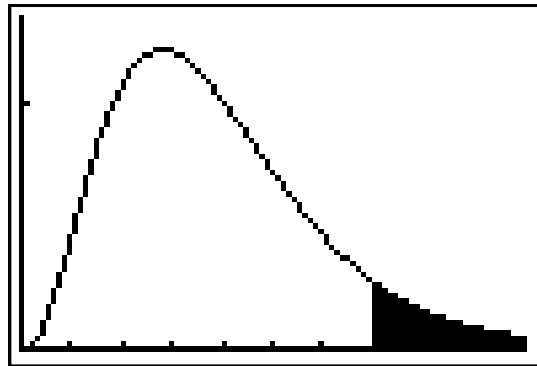


CHAPTER

22



Two Categorical Variables: the Chi-Square Test

22.1	Data Analysis for Two-Way Tables
22.2	Inference for Two-Way Tables
22.3	Goodness of Fit
22.4	Selected Exercise Solutions

Introduction

We saw in Chapter 6 that we can present data on two categorical variables in a two-way table. In that chapter, we discussed creating graphs for these variables and finding marginal and conditional frequencies. In this chapter, we describe how to perform a chi-square test on data from a two-way table. We shall be testing whether there is any association between the row and column variable traits, or whether these row and column traits are independent. We will also perform a test to decide whether data agree with a proposed distribution.

22.1 Data Analysis for Two-Way Tables

Example 22.1 Where do young people live? A sample survey asked a random sample of young adults, “Where do you live now? That is, where do you stay most often?” The table below presents the data from all 2984 people in the sample, classified by their age and where they live. Living arrangement is a categorical variable. Even though age is quantitative, the two-way table treats age as dividing these young people into four categories.

	AGE			
	19	20	21	22
Parents’ home	324	378	337	318
Another person’s home	37	47	40	38
Your own place	116	279	372	487
Group quarters	58	60	49	25
Other	5	2	3	9

- (a) For each age, compute the conditional distribution of living arrangements.
 (b) Draw a bar graph of the conditional distributions.

Solution:

(a) The conditional distributions are the cell counts for each living arrangement divided by the total number of individuals in that age group. Sum down the columns to find there are $324 + 37 + 116 + 58 + 5 = 540$ 19-year-olds in the sample, $378 + 47 + 279 + 60 + 2 = 766$ 20-year-olds, $337 + 40 + 372 + 49 + 3 = 801$ 21-year-olds, and $318 + 38 + 487 + 25 + 9 = 877$ 22-year-olds. Now divide each cell count to find the conditional distributions (read down the columns) as shown below.

	AGE			
	19	20	21	22
Parents’ home	$324/540 = 0.60$	$378/766 = 49.3\%$	$337/801 = 42.1\%$	$318/877 = 36.3\%$
Another person’s home	$37/540 = 6.9\%$	$47/766 = 6.1\%$	$40/801 = 5.0\%$	$38/877 = 4.3\%$
Your own place	$116/540 = 21.5\%$	$279/766 = 36.4\%$	$372/801 = 46.4\%$	$487/877 = 55.5\%$
Group quarters	$58/540 = 10.7\%$	$60/766 = 7.8\%$	$49/801 = 6.1\%$	$25/877 = 2.9\%$
Other	$5/540 = 0.9\%$	$2/766 = 0.3\%$	$3/801 = 0.4\%$	$9/877 = 1.0\%$

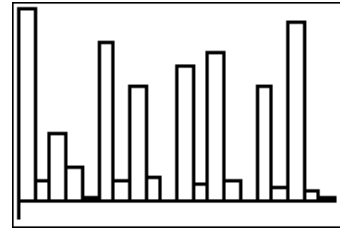
(b) To make a (rough) bar graph of the conditional, we first enter the values 0 through 20 into list L1 in order to represent the 4×5 category combinations, and then enter the conditional distributions calculated above into list L2. Next, we adjust the WINDOW (to ensure that the bars have width 1) and STAT PLOT settings for a histogram of L1 with frequencies L2, then **GRAPH**. Remember that proper bar graphs should not have bars connected if you try to copy this down!

