CHAPTER 6

INTRODUCTION TO INFERENCE

SECTION 6.1

OVERVIEW

A **confidence interval** provides an estimate of an unknown parameter of a population or process along with an indication of how accurate this estimate is and how **confident** we are that the interval is correct. Confidence intervals have two parts. One is an interval computed from our data. This interval typically has the form

estimate ± margin of error

The other part is the **confidence level**, which states the probability that the method used to construct the interval will give a correct answer. For example, if you use a 95% confidence interval repeatedly, in the long run 95% of the intervals you construct will contain the correct parameter value. Of course, when you apply the method only once you do not know if your interval gives a correct value or not. Confidence refers to the probability that the method gives a correct answer in repeated use, not the correctness of any particular interval we compute from data.

Suppose we wish to estimate the unknown mean μ of a normal population with known standard deviation σ based on an SRS of size n. A level C confidence interval for μ is

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

where z^* is such that the probability is C that a standard normal random variable lies between $-z^*$ and z^* and is obtained from the bottom row in Table D.

The margin of error $z^* \frac{\sigma}{\sqrt{n}}$ of a confidence interval decreases when any of the following occur:

- the confidence level C decreases
- the sample size n increases
- the population standard deviation σ decreases.

The sample size needed to obtain a confidence interval for a normal mean of the form

estimate ± margin of error

with a specified margin of error m is

$$n = \left(\frac{z^* \sigma}{m}\right)^2$$

where z^* is the critical point for the desired level of confidence. Many times the n you will find will not be an integer. If it is not, round up to the next larger integer.

The formula for any specific confidence interval is a recipe that is correct under specific conditions. The most important conditions concern the methods used to produce the data. Many methods (including those discussed in this section) assume that our data were collected by random sampling. Other conditions, such as the actual distribution of the population, are also important.

SAMPLE PROBLEMS

GUIDED SOLUTIONS

Exercise 6.5

KEY CONCEPTS - the sampling distribution of $\bar{x}$, confidence intervals

a) We are told that the $n = 100$ invoices are a random sample, and we know that the sampling distribution of $\bar{x}$ has standard deviation $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$. Use the values of σ and n to compute $\sigma_{\bar{x}}$.

$\sigma_{\bar{x}} =$

b) Recall from Section 3 of Chapter 1, the 68 - 95 - 99.7 rule, says that 68% of the area under a normal curve lies within one standard deviation of the mean, 95% within two standard deviations of the mean, and 99.7% within three standard deviations of the mean. Now you should be able to fill in the blank.

c) Use the value of $\sigma_{\bar{x}}$ that you computed in part (a) and your answer to part (b) to fill in the blank.

Exercise 6.19

KEY CONCEPTS - confidence intervals, the sample size required to obtain a confidence interval of specified margin of error

a) The margin of error of a level C confidence interval is $z^* \frac{\sigma}{\sqrt{n}}$, where z^* is such that the probability is C that a standard normal random variable lies between $-z^*$ and z^* and is obtained from the bottom row in Table D. To do this exercise, you must identify

C = the level of confidence required =

z^* = the probability is C that a standard normal random variable lies between $-z^*$ and z^* =

σ = population standard deviation =

n = the sample size used =

Determine the above values and then compute the margin of error

$$m = z^* \frac{\sigma}{\sqrt{n}} =$$

b) The calculations here are the same as in (a) except that n has changed from 100 to 10. Should the margin of error be larger or smaller? Why? Repeat the calculations you did in (a) with this change and see if your intuition was correct.

c) The smallest value of n that will yield a 95% confidence interval with a margin of error = m must satisfy

$$n = \left(\frac{z^* \sigma}{m}\right)^2$$

where z^* and σ are as in (a). Identify m in this case and use the above formula to compute n. Remember to round your answer up to the nearest integer. Is n within the limits of your budget as indicated in (a)?

