Example 14-3 Oxygen at Room Temperature

Calculate (a) the average translational kinetic energy and (b) the root-mean-square speed of an oxygen molecule in air at room temperature (20°C). One mole of oxygen molecules has a mass of 32.0 g $= 32.0 \times 10^{-3}$ kg.

Set Up

Equation 14-13 tells us the average translational kinetic energy, and Equation 14-15 tells us the rms speed of molecules. Note that we're given the mass per mole of O_2, but we'll need to convert this to m, the mass per molecule. We'll also need to convert the temperature from Celsius to Kelvin.

Temperature and average translational kinetic energy of an ideal gas molecule:

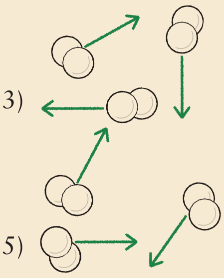
$$K_{\text{translational,average}} = \frac{1}{2}m(v^2)_{\text{average}} = \frac{3}{2}kT \quad (14\text{-}13)$$

Root-mean-square speed of molecules in an ideal gas:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad (14\text{-}15)$$

Conversion between Celsius and Kelvin:

$$T_C = T_K - 273.15 \text{ and } T_K = T_C + 273.15 \quad (14\text{-}2)$$



oxygen molecules

Solve

(a) First find the mass per molecule m and the Kelvin temperature T.

The number of molecules per mole is Avogadro's number, $N_A = 6.022 \times 10^{23}$ molecules/mol. So the mass per O_2 molecule is

$$m = \frac{(32.0 \times 10^{-3}\ \text{kg/mol})}{(6.022 \times 10^{23}\ \text{molecules/mol})} = 5.31 \times 10^{-26}\ \text{kg/molecule}$$

From the second of Equations 14-2, the Kelvin temperature that corresponds to 20°C is

$$T = 20 + 273.15 = 293\ \text{K}$$

Use Equation 14-13 to find the average kinetic energy per molecule.

From Equation 14-13,

$$K_{\text{translational,average}} = \frac{3}{2}kT = \frac{3}{2}(1.381 \times 10^{-23}\ \text{J/K})(293\ \text{K}) = 6.07 \times 10^{-21}\ \text{J}$$

(b) Use Equation 14-15 to find the rms speed of an O_2 molecule.

From Equation 14-15,

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.381 \times 10^{-23}\ \text{J/K})(293\ \text{K})}{5.31 \times 10^{-26}\ \text{kg}}} = 478\ \text{m/s}$$

Reflect

The rms speed of O_2 molecules is tremendous, 478 m/s = 1720 km/h = 1070 mi/h. Note that very few O_2 molecules in the air travel at precisely that speed: Their speeds range from nearly zero to many times faster than 478 m/s.

Note that the nitrogen (N_2) molecules in air at 20°C have the *same* translational kinetic energy (which does not depend on the mass m of the molecules) but a *different* rms speed (which is proportional to the reciprocal of $\sqrt{m}$). As we discussed above, an N_2 molecule has 0.875 the mass of an O_2 molecule, so the rms speed for N_2 is $1/\sqrt{0.875} = 1.07$ times faster than the rms speed for O_2.