L1	L2	L3	3
1	60	██████	
2	6.9		
3	21.5		
4	10.7		
5	9		
6	49.3		
7	6.1		
L3(1)=			

```

WINDOW
Xmin=1
Xmax=21
Xscl=1
Ymin=-5
Ymax=60
Yscl=1
Xres=1

```



We can see that the percent living at home declines with age and the percent living in their own place increases with age. The percent living in group quarters (dorms?) also declines with age.

22.2 Inference for Two-Way Tables

We continue here with an example that shows how to compute the χ^2 test of no association between the row variable and column variable in a two-way table, and obtain the matrix of expected cell counts.

TI Calculators have a built-in χ^2 -Test feature (on the STAT TESTS menu) that will compute the expected counts of a random sample under the assumption that the conditional distributions are the same for each category type. To use this feature, we first must enter data from a two-way table into a matrix in the MATRX EDIT screen.

Example 22.3 Where young people live: the test statistic (and expected counts).

Continuing with our example on where young adults live, under the assumption that there is no relationship between age and living arrangements, find the expected number of individuals living in each type of arrangement by age. Determine whether or not there is an association between age and living arrangements.

TI-83/84 Solution. First, enter the 5×4 table of data (do not enter row or column totals) into matrix [A] in the MATRX EDIT screen. To get to this screen, press $\boxed{2\text{nd}}\boxed{[x^{-1}]}$ for the MATRX menu, then arrow to EDIT. Press $\boxed{\text{ENTER}}$ to select matrix [A]. Enter the size of the body of the matrix (rows \times columns) pressing $\boxed{\text{ENTER}}$ to advance the cursor. Enter the observed counts across the rows, pressing $\boxed{\text{ENTER}}$ each time to advance the cursor. This menu is fussy – you must press $\boxed{2\text{nd}}\boxed{\text{MODE}} = \boxed{\text{QUIT}}$ to exit the editor.

Next, bring up the χ^2 -Test screen from the STAT TESTS menu, and adjust the Observed and Expected names if needed. The calculator defaults for this test are matrix [A] for Observed, and matrix [B] for Expected. If you have used another matrix, press $\boxed{2\text{nd}}\boxed{[x^{-1}]}$ for the MATRX NAMES screen and locate the matrix name you used; press $\boxed{\text{ENTER}}$ to select it. Next, press $\boxed{\text{ENTER}}$ on Calculate.

MATRIX[A] 5 ×4			
[324	378	337	-
[37	47	40	-
[116	279	372	-
[58	60	49	-
[2	3		-
5, 1=5			

EDIT CALC TESTS	
B	2-PropZInt...
8	X ² -Test...
D	X ² GOF-Test...
E	2-SampFTest...
F	LinRegTTest...
G	LinRegTInt...
H	ANOVA(

X ² -Test	
Observed:	[A]
Expected:	[B]
Calculate	Draw

X ² -Test	
X ² =	193.5482798
P=	6.981157E-35
df=	12

MATH EDIT	
1	[A] 5×4
2	[B] 5×4
3	[C]
4	[D]
5	[E]
6	[F]
7	[G]

MATRIX[B] 5 ×4			
[245.57	348.35	364.26	-
[29.316	41.586	43.486	-
[226.93	321.9	336.61	-
[34.745	49.287	51.539	-
[3.4383	4.8773	5.1002	-