Exercise 6.26

KEY CONCEPTS - confidence levels for several confidence intervals simultaneously, binomial probability calculations

We know the following:

- we are interested in a fixed number of intervals (three, to be precise)
- the three intervals are all independent
- either an interval contains the true median household income (success) or it does not (failure)
- the probability that any particular interval will contain the true median household income is 0.95

Let X be the number of intervals (out of the three) that contain the true median household income (i.e., the number of successes). X should remind you of a special type of random variable whose probability distribution we have studied previously (Hint: Look at Section 1 of Chapter 5). How do you calculate probabilities for X?

a) We want the probability of three successes. What is this probability?

b) We want the probability of at least two successes. What is this probability?

Exercise 6.27

KEY CONCEPTS - confidence intervals, interpreting confidence intervals

a) The problem tells us that the *estimate* of the population proportion based on the random sample is 54% and that the *margin of error* is 3% for a 95% confidence interval. Based on the formula,

estimate ± margin of error

give the 95% confidence interval.

b) Use the confidence interval you computed in part (a) to help you formulate your answer.

COMPLETE SOLUTIONS

Exercise 6.5

a) We are given that σ = \$200 and n = 100. Hence $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{\$200}{\sqrt{100}} = \$20$.

b) The probability is 0.95 that $\bar{x}$ is within $2\times\sigma_{\bar{x}}$ of the population mean μ.

c) $2\times\sigma_{\bar{x}}$ = \$40, so about 95% of all samples will capture the true mean of all invoices in the interval $\bar{x}$ plus or minus \$40.

Exercise 6.19

a) From the statement of the problem we see

C = the level of confidence required = 0.95

hence

z^* = the probability is 0.95 that a standard normal random variable lies between $-z^*$ and z^*

= 1.96 (see Table D)

We also see that

σ = population standard deviation = 12, and

n = the sample size used = 100

Thus the margin of error m is

$$m = z^* \frac{\sigma}{\sqrt{n}} = 1.96 \frac{12}{\sqrt{100}} = 2.352$$

b) We change n to 10 in (a) and find that the margin of error m is now

$$m = z^* \frac{\sigma}{\sqrt{n}} = 1.96 \frac{12}{\sqrt{10}} = 7.438$$

c) Again we have $z^* = 1.96$ and $\sigma = 12$. We are told we want $m \le 3$, so we use the maximum possible value we would tolerate, namely $m = 3$, in the formula for n and get

$$n = \left(\frac{z^* \sigma}{m}\right)^2 = \left(\frac{(1.96)(12)}{3}\right)^2 = (7.84)^2 = 61.47$$

Rounding up, we see that the smallest value of n that will accomplish our goal is $n = 62$. Since the budget will allow up to 100 students, this is within the limits of the budget.

Exercise 6.26

Referring to Section 1 of Chapter 5 we see that X has a binomial $B(n=3, p=0.95)$ distribution. We use this distribution to calculate the required probabilities. Table C in your text gives binomial probabilities.

a) Here we want the probability that $X = 3$. Table C only gives binomial probabilities for $p \le 0.50$. Thus to use Table C we have to rewrite the desired probability in terms of the number of failures (which has $p = 0.05$, a value that is given in Table C) We have (look in the portion of Table C with $n = 3$ and $p = 0.05$)

$$P(X = 3) = P(\text{number of failures} = 0) = 0.8574$$

b) Again, rewriting the desired probability (at least 2 successes) in terms of number of failures we have

$$P(X \ge 2) = P(\text{number of failures} \le 1) = P(0 \text{ failures}) + P(1 \text{ failure})$$

$$= 0.8574 + 0.1354 = 0.9928$$

Exercise 6.27

a) Based on the information given the 95% confidence interval is

$$54\% \pm 3\%$$

which corresponds to the range 51% to 57%.

b) The 95% tells us the probability that the *method* used to construct the interval will produce an interval containing the true value of the proportion of fans that prefer a play-off as an alternative to the BCS. We can be 95% confident that if we knew all the opinions of adults 18 years and over and computed the proportion favoring a play-off, the interval 51% to 57% would contain this proportion. In interpreting this, one should remember that

• This is a 95% confidence interval, hence we cannot be absolutely certain that the majority of adults 18 years and older favor a play-off. However, there is a good evidence that a majority do, but this may only be a slight majority. because the percentage can be as low as 51%.

