)\, d\lambda \end{aligned} \qquad \textbf{RP-7}$$

In the case of a gas in a container, the pressure of the gas exists throughout the container, not just at the walls. In an isotropic radiation field, the radiation analog of the gas-filled container, radiation pressure exists everywhere in the field, not just on the surface dA. The 2 in Equation RP-5 arose because of the reflection of the photons from the perfectly reflecting surface dA. If we replace the reflecting surface with a mathematical surface, the factor 2 disappears and photons impinge on and pass through the surface dA from both above and below. In that case the magnitude of the radiation pressure due to photons with wavelengths between λ and $\lambda + d\lambda$ impinging on the area dA becomes

$$P(\lambda)\,d\lambda = \frac{1}{c}\int_0^{2\pi}\int_0^{\pi} u(\lambda)\,d\lambda \cos^2\vartheta \sin\vartheta\, d\vartheta\, d\varphi$$
$$= \frac{4\pi}{3c}u(\lambda)\,d\lambda \qquad \textbf{RP-8}$$

The total radiation pressure P_{rad} is then

$$P_{\text{rad}} = \int_0^{\infty} P(\lambda)\,d\lambda = \frac{4\pi}{3c}\int_0^{\infty} u(\lambda)\,d\lambda = \frac{4\sigma T^4}{3c} = \frac{1}{3}U \qquad \textbf{RP-9}$$

where U is the total energy density of the radiation. (To confirm that the units of U are pressure, note that $\frac{\text{J}}{\text{m}^3} = \frac{\text{N}\cdot\text{m}}{\text{m}^3} = \frac{\text{N}}{\text{m}^2} = \text{Pa}$.)

EXAMPLE RP-1 Radiation Pressure from Sunlight What is the magnitude of the radiation pressure produced by sunlight at Earth's distance from the Sun? How does it compare with atmospheric pressure?

SOLUTION

The power per unit area R of sunlight arriving at the top of Earth's atmosphere, called the *solar constant*, is 1.36×10^3 W/m^2. It is related to the energy density U by Equation 3-6:

$$R = \frac{1}{4}cU \quad \Rightarrow \quad U = \frac{4}{c}R$$

Substituting U into Equation RP-9 yields

$$P_{\text{rad}} = \frac{4}{3c}R = \frac{4(1.36 \times 10^3\ \text{W/m}^2)}{3(3.00 \times 10^8\ \text{m/s})}$$
$$P_{\text{rad}} = 6.04 \times 10^{-6}\ \text{Pa}$$

For comparison, about 99 percent of Earth's atmosphere lies below 35 km. At that altitude atmospheric pressure is about 10^3 Pa. The radiation pressure of sunlight there is about nine orders of magnitude smaller.