TI-89 Solution. On TI-89 calculators, the Matrix Editor is an application. Press [APPS]. Either press [6] or arrow down and press the right arrow to expand the options for what to edit. If you want to reuse a matrix name with a matrix of the same size, select option 2:Open or select 3:New here. You next expand the Type: box to select 2:Matrix. In a similar way, select a folder (usually Main) and give the matrix a name (here aa) and its dimensions (ours is a 2x2). The matrix editor screen appears. Enter the observed counts across the rows, pressing [ENTER] each time to advance the cursor. This menu is fussy – you must press [2nd][ESC] = [QUIT] to exit the editor. Return to the Stats/List Editor application and press [2nd][F1] = [F6] for the Tests menu. The test we want is option 8:Chi2 2-way. Select it and use [2nd][=] = [VAR-LINK] to locate and enter your matrix name in the Observed Mat box. One can normally use the defaults for the Expected and CompMat matrices. As with the TI-83/84 series, there is a Calculate or Draw option for the output. Output is similar to the TI-83/84, but the first portion of the expected counts and components of the χ^2 statistic are shown. To observe these matrices fully, return to the Matrix Editor application and use the Open option to view them. Below, the contents of Expmat (the expected counts) and Compmat (contributions from each cell to the χ^2 statistic) are shown.

APPLICATIONS			
1:	FlashApps...	APPS	
2:	V= Editor		
3:	Window Editor		
4:	Graph		
5:	Table		
1:	Current Matrix Editor		
2:	Open... am Editor		
3:	New... Editor		
Enter=OK			
ESC=Cancel			
MAIN RAD APPRX FUNC 3/30			

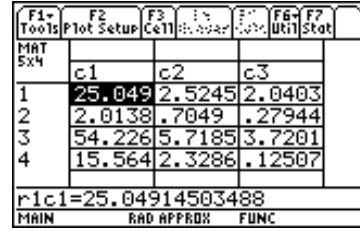
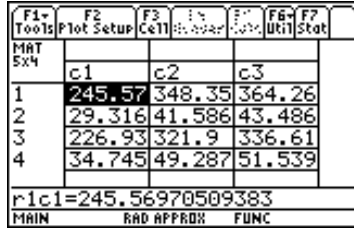
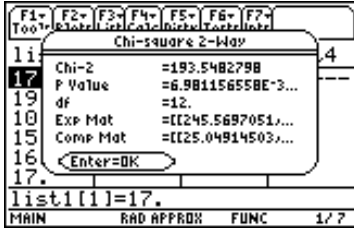
NEW			
Type:	1:Data		
Folder:	2:Matrix		
Variable:	3>List		
Row dimension:	2		
Col dimension:	2		
Enter=OK			
ESC=CANCEL			
MAIN RAD APPRX FUNC			

NEW			
Type:	Matrix		
Folder:	main		
Variable:	aa		
Row dimension:	2		
Col dimension:	2		
Enter=OK			
ESC=CANCEL			
MAIN RAD APPRX FUNC			

MATH EDIT			
1	[A] 5×4		
2	[B] 5×4		
3	[C]		
4	[D]		
5	[E]		
6	[F]		
7	[G]		
r5c4=9.			
MAIN RAD APPRX FUNC			

TESTS			
1	[A] 5×4		
2	[B] 5×4		
3	[C]		
4	[D]		
5	[E]		
6	[F]		
7	[G]		
8	Chi2 2-way...		
list1[20]=20.9			
MAIN RAD APPRX FUNC 1/6			

Chi-square 2-Way			
Observed Mat:	aa		
Store Expected to:	Expmat		
Store CompMat to:	Compmat		
Results:	Calculate		
Enter=OK			
Draw			
list1[20]=20.9			
MAIN RAD APPRX FUNC 1/6			

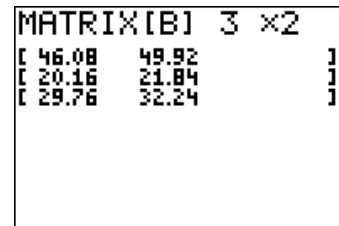
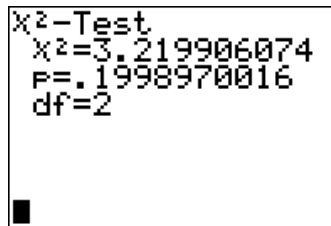
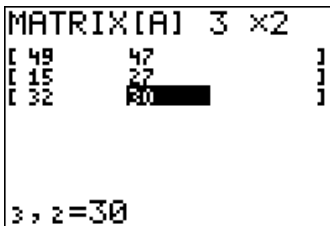