• This was a survey of adults 18 years and over. Not all of those sampled are necessarily football fans. This makes it difficult to determine if the survey results are an accurate reflection of the attitudes of football fans.

SECTION 6.2

OVERVIEW

Tests of significance and confidence intervals are the two most widely used types of formal statistical inference. A test of significance is done to assess the evidence against the **null hypothesis** H_0 in favor of an **alternative hypothesis** H_a. Typically the alternative hypothesis is the effect that the researcher is trying to demonstrate, and the null hypothesis is a statement that the effect is not present. The alternative hypothesis can either be **one** or **two-sided**.

Tests are usually carried out by first computing a **test statistic**. The test statistic is used to compute a ***P*-value** which is the probability of getting a test statistic at least as extreme as the one observed, where the probability is computed when the null hypothesis is true. The *P*-value provides a measure of how incompatible our data is with the null hypothesis, or how unusual it would be to get data like ours if the null hypothesis were true. Since small *P*-values indicate data that is unusual or difficult to explain under the null hypothesis, we typically reject the null hypothesis in these cases. In this case, the alternative hypothesis provides a better explanation for our data.

Significance tests of the null hypothesis H_0: $\mu = \mu_0$ with either a one or two-sided alternative are based on the test statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

The use of this test statistic assumes that we have an SRS from a normal population with known standard deviation σ. When the sample size is large, the assumption of normality is less critical because the sampling distribution of $\overline{x}$

is approximately normal. P-values for the test based on z are computed using Table A.

When the P-value is below a specified value α, we say the results are statistically significant at level α, or we reject the null hypothesis at level α. Tests can be carried out at a fixed significance level by obtaining the appropriate critical value z^* from the bottom row in Table D.

GUIDED SOLUTIONS

Exercise 6.31

KEY CONCEPTS - null and alternative hypotheses

Typically, H_0 is of the form

$$H_0: \mu = \text{constant}$$

and H_a is of the form

$$H_a: \mu \neq \text{ constant (two-sided alternative)}$$

or

$$H_a: \mu > \text{constant (one-sided alternative)}$$

or

$$H_a: \mu < \text{constant (one-sided alternative)}$$

Remember that in many instances (especially with one-sided alternatives) it is easier to begin with H_a, the effect that we are concerned about, and then to set up H_0 as the statement that the effect is absent.

In each example, think carefully about whether H_a should be one-sided or two-sided.

a) We are interested in determining whether or not the scanner accurately measures a phantom that has known mineral density 1.4. The scanner will be accurate if the mean of the population of all possible measurements on the phantom is 1.4. It will be inaccurate if this mean differs from 1.4. Is H_a one- or two-sided? What is H_a? What is H_0?

b) Why was the old form redesigned? What do you hope to show about the new form? Use this to set up H_a and H_0. Is the alternative one or two-sided?

c) What are you concerned about in this example? Set up H_0 and H_a.

Exercise 6.41

KEY CONCEPTS - interpreting *P*-values

Write your explanation in the space. Refer to the Section Overview in this Study Guide or Section 6.2 in the text if you need a hint.

Exercise 6.49

KEY CONCEPTS - null and alternative hypotheses, carrying out a significance test about a mean

a) What do the researchers hope to show? This will be the alternative. Write down the null and alternative hypotheses.

b) The first step in carrying out the test is to compute the test statistic $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ which measures how far the sample mean is from the hypothesized value μ_0. To find the numerical value of z, you need to determine μ_0, σ and n from the problem, and then compute $\bar{x}$, the mean DRP score from the data given.

$z =$

Once the value of the test statistic z has been determined, the P-value can be computed. The P-value is the probability that the test statistic takes a value at least as extreme as the one observed. In the space provided, write this down as a probability in terms of Z, the standard normal, and then use Table A to evaluate this probability.

P-value = $P(Z \quad\quad)$ =

If you are having trouble doing this directly from the meaning of the P-value, refer to the rules for computing P-values given in Section 6.2 in the text and try to understand the rationale behind them.

Now try to interpret your result in plain language.

Exercise 6.53

KEY CONCEPTS - testing hypotheses at a fixed significance level

The value of the test statistic is given as $z = 2.42$. When carrying out the test at a fixed significance level, you can first compute the P-value and then reject the null hypothesis if the P-value is smaller than the significance level given. However, it is more direct to look up the critical value z^* in Table D and compare the value of the test statistic directly to the critical value.

a) The alternative is $\mu > 1.4$, so the P-value is $P(Z > 2.42)$, which represents the probability of a test statistic at least as extreme as the one observed. Compare the P-value to 0.05. What do you conclude?

Using Table D, we look up the critical value corresponding to the tail probability 0.05 (it does not need to be doubled since the test is one-sided), and find the critical value $z^* = 1.645$. We reject if the computed value of the test statistic exceeds this critical value. What do you conclude? When the significance level is fixed, it is easier to use Table D directly.

b) Follow the procedure in (a), but with significance level 1%.

Exercise 6.61

KEY CONCEPTS - calculating *P*-values

We need to compare the value $z = 1.12$ to the critical values in Table D. Since the test is two-sided, we double the tail area. Its value is between the critical values $z^* = 1.036$ and $z^* = 1.282$. What two significance levels do these critical values correspond to? Remember that the test is two-sided. What can we say about the *P*-value?

Now use Table A to determine the *P*-value by computing the area to the left of -1.37. Is this area the *P*-value? Why or why not? If not, what do you need to do to find the *P*-value?

Exercise 6.65

KEY CONCEPTS - relationship between two-sided tests and confidence intervals

a) We are interested in determining whether or not the mean amount of sugar, μ, in the hindguts under these conditions is 7 mg. State the null and alternative hypotheses.

H_0: H_a:

A level α two-sided significance test rejects a hypothesis H_0: $\mu = \mu_0$ when μ_0 falls outside a level 1 - α confidence interval for μ. In this exercise, we are told that a 95% confidence interval for μ is 4.2 ± 2.3. Thus α corresponds to 0.05 in this case and we can use the confidence interval to conduct a significance test at the 5% level. What is the value of μ_0 and is it outside the 95% confidence interval? Do we reject at the 5% significance level?

b) What is μ_0 in this case? Can we reject H_0: $\mu = 5$ at the 5% significance level?

COMPLETE SOLUTIONS

Exercise 6.31

a) We are interested in determining whether the mean of the population of all possible measurements on the phantom is equal to 1.4 or not. Thus, the alternative hypothesis is two-sided and of the form H_a: $\mu \neq 1.4$. The null hypothesis is H_0: $\mu = 1.4$.

b) The reason for redesigning the form was to reduce the time to complete it. An effect is present if the mean time μ to complete it is less than for the old form. This corresponds to scores below 0. Hence, an effect is present if μ is less than 0 and so the alternative hypothesis is H_a: $\mu < 0$. The null hypothesis is thus H_0: $\mu = 0$ (but one could also express the null hypothesis as H_0: $\mu \geq 0$.)

c) There is no problem if the mean wattage μ is 60. You will be concerned if μ differs from 60. Thus our hypotheses are H_0: $\mu = 60$ and H_a: $\mu \neq 60$.

Exercise 6.41

In the sample selected by the psychologist, ethnocentrism among church attendees was higher than among nonattenders. Furthermore, the chance of obtaining a difference as large as that observed by the psychologist is less than 0.05 if, in fact, there is no real difference in the population from which the sample was selected. We would take this as strong evidence that ethnocentrism is higher among church attendees than among nonattenders in the population from which the sample was selected.