Conclusions. The χ^2 -Test feature on all these calculators displays the chi-square test statistic, the p -value, and the degrees of freedom for the chi-square test of no association between the row and column variables. In this case, the low p -value of 7×10^{-35} (essentially 0) gives extremely strong evidence to reject the claim that there is no relationship between age and living arrangements of young adults. The TI-89 CompMat (the last matrix shown above) shows us that the largest component to the overall χ^2 statistic is from 19-year-olds living in their own place – we actually had 116 of them in the sample and 226.93 were expected. Those are most unlike what is expected under an assumption of no relationship.

Example 22.6 Are cell-only telephone users different? Random digit dialing telephone surveys do not call cell phone numbers. If the opinions of people who have only cell phone differ from those of people who still have land lines, the poll results may not represent the entire adult population. The Pew Research Center interviewed separate random samples of cell-only and landline telephone users. We will compare the 96 cell-only users and the 104 landline users who were less than 30 years old. What do the data below say about type of phone and political leaning?

	Cell-only Sample	Landline Sample
Democrat or lean Democrat	49	47
Refuse to lean either way	15	27
Republican or lean Republican	32	30
Total	96	104

Solution: We test a null hypothesis that there is no relationship between phone type and political leaning against an alternative that there is a relationship. Enter the data into a matrix, then use the χ^2 test.



The test statistic is $\chi^2 = 3.22$ with p -value 0.1999. This large p -value indicates there is no significant difference in political leanings in this age group between those with only cell-

phones and those with landlines. Notice that the largest differences between the observed and expected counts are for those who don't lean either way.

22.3 Goodness of Fit

TI-84 and -89 calculators have a built-in function to perform the goodness of fit for a discrete distribution. The commands for a TI-83 are simple enough, but we provide a small program below that also will accomplish the task.

Before executing the **FITTEST** program that follows, enter the specified proportions into list L1 and enter the observed cell counts into list L2. The expected cell counts are computed and stored in list L3, and the individual contributions to the chi-square test statistic are stored in list L4. The program displays the test statistic and the p -value.

The FITTEST Program

Program:FITTEST	:1- χ^2 cdf(0,X,dim(L2)-1) [STO] P
:sum(L2) [STO] N	:ClrHome
:N*L1 [STO] L3	:Disp "CHI SQ STAT"
:(L2-L3) /L3 [STO] L4	:Disp X
:sum(L4) [STO] X	:Disp "P VALUE"
	:Disp P

Example 22.8 Never on Sunday? Births are not evenly distributed across the days of the week. Fewer babies are born on Saturday and Sunday than on other days, probably because doctors find weekend births inconvenient. A random sample of 140 births from local records shows this distribution across the days of the week:

Day	Sun.	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.
Births	13	23	24	20	27	27	18

Do these data give significant evidence that local births are not equally likely on all days of the week?

TI-83 Solution. If each day were equally likely, then $1/7$ of all births should occur on each day. To test the fit of this distribution, we shall use the **FITTEST** program. We first enter $1/7$ seven times into list L1 (the calculator displays a decimal equivalent) to specify the expected distribution, and enter the given frequencies from the table into list L2. Next, we run the **FITTEST** program to obtain a p -value of 0 from a chi-square test statistic of 0.2689.