Exercise 6.49

a) The researchers hope to show that their district mean exceeds the national mean of 32. The alternative is $\mu > 32$, so the hypotheses of interest to the researchers are H_0: $\mu = 32$ and H_a: $\mu > 32$.

b) The value of the test statistic is $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{35.091 - 32}{11 / \sqrt{44}} = 1.86$. Since the alternative is $\mu > 32$, the P-value is the chance of getting a value of $\bar{x}$ at least as large as 35.091 if the true mean were 32. In terms of the test statistic, this is equivalent to computing $P(Z \geq z) = P(Z \geq 1.86) = 1 - 0.9686 = 0.0314$.

There is evidence that the mean score of all third graders in this district exceeds the national mean of 32.

Exercise 6.53

a) The value of the test statistic is given as $z = 2.42$. The P-value is $P(Z > 2.42) = 1 - 0.9922 = 0.0078$. Since the value is below 0.05, we reject at the 5% level of significance.

Using Table D, we look up the critical value corresponding to the tail probability 0.05 and find the critical value $z^* = 1.645$. A 5% level test rejects if $z > 1.645$. Since 2.42 exceeds this value, we reject at the 5% level of significance.

b) The P-value was computed in (a) to be 0.0078. Since 1% = 0.01 exceeds this value, we also reject at the 1% level of significance. This is the advantage of giving the P-value. It allows us to assess significance at any level.

Using Table D, we look up the critical value corresponding to the tail probability 0.01 and find the critical value $z^* = 2.326$. Since $z = 2.42$ exceeds the critical value, we reject at the 1% level of significance.

Exercise 6.61

From Table D, $z^* = 1.036$ corresponds to a significance level of 2 x 0.15 = 0.30 since the test is two-sided. The tabled value $z^* = 1.282$ corresponds to a significance level of $2 \times 0.10 = 0.20$. We would reject at $\alpha = .30$ since $|z| = 1.12 > 1.036$, but not at $\alpha = 0.20$ since $|z| = 1.12 < 1.282$. So the P-value lies between 0.30 and 0.20.

Using Table A the area to the right of 1.12 is 0.1314 and the P-value is $2 \times 0.1314 = 0.2628$ since the test is two-sided. Clearly this lies between the values we found in Table D.

Exercise 6.65

a) The hypotheses are

$$H_0\colon \mu = 7 \qquad\qquad H_a\colon \mu \neq 7$$

In this case $\mu_0 = 7$ falls outside the level 95% confidence interval for μ which is (1.9, 6.5). So we reject H_0 at significance level 5%.

c) Since H_0: $\mu = 5$, $\mu_0 = 5$ which doesn't fall outside the 95% confidence interval for μ. So we fail to reject H_0 at significance level 5%.

SECTION 6.3

OVERVIEW

When describing the outcome of a hypothesis test it is more informative to give the *P*-value than just the reject or not decision at a particular significance level α. The traditional levels of 0.01, 0.05 and 0.10 are arbitrary and serve as rough guidelines.

When testing hypotheses with a very large sample, the *P*-value can be very small for effects that may not be of practical interest. Don't confuse small *P*-values with large or important effects. Plot the data to display the effect you are trying to show, and also give a confidence interval which says something about the size of the effect.

Just because a test is not statistically significant doesn't imply that the null hypothesis is true. This may occur when the test is based on a small sample size and has low power. Finally, if you run enough tests, you will invariably find statistical significance for one of them. Be careful in interpreting the results when testing many hypotheses on the same data.

GUIDED SOLUTIONS

Exercise 6.69

KEY CONCEPTS - significance levels vs. *P*-values

a) In this case, we reject the null hypothesis if the value of the *z* statistic exceeds 1.645 since the significance level is 0.05. Compute the test statistic when $\bar{x}$ = 541.4. What do you conclude at $\alpha = 0.05$?

b) Compute the test statistic when $\bar{x}$ = 541.5. What do you conclude at α = 0.05?

Exercise 6.73

KEY CONCEPTS - statistical significance vs. practical importance

What would you consider to be a "strong" (practically important) effect? Is the *P*-value sufficient to assess whether the effect is strong?

Exercise 6.79

KEY CONCEPTS - multiple tests, the *Bonferroni* procedure

If you perform k tests and want protection at level α, use α/k as your cutoff for statistical significance. In our case, $k = 6$ and $\alpha = 0.05$ so each test is conducted at the 0.05/6 = 0.00833 significance level. Which P-values given lead to rejection at the 0.00833 level?

COMPLETE SOLUTIONS

Exercise 6.69

In this problem we see that the P-value is more informative than just stating whether we accept or reject at $\alpha = 0.05$. In both (a) and (b) the evidence against the null hypothesis is almost the same, yet at the 5% level of significance we reject in (b) but not in (a). Don't attach too much importance to the 5% level of significance. Use the P-value and think about the problem in the context of the field and in terms of other data that may have been collected.

a) $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{541.4 - 525}{100 / \sqrt{100}} = 1.64$, so we don't reject at $\alpha = 0.05$. The P-value is 0.0505.

b) $z = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{541.5 - 525}{100 / \sqrt{100}} = 1.65$, so we reject at $\alpha = 0.05$. The P-value is 0.0495.

Exercise 6.73

Intuitively, we would consider an effect to be "strong" if there were a large difference in the percents of subjects in the two groups who were free of colds. For most people, "large" probably means a difference of more that just a few percent Unfortunately, we are not given the size of the difference, only the P-value. Hence, it is not possible to conclude (from the information given) that vitamin C has a strong effect in preventing colds. All we can conclude is that we are not likely to have observed a difference as large or larger than was actually observed (whatever its magnitude) by chance.

Exercise 6.79

If the *P*-value is less than α/k = 0.05/6 = 0.00833, then we reject using the *Bonferroni* procedure. In this case we reject the null hypothesis for two of the tests - the ones with *P*-values of 0.008 and 0.001 because these *P*-values are smaller than 0.00833.

SECTION 6.4

OVERVIEW

The **power** of a significance test is always calculated at a specific alternative hypothesis and is the probability that the test will reject H_0 when that alternative is true. This calculation requires knowledge of the sampling distribution under the specific alternative hypothesis of the test statistic used. Power is usually interpreted as the ability of a test to detect an alternative hypothesis or as the sensitivity of a test to an alternative hypothesis. The power of a test can be increased by increasing the sample size when the significance level remains fixed.

To compute the power of a significance test about a mean of a normal population, we need to:

- state H_0, H_a (the particular alternative we want to detect), and the significance level α,
- find the values of $\bar{x}$ that will lead us to reject H_0,
- calculate the probability of observing these values of $\bar{x}$ when the alternative is true.

Statistical inference can be regarded as giving rules for making decisions in the presence of uncertainty. From this **decision theory** point of view, H_0 and H_a are just two statements of equal status that we must decide between. Decision analysis chooses a rule for deciding between H_0 and H_a on the basis of the probabilities of the two types of errors that we can make. A **Type I error** occurs if H_0 is rejected when it is in fact true. A **Type II error** occurs if H_0 is accepted when in fact H_a is true.

There is a clear relation between α level significance tests and testing from the decision making point of view. The significance level α is the probability of a Type I error and the power of the test against a specific alternative is 1 minus the probability of a Type II error for that alternative.

SAMPLE PROBLEMS

GUIDED SOLUTIONS

Exercise 6.83

KEY CONCEPTS - power of a significance test

To compute the power of a significance test about a mean, we need to:

(i) state H_0, H_a (the particular alternative we want to detect), and the significance level α,

(ii) find the values of $\bar{x}$ that will lead us to reject H_0,

(iii) calculate the probability of observing these values of $\bar{x}$ when the alternative is true.