L1	L2	L3	3
.14286	13		
.14286	23		
.14286	24		
.14286	20		
.14286	27		
.14286	18		
.14286	15		

L3(1)=

CHI SQ STAT	7.6
P VALUE	.2688966777
	Done

L1	L2	L3	3
.14286	13	20	
.14286	23	20	
.14286	24	20	
.14286	20	20	
.14286	27	20	
.14286	18	20	
.14286	15	20	

L3(1)=20

TI-84/89 Solution. Both of these calculators have built-in Chi-square goodness-of-fit tests and they function similarly. In both cases, we enter the observed and expected counts into two lists. The expected count under the null hypothesis (births are equally likely each day of the week) is $1/7 \cdot 140 = 20$. Our observed counts have been entered into L1 and the expected counts in L2. From the STAT TESTS menu select option χ^2 GOF-Test (Chi² GOF on a TI-89). Enter the list names for your observed and expected counts and the degrees of freedom for the test which are $k - 1$, where k is the number of categories. Since there are 7 days per week, for this example $k = 7$, so we have $df = 6$. The TI-84 output seen below shows the contributions to the χ^2 statistic. To see all of them, use the right arrow key. The TI-89 creates a new list of components which can be viewed in the List editor.

L1	L2	L3	3
13	20		
23	20		
24	20		
20	20		
27	20		
18	20		
15	20		
L3(1)=			

```

χ²GOF-Test
Observed:L1
Expected:L2
df:6
Calculate Draw
    
```

```

χ²GOF-Test
X²=7.6
P=.2688966777
df=6
CNTRB=(2.45 .4...
    
```

Conclusions. With a p -value of 0.2689, we cannot reject the null hypothesis of no relationship between day of week and the number of births. At least in this locality, there is no relationship.

22.4 Selected Exercise Solutions

22.13 Use program **FITTEST** (TI-83) or χ^2 GOF-Test (TI-84/89) to calculate the test. In any case, the expected counts are $53 \cdot 1/3 = 17.6667$, since if the tilts made no difference, there should be an equal number of strikes on each type of window. If you are using **FITTEST**, enter the expected proportions in L1 and the observed counts in L2. With the other two calculators, enter the observed and expected counts in two lists (you will be asked for them). Note that for the TI-84 or -89 procedures, you need to give the degrees of freedom which are categories-1. With a P -value of 0.0003, we conclude that tilting the windows does make a difference in birds striking them.

L1	L2	L3	3
.33333	31		
.33333	14		
.33333	8		
L3(1)=			

```

CHI SQ STAT
16.11320755
P VALUE
3.170015883E-4
Done
    
```

```

χ²GOF-Test
Observed:L2
Expected:L3
df:2
Calculate Draw
    
```

```

χ²GOF-Test
X²=16.11320755
P=3.1700159E-4
df=2
CNTRB=(10.0628...
    
```

22.43 This is a test of homogeneity (do all three sets of parents have the same types of opinions). We enter the data from the body of the table into matrix [A], then use χ^2 -Test. The differences in the distributions are statistically significant ($P = 0.0042$).

```

MATRIX[A] 5 x3
[ 12   34   22 ]
[ 68   55   81 ]
[ 75   61   60 ]
[ 24   24   24 ]
[ 22   28   10 ]
5, 3=14

```

```

χ2-Test
Observed: [A]
Expected: [B]
Calculate Draw

```

```

χ2-Test
χ2=22.42614287
P=.0041846941
df=8

```

To see the departures from the null hypothesis, examine matrix [B]. Blacks are less likely to call schools Excellent than expected (12 observed versus 22.7 expected) while Hispanics are more likely to call them Excellent (34 observed and 22.7 expected) and less likely to call them Good (55 versus 68). Blacks are more likely to call them Good (75 versus 65.4). There seems to be no real differences among the ethnicities on calling the schools Poor.

```

MATRIX[B] 5 x3
[ 22.704 22.704 22.592 ]
[ 68.446 68.446 68.107 ]
[ 65.441 65.441 65.117 ]
[ 24.04 24.04 23.921 ]
[ 21.369 21.369 21.263 ]

```

Continue your practice with these exercises:

- 22.1 Facebook at Penn State.
- 22.3 Attitudes towards recycled products.
- 22.5 Facebook at Penn State.
- 22.15 Police harassment?
- 22.17 What's your sign?
- 22.29 Free speech for racists?
- 22.45 Market research.
- 22.47 Party support in brief.