Following these steps we notice

(i) In this problem the hypotheses are

$$H_0\text{: } \mu = 450 \text{ vs. } H_a\text{: } \mu > 450$$

The particular alternative we want to detect is $\mu = 462$. The significance level is $\alpha = 0.01$.

(ii) The values of $\bar{x}$ that will lead us to reject H_0 are indicated in the problem and are those for which

$$z = \frac{\bar{x} - 450}{100 / \sqrt{500}} \geq 2.326$$

since $\sigma = 100$ and the sample size $n = 500$. Solve the above inequality for one of the form $\bar{x} \geq$ __________.

(iii) The probability of observing these values of $\bar{x}$ when the alternative is true is

$$P(\bar{x} \geq 460.4 \text{ when } \mu = 462) = P(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \geq \frac{460.4-462}{100/\sqrt{500}})$$

Now complete the calculation of this probability using Table A. The result will be the desired power.

Exercise 6.87

KEY CONCEPTS - Type I and Type II error probabilities

a) From the information given in the problem we are told that our population is normal, our sample size is $n = 16$, $\sigma = 1$, we are testing

$$H_0: \mu = 0 \text{ vs. } H_a: \mu > 0$$

and we reject H_0 if $\bar{x} > 0$. Thus

$$P(\text{Type I error}) = P(\text{our test rejects } H_0 \text{ when in fact } \mu = 0)$$

$$= P(\bar{x} > 0 \text{ when in fact } \mu = 0)$$

Now compute this probability.

b) In this part we want (since we accept H_0 if $\bar{x} \leq 0$)

$$P(\text{Type II error}) = P(\text{our test accepts } H_0 \text{ when in fact } \mu = 0.2)$$
$$= P(\bar{x} \leq 0 \text{ when in fact } \mu = 0.2)$$

Now compute this probability.

c) See if you can do this on your own. Repeat the argument in (b) with $\mu = 0.6$ in place of $\mu = 0.2$.

COMPLETE SOLUTIONS

Exercise 6.83

We follow the three steps indicated in the Guided Solutions.

(i) This step was completely discussed in the Guided Solutions.

(ii) In terms of $\bar{x}$, after solving the inequality given in the Guided Solutions, we reject H_0 if $\bar{x} \geq (2.326\text{x}100/\sqrt{500}) + 450 = 460.4$.

(iii) We find

$$P(\bar{x} \geq 460.4 \text{ when } \mu = 462) = P(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \geq \frac{460.4-462}{100/\sqrt{500}})$$

$$= P(Z \geq -0.36) = 0.6406$$

where we have used Table A to compute $P(Z \geq -0.36)$. The desired power is 0.6406.

Exercise 6.87

a) We have

$$P(\text{Type I error}) = P(\text{our test rejects } H_0 \text{ when in fact } \mu = 0)$$

$$= P(\bar{x} > 0 \text{ when in fact } \mu = 0)$$

$$= P(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} > \frac{0-0}{1/\sqrt{16}})$$

$$= P(Z > 0)$$

which we know is 0.5.

b) We have

$$P(\text{Type II error}) = P(\text{our test accepts } H_0 \text{ when in fact } \mu = 0.2)$$

$$= P(\bar{x} \leq 0 \text{ when in fact } \mu = 0.2)$$

$$= P(\frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \leq \frac{0-0.2}{1/\sqrt{16}})$$

$= P(Z \leq -0.8)$

$= 0.2119$

c) We have

$P($Type II error$)$ $= P($our test accepts H_0 when in fact $\mu = 0.6)$

$= P(\overline{x} \leq 0$ when in fact $\mu = 0.6)$

$$= P\left(\frac{\overline{x} - \mu}{\sigma / \sqrt{n}} \leq \frac{0 - 0.6}{1/\sqrt{16}}\right)$$

$= P(Z \leq -2.4)$

$= 0.0082$